Modeling in the Spirit of Markowitz Portfolio Theory in a Non Gaussian World

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Summary

- History
- Diversification
- Markowitz mean variance
- Roy’s Safety First
- Sharpe Ratio
- Risk Return Tradeoff Ratio
- Example

Diversification

- Merchant of Venice, William Shakespeare had Antonio say:
- “My ventures are **not in one bottom trusted**, Nor to one place; nor is my whole estate Upon the fortune of this present year.”
- Markowitz (1999) noted, even Shakespeare knew about covariance at an intuitive level.
- Shakespeare understood time diversification

Diversification

- R.L. Stevenson in Treasure Island (1883) where Long John Silver commented on where he keeps his wealth, “I puts it all away, some here, some there, and none too much anywheres…”
- Mark Twain Pudd’nhead Wilson
- “Put all your eggs in the one basket and – watch that basket.”
- Twain practiced it, investing $8m in typesetting technology that was obsolete before it came to the market bankrupting him

Standard deviation as a measure of risk

- It has the advantage of being measured in the same units as the original variable.
- Surprisingly, the measure for evaluating financial assets using the standard deviation is relatively new
- This measure of risk for evaluating single assets in isolation was suggested by Irving Fisher (1906)

Markowitz

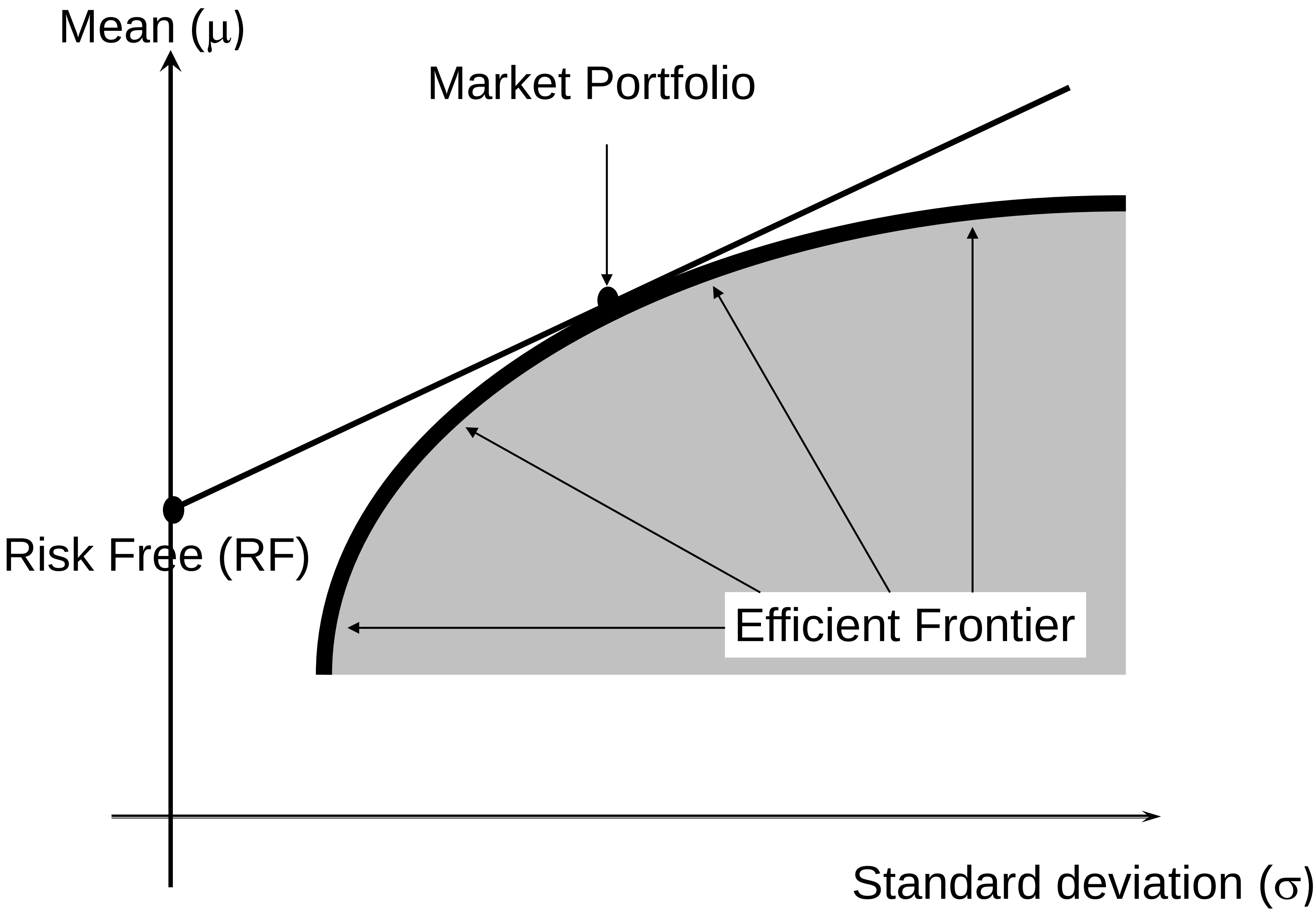
- “We next consider the rule that the investor does (or should) consider the expected return a desirable thing and the variance of return an undesirable thing. This rule has many sound points, both as a maxim for, and hypothesis about, investment behavior.”

