Surrender risk models: impact of correlation crises on economic capital

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Massive surrenders

• Two main categories:
  – Reputation risk
    • Bank runs (Northern Rock)
    • ERGO
  – Herd behaviour due to guru opinions, or development of new consultancy firms specialized in advice to VA customers.

Analogy with the UK mortgage market: customers open the newspaper, find recommendations and massively follow them..
How can independent risks become suddenly strongly correlated?

Endogenous uncertainty:
- Uncertainty is generated/modified by response of individual entities to events
- Feedback loop: outcomes → forecasts → decisions → outcomes → revised forecasts → revised decisions → … (Millennium Bridge)
- Statistical relationships are endogenous to the model, and may undergo structural shifts (Goodhart’s Law: Any observed statistical regularity will tend to collapse once pressure is placed upon it for control purposes)
- Relevant when individual entities are similar in terms of forecasts and likely reactions to events
- Relevant when outcomes are sensitive to concerted actions
How can independent risks become suddenly strongly correlated?

**Endogenous Risk**
- Risk from shocks generated and amplified within the system

**Exogenous Risk**
- Risk from shocks from outside the system

**Millennium Bridge**
- New design
- Tested with extensive simulations
- All angles covered
- No endogenous shocks

**What Endogeneity?**
- Pedestrians had some problems
- Bridge closed
How can independent risks become suddenly strongly correlated?

Bridge moves → Adjust stance

↑ Further adjust stance ↓

← Push bridge
Runs / massive surrenders

Fitch downgrades

Life insurance industry

More downgrades
From S-shaped deterministic models to stochastic approach

Mean surrender rate = f(int. rate spread)
Conditional surrender rate pdf for different spread levels

(a) Normal distribution vs. Bernoulli trials
(b) Distribution for spread levels
(c) High surrender rate distribution
(d) Low surrender rate distribution
A guru-type model

The decision to surrender by policyholder $k$ is modeled thanks to a Bernoulli law with parameter $p$ (historical/empirical prob.)

$$I_k = \begin{cases} 1 & \text{avec proba } p \ (\text{rachat}) \\ 0 & \text{avec proba } (1-p) \end{cases}$$

To have an independent behavior from others (or the market) is also modeled by a Bernoulli law with parameter $p_0$:

$$J_k = \begin{cases} 1 & \text{avec proba } p_0 \\ 0 & \text{avec proba } (1 - p_0) \ (\text{independance}) \end{cases}$$

Finally the decision of the $k$th policyholder to surrender is:

$$I_k = J_k I_0 + (1 - J_k) I_k^\perp$$
Model assumptions

Independence between $I_k$ and $J_k$

Independence between $I_0, I_1^\perp, I_2^\perp, ..., I_n^\perp$

$p$ increases as delta (difference between two rates) increases

$p_0$ increases as delta increases...

Question: At what specific delta do we observe the change of the distribution and density of the surrender rate?
Message 7: Correlation crises and endogenous risk may occur

- Bi-modal world instead of Gaussian world
- Values close to conditional best estimate are then unlikely to be observed (even if model was perfect)
- Huge potential impact on 99.5% - VaR:
  - Soft economic context (lapse prob. $p=8.08\%$)
  - Proba. $p_0=5\%$ for each individual to follow guru
  - Impact: +50% on 99.5%-VaR of lapse rate!
We could define the *Value-at-Risk on surrender rate*. It would represent the highest surrender rate that the insurer could accept with a certain confidence threshold.

In the literature, *VaR* is defined by

\[ \text{VaR}_\alpha(X) = \inf \{ x \in X, \ F_X(x) \geq \alpha \} \]

where \( X \) is the random variable of surrender rate, and usually equals 99.5% in Life Insurance.

