Ruin Probability as a Prudential Criteria for Catastrophic Risks

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I. Introduction

What is AMIS?

The Mexican Insurance Association that voluntarily groups 80 of the 101 companies that operate in the Mexican insurance market, and represent 99.3% of the written premiums.

Objective: To promote the development of the insurance industry, to represent their interests upon the authorities, and to give technical support to their associate members.

Founding member of the Enterprise Coordinator Council (CCE):

Active associate of FIDES
Holds the General Secretary…

In 2010 we celebrated 70 years of uninterrupted activities.
I. Introduction

According to the Solvency II glossary from the European Insurance Committee (CEA)\(^1\), a CATASTROPHIC RISK is the risk that un just one, or in a series of events of real magnitude, generally occurred within a short period of time (72 hours), can cause a significant deviation of the claims occurred with respect to the expected claims.

\(^1\) CEA, Groupe Consultatif Actuariel Européen, (2007), Solvency II Glossary, Bruselas
I. Introduction

MÉXICO: A country with Great Natural Risks

Source: Maps from National Geographic
II. Summary of the Mexican Catastrophic Insurance Regulation.
II. Summary of the Mexican Catastrophic Insurance Regulation

Model, run monthly (similar to EQECAT, AIR, RMS)

Database with information of each of the buildings insured:

a) Financial information
   - Sum insured,
   - Deductibles, coinsurance
   - Policy limits and definitions
   - Reinsurance structure (proportional, layers)

b) Localization
   - State/municipality, zip code, geo coordinates

c) Building characteristics
   - Number of stories, irregularities in height / floor
   - Year of construction, previous damages, reinforcement
   - Type of structure (building materials, existence of short columns, in corner, % of glass in building face, etc.)
II. Summary of the Mexican Catastrophic Insurance Regulation

Generates:

a) retained pure premium per policy
b) Aggregated portfolio PML

PML and Pure Premium Model

<table>
<thead>
<tr>
<th>Utility</th>
<th>Commisions</th>
<th>XL Cost</th>
<th>Administrative Costs</th>
<th>Pure Premium</th>
</tr>
</thead>
</table>

“Sufficient” Unearned Premium Reserve

Issued Premiums (retention)
II. Summary of the Mexican Catastrophic Insurance Regulation

\[ INC_{RCAT} = PR \times FD_m \]

\[ FD_m = \frac{D_m}{D_v} \]

PML and Pure Premium Model

Retained*/
Pure Premium

*/ Just from proportional contracts

Monthly Earned premium

Catastrophic Reserve
II. Summary of the Mexican Catastrophic Insurance Regulation

Solvency Margin Rules

a) Solvency Requirement (R12)

\[ R12 = RT_1 + RT_2 \]

\[ RT_1 = PML_T \]

\[ RT_2 = PML_T \times (Iroenr - 1) \]

b) Eligible Assets (EA)

\[ RRCAT + CXLAr \leq RT_1 \]

c) Solvency Margin (SM)
II. Summary of the Mexican Catastrophic Insurance Regulation

Decision:

- Option a): Cesions to Not Registered Reinsurers
- Option b): Catastrophic Reserve
- Option c): Capital Catastrophic Reserve
- Cat XL Protection
- Retained PML

(1) Requirement Option a) Option b) Option c)
II. Summary of the Mexican Catastrophic Insurance Regulation

Technical Limit:

\[ LT_{RCAT} = 0.9 \left( \overline{PML_t} \right) \]

where:

\[ \overline{PML_t} = \overline{F}_{PML} \times \overline{SA} \times \overline{FR} \]

where:

\[ \overline{F}_{PML} = \frac{1}{5} \sum_{t=1}^{5} \frac{PML_t}{SA_t} \]

\[ \overline{SA} = \frac{\sum_{t=1}^{5} \prod_{j=t}^{5} (1 + \Delta INPC_j) \times (SA_t)}{5} \]

\[ \overline{FR} = \frac{1}{5} \sum_{t=1}^{5} \frac{SAR_t}{SAT_t} \]
III. Mexican industry concerns.
### III. Mexican Industry Concerns