Question

- Why not consider *other* measures of central tendency or *other* measures of variability?
- Usual answer: Mean and variance have nice properties
- If they are symmetric with finite moments, then, other common measures of central tendency like median and mode are the same

What if the distributions are skewed

- Once we permit skewed distribution, alternatives such as the median as a measure of central tendency and general quantile measures or specific measures such as the Value at Risk (VaR) or Tail VaR may be more reasonable measures of risk.
- These are essentially measures of risk beyond (or below) a certain threshold.


Mean (μ)
Market Portfolio
Risk Free (RF)
Efficient Frontier
Standard deviation (σ)

What do we need for Markowitz

- (1) The distribution of the returns should be multivariate normal and the utility function is exponential or (2) the utility function of the investor is quadratic.
- Return structure in most countries most of the time do not have multivariate normal structure
- Quadratic utility function is not appealing

Roy's Safety First Principle

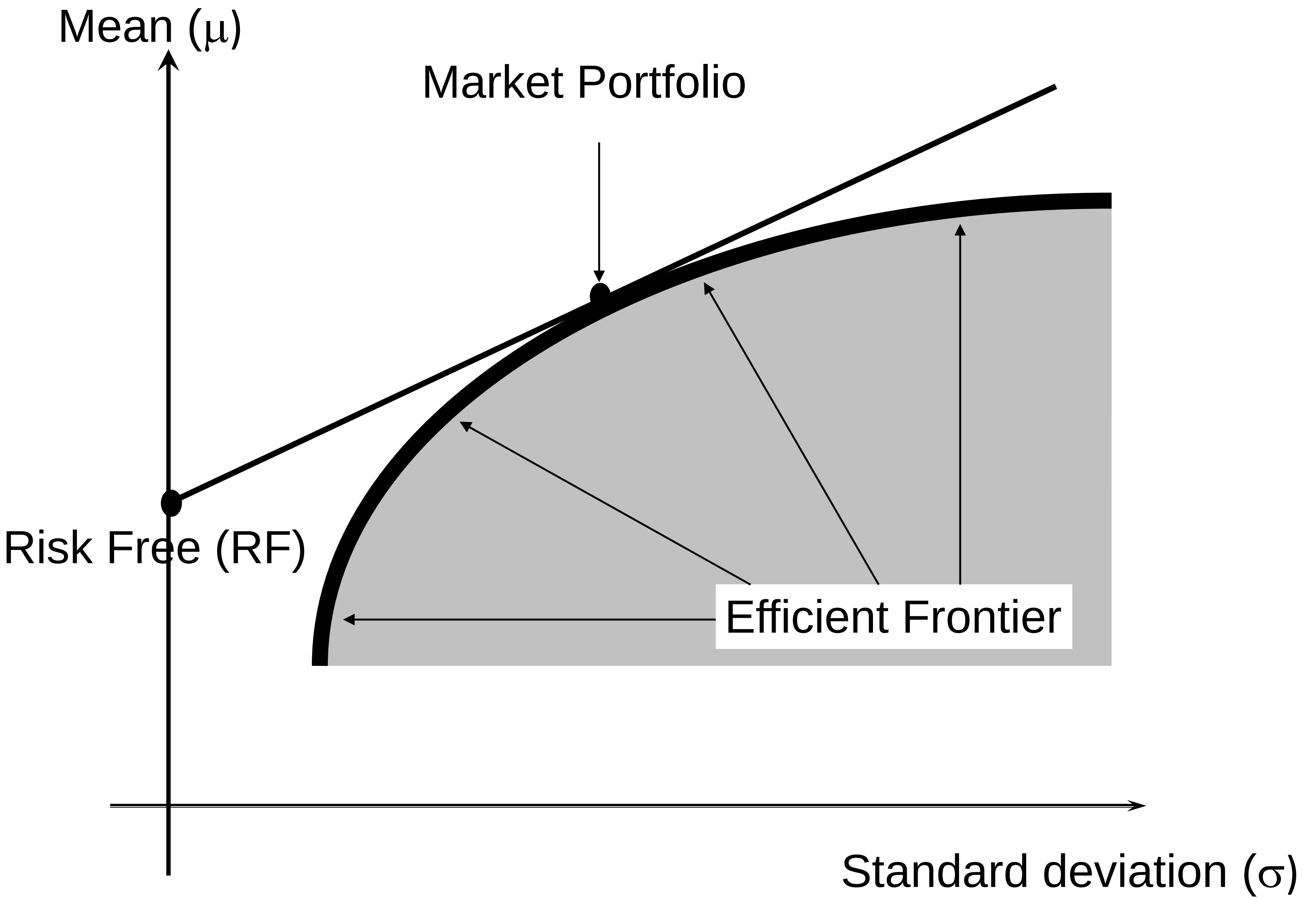
- Maximize $[\mu - d]/\sigma$ where d is some fixed minimum acceptable level of return.
- Roy called it the "Safety First Principle"
- Roy postulated that a sufficient condition is to minimize the probability of going below d.
- If the distribution of the portfolio returns is multivariate normal, then, the two conditions

