Modeling in the Spirit of Markowitz Portfolio Theory in a Non Gaussian World

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Summary

- History
- Diversification
- Markowitz mean variance
- Roy’s Safety First
- Sharpe Ratio
- Risk Return Tradeoff Ratio
- Example
Diversification

• Merchant of Venice, William Shakespeare had Antonio say:
  “My ventures are not in one bottom trusted, Nor to one place; nor is my whole estate Upon the fortune of this present year.”

• Markowitz (1999) noted, even Shakespeare knew about covariance at an intuitive level.

• Shakespeare understood time diversification
Diversification

- R.L. Stevenson in Treasure Island (1883) where Long John Silver commented on where he keeps his wealth, “I puts it all away, some here, some there, and none too much anywheres…”
- Mark Twain Pudd’nhead Wilson
- “Put all your eggs in the one basket and – watch that basket.”
- Twain practiced it, investing $8m in typesetting technology that was obsolete before it came to the market bankrupting him
Standard deviation as a measure of risk

- It has the advantage of being measured in the same units as the original variable.
- Surprisingly, the measure for evaluating financial assets using the standard deviation is relatively new.
- This measure of risk for evaluating single assets in isolation was suggested by Irving Fisher (1906)
Markowitz

• “We next consider the rule that the investor does (or should) consider the expected return a desirable thing and the variance of return an undesirable thing. This rule has many sound points, both as a maxim for, and hypothesis about, investment behavior.”
Question

• Why not consider other measures of central tendency or other measures of variability?
• Usual answer: Mean and variance have nice properties
• If they are symmetric with finite moments, then, other common measures of central tendency like median and mode are the same
What if the distributions are skewed

- Once we permit skewed distribution, alternatives such as the median as a measure of central tendency and general quantile measures or specific measures such as the Value at Risk (VaR) or Tail VaR may be more reasonable measures of risk.

- These are essentially measures of risk beyond (or below) a certain threshold.
Risk Free (RF)

Market Portfolio

Mean ($\mu$)

Efficient Frontier

Standard deviation ($\sigma$)
What do we need for Markowitz

• (1) The distribution of the returns should be multivariate normal and the utility function is exponential or (2) the utility function of the investor is quadratic.
• Return structure in most countries most of the time do not have multivariate normal structure
• Quadratic utility function is not appealing
Roy’s Safety First Principle

• Maximize \(\frac{\mu - d}{\sigma}\) where \(d\) is some fixed minimum acceptable level of return.

• Roy called it the “Safety First Principle”

• Roy postulated that a sufficient condition is to minimize the probability of going below \(d\).

• If the distribution of the portfolio returns is multivariate normal, then, the two conditions
Computational burden

• To apply the Markowitz model, one needs to estimate from the data the mean, variance and covariance of every pair of rates of return that one has.

• 1,000 stocks needs 1,000 means and 1,000 variances and 499,500 covariances.

• Rebalancing a portfolio requires half a million calculations each time
Existence of a risk free asset

- If there is a risk free asset
- Two fund theorem
- Just two assets are needed
Risk Free (RF)

Market Portfolio

Efficient Frontier

Mean ($\mu$)

Standard deviation ($\sigma$)
Sharpe Ratio

• How do we get to the efficient frontier?
• We calculate the Sharpe Ratio
• Maximize the following for a portfolio $P$ and a riskless return $r$

\[
s(P) = \frac{\mu(P) - r}{\sigma(P)}
\]

• This is very much like Roy’s measure except instead of a threshold value $d$, we have $r$
Sharpe Ratio to RT Ratio

• Instead of the mean, we use the median
• Instead of the standard deviation we use the Value at Risk
• Return Risk Tradeoff Ratio
• RT ratio for short defined as:
  \[ \text{RT}(P) = \frac{\text{Median}(P) - R}{\text{VaR}(P)} \]
• If \( P^* \) is the portfolio such that \( \text{RT}(P) < \text{RT}(P^*) \) for all \( P \), then we can call \( P^* \) the optimal portfolio
This is NOT merely changing minor things

• Example
• Let $X \sim \text{Laplace}(0.3,1)$ (also called Double exponential)
• Let $Y \sim \text{Laplace}(0.3,1.1)$
• By Mean-Variance Criterion, $Y$ is more risky
• But using Median (in this case the same as mean) and VaR criterion, $X$ is more risky if we use 99% VaR
• VaR($X$)=2.466 and VaR($Y$)=2.357
• Mexican Blue Chip Companies have Laplace distribution for daily returns
Estimation strategy

• Have historical data
• Estimate the rank correlation among the stock returns and use a t-copula with the estimated rank correlation matrix as the copula.
• The marginal distributions can be fitted from among a family of distributions that include the well-known distributions (or the empirical marginal distribution).
Estimation strategy

• Once the marginal distributions and copula are chosen, we can simulate observations from the chosen joint distribution using Monte Carlo techniques

• 1% VaR as the risk measure, at least 10,000 observations from the joint distribution
Estimation strategy

• We do not have an algebraic way of dealing with the question of obtaining the market portfolio P*
• Consider all possible portfolios p1, p2, .., pn where each pj is a multiple of a fixed number $d = 1/M$ where n is the number of stocks under consideration.
• For a given portfolio, one can determine the median as well as the VaR by first computing the returns for the portfolio (in the simulated data) and then sorting them.
Estimation strategy

• This modest proposal can bring an exploding computational difficulty.

• For example, when $n = 30$ and we only take $d=0.01$, the total number of portfolios exceeds a trillion trillion and so an exhaustive search is ruled out. Indeed, the total number of portfolios with only integral percentage components exceeds $2^{95}$. 
Estimation strategy

• Even if one had a billion computers each working at 1000 Gigahertz and an efficient code that computes $RT(P)$ in a single clock cycle, it will take over 1900 years to compute $RT(P)$ for all possible portfolios (where each $p_i$ is a multiple of 0.01).

• So we propose an alternate method for doing such calculations.
Our method

• Random perturbation
• Method is described in the paper
• We apply it for simulated data
• The joint distribution is then determined by a t-copula with 1 degree of freedom
• Our method gives a 6 percent improvement
Thank you!

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