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AFIR/ERM - Financial Risk Section

Interest Rate Derivatives under the Standard Market Model



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Introduction

- ... Three types of interest rate financial derivatives traded in OTC markets: bond options, caps - floors i.r. , and swap options
- ... Financial characteristics
- ... Pricing from suitable martingale under the Standard Market Model
- ... Cap ir. is valued using DerivaGem Software
- ... Provide reliable information for either the hedger or the issuer

Introduction

Bond option

Long ... gives the right to buy the bond at a given date and at a given price.

Cap-Floor i.r.

A cap (floor) contract provides hedging in the case that the floating rate, in a floating rate note, is greater (less) than a particular level of interest rate.

Swap option

Holder ... right to enter into a swap on a given date, to exchange a fixed rate for a floating rate, both rates applied over a certain capital.

Introduction

... a stochastic process: The Martingale

a stochastic process, X , is a martingale if it has a zero drift, i.e., $dX = \sigma dw$, for a constant σ and w given by a Wiener process or a Brownian motion.

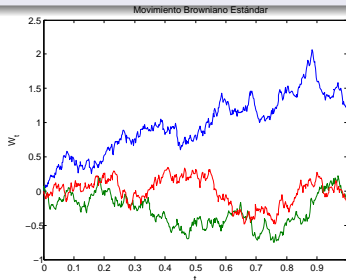


Figure : Three realizations of Brownian Motion

I.R. Derivatives - Std Market

Risk premium λ

- Underlying asset θ : $d\theta = m\theta dt + s\theta dw$.
- Derivative f : $df = \mu f dt + \sigma f dw$.
- λ : any derivative f satisfies the relation $(\mu - r) = \lambda\sigma$. The constant λ is called the market price of risk of θ , and this is used to define the derivative expected excess return regarding the risk free rate, per unit of risk σ .

I.R. Derivatives - Std Market

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Traditional risk neutral measure

The one where λ is zero, and therefore, $\mu = r$.

I.R. Derivatives - Std Market

Equivalent Martingale Measure (EMM)

- Numeraire g : ... a unit of measure, which is given by the price of a financial asset g (choice of $g \sim$ change of measure \sim define a *world*)
- EMM states that, if λ is the volatility of the numeraire g , then the ratio f/g is a martingale for all f , i.e.,

$$f_0 = g_0 E_g \left[\frac{f_T}{g_T} \right] \quad (1)$$

(... Itô's lemma: $\ln f, \ln g, y = \ln(f/g), \exp(y)$)

- ... such *world* is called forward risk neutral (f.r.n.) measure w.r.t. the numeraire g (... travel through different worlds!).

I.R. Derivatives - Std Market

- ... g is taken as a monetary account: $dg = rgdt$, $g_0 = 1$.
Martingales is f/g , i.e., $f_0 = \hat{E}[\exp(-\bar{r}T) f_T]$.
... stock exchange: futures, options on either stocks, indices, currencies or futures.
- ... g as a zero coupon bond $P(t, T)$... $f/P(t, T)$ is a martingale in a f.r.n. world wrt $P(t, T)$, i.e.,
 $f_0 = P(0, T)E_T[f_T]$.
... otc market: forwards, bond option.

I.R. Derivatives - Std Market

- ... g as a zero coupon bond $P(t, T^*)$. Let $P(t, T^*)/P(t, T)$ the forward price of a zero bond in period $[T, T^*]$... The forward rate $R(t, T, T^*) = f/P(t, T^*)$ is a martingale in a f.r.n. world wrt $P(t, T^*)$, i.e., $R(0, T, T^*) = E_{T^*}[f_T/g_T]$, where $f_t = [P(t, T) - P(t, T^*)]/(T^* - T)$... (... f is artificial!) ... otc market: caps, floors.
- ... g as $A(t) = \sum_{i=0}^N (T_{i+1} - T_i) P(t, T_{i+1})$. Let $s(t) = [P(t, T_0) - P(t, T_N)]/A(t)$ the forward swap rate... So $s(t)$ and $f/A(t)$ are both martingales in a f.r.n. world wrt numeraire $A(t)$, and $f_0 = A(0)E_A[f_T/A(T)]$
... otc market: swap options

Cap rate & DerivaGem

Details for pricing a cap i.r.

- Structuring the contract: R_K (cap rate), Tenor, R_k (*LIBOR*), T (maturity).
- Payoff at t_{k+1} is called *caplet* and it is given by $c_{k+1,k+1} = L\tau (R_k - R_K)^+$, $k = 1, \dots, n$, $\tau = t_{k+1} - t_k$, L is the principal on which interest is earned, and $R_k = R(t_k, t_k, t_{k+1})$.
- The cap rate contract is a portfolio of n caplets (european call options).
- $c_{0,k+1} = L\tau P(0, t_{k+1}) [F_k N(d1) - R_K N(d2)]$;
where $F_k = R(t_0, t_k, t_{k+1})$ (*LIBOR/SWAP*), ...

Cap rate & DerivaGem

Cap in DerivaGem

- DerivaGem: derivatives free calculator from Prof. J.Hull's web site
- Numerical illustration: $L = 100$, $t_1 = 0$, $T = 5$ years, $R_K = 5\%$, $\sigma_k = 20\%$, and quarterly tenor. Zero curve assumed $(t, r(t))$: $(0, 3.0\%)$, $(1, 3.6\%)$, $(2, 4.0\%)$, $(3, 4.6\%)$, $(4, 4.9\%)$, $(5, 5.20\%)$.
- Outputs: cap price, price sensitivity respect the variation to the cap rate R_K , to time, to the zero curve when it is disturbed by a slight parallel movement, and to the volatility σ_k .