Computational burden

- To apply the Markowitz model, one needs to estimate from the data the mean, variance and covariance of every pair of rates of return that one has.
- 1,000 stocks needs 1,000 means and 1,000 variances and 499,500 covariances.
- Rebalancing a portfolio requires half a million calculations each time

Existence of a risk free asset

- If there is a risk free asset
- Two fund theorem
- Just two assets are needed



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Sharpe Ratio

- How do we get to the efficient frontier?
- We calculate the Sharpe Ratio
- Maximize the following for a portfolio P and a riskless return r
- $s(P) = (\mu(P)-r)/\sigma(P)$
- This is very much like Roy's measure except instead of a threshold value d, we have r

Sharpe Ratio to RT Ratio

- Instead of the mean, we use the median
- Instead of the standard deviation we use the Value at Risk
- Return Risk Tradeoff Ratio
- RT ratio for short defined as:
- **RT(P) = (Median(P)-R)/ (VaR(P))**
- **If P* is the portfolio such that RT(P) < RT(P*) for all P, then we can call P* the optimal portfolio**

This is NOT merely changing minor things

- Example
- Let X~Laplace(0.3,1) (also called Double exponential)
- Let Y~Laplace(0.3,1.1)
- By Mean-Variance Criterion, Y is more risky
- But using Median (in this case the same as mean) and VaR criterion, X is more risky if we use 99% VaR
- VaR(X)=2.466 and VaR(Y)=2.357
- Mexican Blue Chip Companies have Laplace distribution for daily returns

Estimation strategy

- Have historical data
- Estimate the rank correlation among the stock returns and use a t-copula with the estimated rank correlation matrix as the copula.
- The marginal distributions can be fitted from among a family of distributions that include the well-known distributions (or the empirical marginal distribution).

Estimation strategy

- Once the marginal distributions and copula are chosen, we can simulate observations from the chosen joint distribution using Monte Carlo techniques
- 1% VaR as the risk measure, at least 10,000 observations from the joint distribution

Estimation strategy

- We do not have an algebraic way of dealing with the question of obtaining the market portfolio P*
- Consider all possible portfolios p1,p2,..,pn where each pj is a multiple of a fixed number d = 1/M where n is the number of stocks under consideration.
- For a given portfolio, one can determine the median as well as the VaR by first computing the returns for the portfolio (in the simulated data) and then sorting them.