Enables us to have an estimated quantitative difference in terms of surrender risk with the change of distribution from Normal to Bi-modal.
<table>
<thead>
<tr>
<th>Economic context</th>
<th>Correlation value</th>
<th>$VaR_{90%}$</th>
<th>$VaR_{95%}$</th>
<th>$VaR_{99.5%}$</th>
<th>$\Delta VaR_{99.5%}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>soft ($p = 8.08%$)</td>
<td>$p_0 = 0$</td>
<td>8.704763</td>
<td>8.784052</td>
<td>9.055899</td>
<td>0 %</td>
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<tr>
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<td>$p_0 = 0.01$</td>
<td>9.441015</td>
<td>9.571275</td>
<td>9.735516</td>
<td>+7.5 %</td>
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<td>$p_0 = 0.02$</td>
<td>10.33584</td>
<td>10.48309</td>
<td>10.72662</td>
<td>+18.5 %</td>
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<tr>
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<td>$p_0 = 0.05$</td>
<td>13.15059</td>
<td>13.31483</td>
<td>13.54137</td>
<td>+49.5 %</td>
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<tr>
<td></td>
<td>$p_0 = 0.15$</td>
<td>22.46135</td>
<td>22.63125</td>
<td>22.87478</td>
<td>+152.6 %</td>
</tr>
<tr>
<td></td>
<td>$p_0 = 0.3$</td>
<td>36.34819</td>
<td>36.52376</td>
<td>36.80693</td>
<td>+306.4 %</td>
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<td>$p_0 = 0.5$</td>
<td>54.81679</td>
<td>55.00368</td>
<td>55.28119</td>
<td>+510.4 %</td>
</tr>
<tr>
<td>Medium ($p = 20%$)</td>
<td>$p_0 = 0$</td>
<td>20.89823</td>
<td>21.01150</td>
<td>21.27768</td>
<td>0 %</td>
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<tr>
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<td>$p_0 = 0.01$</td>
<td>21.50988</td>
<td>21.66846</td>
<td>21.93464</td>
<td>+3.1 %</td>
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<tr>
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<td>$p_0 = 0.02$</td>
<td>22.22914</td>
<td>22.44436</td>
<td>22.72187</td>
<td>+6.8 %</td>
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<tr>
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<td>$p_0 = 0.05$</td>
<td>24.64745</td>
<td>24.86266</td>
<td>25.13451</td>
<td>+18.1 %</td>
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<tr>
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<td>$p_0 = 0.15$</td>
<td>32.70658</td>
<td>32.92745</td>
<td>33.20496</td>
<td>+56.1 %</td>
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<td>$p_0 = 0.3$</td>
<td>44.78111</td>
<td>45.00198</td>
<td>45.27383</td>
<td>+112.8 %</td>
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<td>$p_0 = 0.5$</td>
<td>60.74645</td>
<td>60.95599</td>
<td>61.24483</td>
<td>+187.8 %</td>
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<tr>
<td>Hard ($p = 42%$)</td>
<td>$p_0 = 0$</td>
<td>43.11604</td>
<td>43.25763</td>
<td>43.46152</td>
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<tr>
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<td>$p_0 = 0.01$</td>
<td>43.49550</td>
<td>43.67673</td>
<td>43.93725</td>
<td>+1.1 %</td>
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<td>$p_0 = 0.02$</td>
<td>43.98822</td>
<td>44.22042</td>
<td>44.52059</td>
<td>+2.4 %</td>
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<td>$p_0 = 0.05$</td>
<td>45.69859</td>
<td>45.95911</td>
<td>46.32724</td>
<td>+6.6 %</td>
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<td>$p_0 = 0.15$</td>
<td>51.51498</td>
<td>51.77550</td>
<td>52.16062</td>
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<tr>
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<td>$p_0 = 0.3$</td>
<td>60.16877</td>
<td>60.41796</td>
<td>60.75777</td>
<td>+39.8 %</td>
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<tr>
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<td>$p_0 = 0.5$</td>
<td>71.74492</td>
<td>71.97712</td>
<td>72.28295</td>
<td>+66.3 %</td>
</tr>
</tbody>
</table>
Assumptions:

- Initial account value: 1M$
- Term: 1 year
- Yearly benefit: 10%
- Account Value dynamics:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>BE</th>
<th>EC / AV($VaR_{99.5%}^{Normal}$)</th>
<th>Correlation</th>
<th>EC ($VaR_{99.5%}^{Bi-modal}$)</th>
<th>ΔEC</th>
<th>ΔAV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft</td>
<td>8.06%</td>
<td>0.84 / 1 091 029 (p0 = 0)</td>
<td>p0 = 0.05</td>
<td>5.45</td>
<td>4.61%</td>
<td>5506.7</td>
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<tr>
<td></td>
<td>(p = 8.08%)</td>
<td>(p0 = 0)</td>
<td>p0 = 0.15</td>
<td>14.85</td>
<td>14.01%</td>
<td>14857.1</td>
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<tr>
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<td>(p = 20%)</td>
<td>(p0 = 0)</td>
<td>p0 = 0.3</td>
<td>28.7</td>
<td>27.86%</td>
<td>28743.9</td>
</tr>
<tr>
<td>Medium</td>
<td>20.03%</td>
<td>1.18 / 1 078 824 (p0 = 0)</td>
<td>p0 = 0.05</td>
<td>5.06</td>
<td>3.88%</td>
<td>5179.8</td>
</tr>
<tr>
<td></td>
<td>(p = 20%)</td>
<td>(p0 = 0)</td>
<td>p0 = 0.15</td>
<td>13.15</td>
<td>11.97%</td>
<td>13182.3</td>
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<tr>
<td></td>
<td>(p = 42%)</td>
<td>(p0 = 0)</td>
<td>p0 = 0.3</td>
<td>25.35</td>
<td>24.17%</td>
<td>25290.8</td>
</tr>
<tr>
<td>Hard</td>
<td>42.02%</td>
<td>1.48 / 1 056 459 (p0 = 0)</td>
<td>p0 = 0.05</td>
<td>4.28</td>
<td>2.8%</td>
<td>4253.6</td>
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<tr>
<td></td>
<td>(p = 42%)</td>
<td>(p0 = 0)</td>
<td>p0 = 0.15</td>
<td>10.13</td>
<td>8.65%</td>
<td>10036</td>
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<tr>
<td></td>
<td>(p = 42%)</td>
<td>(p0 = 0)</td>
<td>p0 = 0.3</td>
<td>18.76</td>
<td>17.28%</td>
<td>18769.1</td>
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<tr>
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<td>(p = 42%)</td>
<td>(p0 = 0)</td>
<td>p0 = 0.5</td>
<td>30.21</td>
<td>28.73%</td>
<td>30260.3</td>
</tr>
</tbody>
</table>

Table 2
Estimations of differences of economic capital (in terms of surrender rate) and account value with different correlations in various scenarios.

\[ AV_{adj}(t) = AV_{adj}(t - 1) \times (1 + \text{yearly beneff} \times (1 - \text{surrender rate})) \]