Cap rate & DerivaGem

Cap in DerivaGem

Cap price	3.7849
DV01 (per b.p.)	0.0214
Gamma01 (per %)	0.006819
Vega (per %)	0.1049

- Cap: could be a hedging instrument for the hedger ...
- Cap: has a high delta and vega risk for the issuer ...

Cap rate & DerivaGem

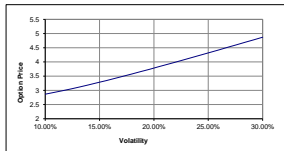
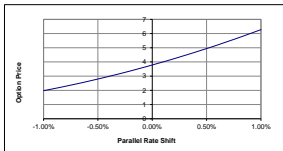
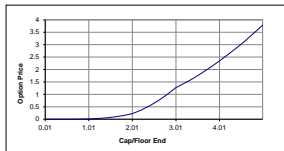
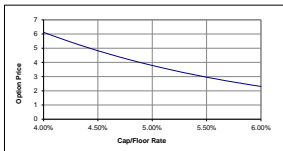


Figure : Cap price sensitivities

Agenda & Conclusion

Cap in DerivaGem ... Other Possibilities?

- Cap Price vs Instantaneous Variation of one point in the zero curve ... What point in the zero curve is more sensible for the cap? ...
- Get some simulations of the zero curve in a future time τ by using a short rate model ... would allow us to get a possible density function of the cap price at time τ ... Vasicek or CIR calculator in John Hull's web site could be worked together DerivaGem software in order to obtain such simulations ...

Cap in DerivaGem ... Other Possibilities?

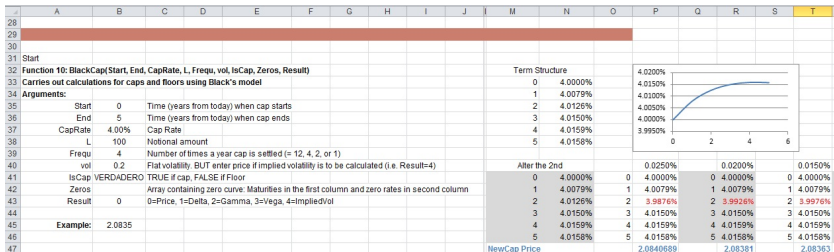


Figure : A new example ... Study of Cap price sensibility to the zero curve at two years

Cap in DerivaGem ... Other Possibilities?

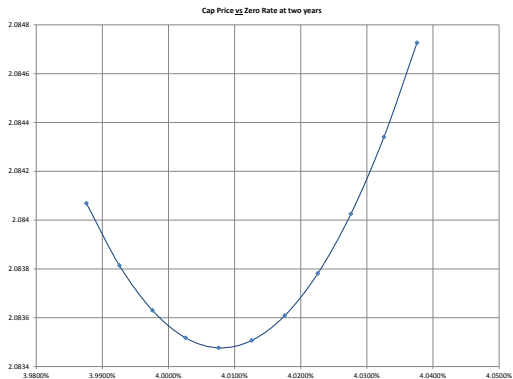


Figure : Cap price sensibility to the zero curve at two years

Cap in DerivaGem ... Other Possibilities?

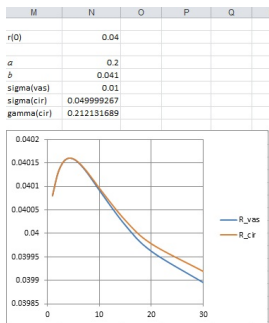



Figure : ... & other example ... use of Vasicek short rate model ($dr = a(b - r)dt + \sigma dw$) to generate a zero curve at time zero

Cap in DerivaGem ... Other Possibilities?

50	Function 10: BlackCap(Start, End, CapRate, L, Frequ, vol, IsCap, Zeros, Result)			
51	Carries out calculations for caps and floors using Black's model			
52	Arguments:			
53	Start	0	Time (years from today) when cap starts	
54	End	4.98076923	Time (years from today) when cap ends	
55	CapRate	4.00%	Cap Rate	
56	L	100	Notional amount	
57	Frequ	4	Number of times a year cap is settled (= 12, 4, 2, or 1)	
58	vol	0.2	Fiat volatility. BUT enter price if implied volatility is to be calculated (i.e. Result=4)	
59	IsCap	VERDADERO	TRUE if cap, FALSE if Floor	
60	Zeros		Array containing zero curve: Maturities in the first column and zero rates in second column	
61	Result	0	0=Price, 1=Delta, 2=Gamma, 3=Vega, 4=ImpliedVol	
62				
63	Example:	1.8987		
64				Simulate!!
65				
66	Cap_Prices_Simulated_1Week_Vasicek			
67	1	1.9500		
68	2	2.16922935	Term Structure 1 week	
69	3	1.99506548	0	3.8776%
70	4	2.04223602	1	3.8966%
71	5	1.87364016	2	3.9113%
72	6	2.27511118	3	3.9226%
73	7	1.93680591	4	3.9313%
74	8	2.19067963	5	3.9381%
75	9	2.2560455		

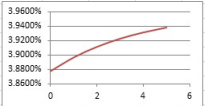


Figure : Cap prices simulated by using the previous Vasicek model

...

Agenda & Conclusion

... a logic argument ... value three i.r. derivatives ... bond options, caps - floors rates, and swap options, under suitable forward risk neutral measures and starting from the equivalent martingale measure principle ...

... In particular, it has shown an argument that begins from a forward risk neutral measure, with respect to a zero coupon bond, and provides an analytical and straightforward formula for cap rate pricing ...

... The DerivaGem software was employed to figure out the price of a given cap interest rate derivative, and to explore some measures of price sensitivity The results suggest high delta and vega risk, all of which can provide valuable information for either the hedger or the derivative issuer ...



Beyond Boundaries

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THANKS ...