Estimation strategy

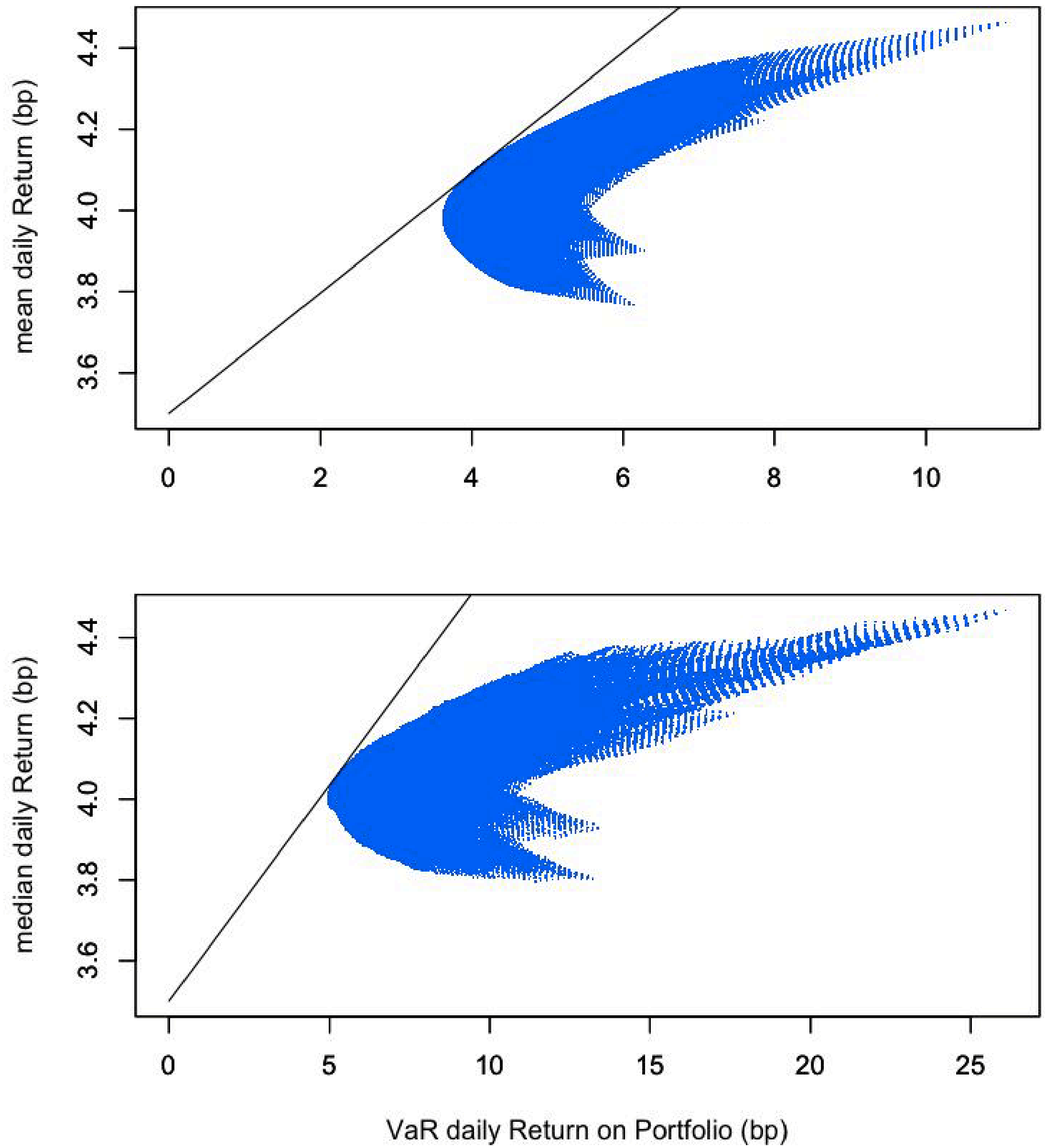
- This modest proposal can bring an exploding computational difficulty.
- For example, when n = 30 and we only take d=0.01, the total number of portfolios exceeds a trillion trillion and so an exhaustive search is ruled out. Indeed, the total number of portfolios with only integral percentage components exceeds 2^{95}.

Estimation strategy

- Even if one had a billion computers each working at 1000 Gigahertz and an efficient code that computes RT(P) in a single clock cycle, it will take over 1900 years to compute RT(P) for all possible portfolios (where each pi is a multiple of 0.01).
- So we propose an alternate method for doing such calculations.

Our method

- Random perturbation
- Method is described in the paper
- We apply it for simulated data
- The joint distribution is then determined by a t-copula with 1 degree of freedom
- Our method gives a 6 percent improvement



mean daily Return (bp)
3.6
3.8
4.0
4.2
4.4
0
2
4
6
8
10
median daily Return (bp)
3.6
3.8
4.0
4.2
4.4
0
5
10
15
20
25
VaR daily Return on Portfolio (bp)

Thank you!

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