#### Hydrometeorological Perils

**Profit & Loss Statement**

<table>
<thead>
<tr>
<th>Millions of Current Pesos</th>
<th>Hydrometeorological Perils</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2006</td>
</tr>
<tr>
<td><strong>Premiums Issued</strong></td>
<td>4,184.33</td>
</tr>
<tr>
<td>Ceded Premiums</td>
<td>2,982.76</td>
</tr>
<tr>
<td>Retention Premiums</td>
<td>1,201.57</td>
</tr>
<tr>
<td>Unearned Premium Reserve</td>
<td>-13.66</td>
</tr>
<tr>
<td>Ratained Earned Premiums</td>
<td>1,215.24</td>
</tr>
<tr>
<td>Excess Loss Coverage Cost</td>
<td>303.67</td>
</tr>
<tr>
<td><strong>Acquisition Costs</strong></td>
<td></td>
</tr>
<tr>
<td>Direct</td>
<td>334.95</td>
</tr>
<tr>
<td>Taken from other institutions</td>
<td>13.48</td>
</tr>
<tr>
<td>Commissions Recovered</td>
<td>334.30</td>
</tr>
<tr>
<td>Retained</td>
<td>14.12</td>
</tr>
<tr>
<td><strong>Claims Cost</strong></td>
<td></td>
</tr>
<tr>
<td>Direct</td>
<td>2,286.05</td>
</tr>
<tr>
<td>Taken from other institutions</td>
<td>703.34</td>
</tr>
<tr>
<td>Ceded</td>
<td>2,294.89</td>
</tr>
<tr>
<td>Retained</td>
<td>694.51</td>
</tr>
<tr>
<td><strong>Technical Result</strong></td>
<td>202.94</td>
</tr>
<tr>
<td>Operation Costs</td>
<td>459.15</td>
</tr>
<tr>
<td>Operation Result</td>
<td>-256.22</td>
</tr>
<tr>
<td>Other Expenses and financial products</td>
<td>288.95</td>
</tr>
<tr>
<td>Catastrophic Reserve</td>
<td>492.10</td>
</tr>
<tr>
<td><strong>Net Result before taxes</strong></td>
<td>-459.16</td>
</tr>
<tr>
<td>Taxes</td>
<td>6.96</td>
</tr>
<tr>
<td><strong>Net Result</strong></td>
<td>-436.85</td>
</tr>
</tbody>
</table>
### III. Mexican Industry Concerns

#### Earthquake

#### Profit & Loss Statement

<table>
<thead>
<tr>
<th></th>
<th>Millions of Current Pesos</th>
<th>Earthquake</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2006</td>
<td>2007</td>
</tr>
<tr>
<td><strong>Premiums Issued</strong></td>
<td>3,118.59</td>
<td>3,564.01</td>
</tr>
<tr>
<td><strong>Ceded Premiums</strong></td>
<td>2,242.21</td>
<td>2,540.90</td>
</tr>
<tr>
<td><strong>Retention Premiums</strong></td>
<td>876.38</td>
<td>1,023.11</td>
</tr>
<tr>
<td><strong>Unearned Premium Reserve</strong></td>
<td>-42.39</td>
<td>-97.72</td>
</tr>
<tr>
<td><strong>Ratained Earned Premiums</strong></td>
<td>918.77</td>
<td>1,120.83</td>
</tr>
<tr>
<td><strong>Excess Loss Coverage Cost</strong></td>
<td>535.76</td>
<td>562.69</td>
</tr>
<tr>
<td><strong>Acquisition Costs</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct</td>
<td>223.24</td>
<td>235.09</td>
</tr>
<tr>
<td>Taken from other institutions</td>
<td>34.38</td>
<td>26.28</td>
</tr>
<tr>
<td>Commisions Recovered</td>
<td>293.70</td>
<td>321.16</td>
</tr>
<tr>
<td>Retained</td>
<td>-36.08</td>
<td>-59.78</td>
</tr>
<tr>
<td><strong>Technical Result</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Operation Costs</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Operation Reult</td>
<td>468.68</td>
<td>242.74</td>
</tr>
<tr>
<td>Other Expenses and finantial products</td>
<td>834.44</td>
<td>967.85</td>
</tr>
<tr>
<td><strong>Catastrophic Reserve (from de p&amp;L statement)</strong></td>
<td>-519.52</td>
<td>-969.55</td>
</tr>
<tr>
<td>Liberation Efect</td>
<td>834.49</td>
<td>1,363.15</td>
</tr>
<tr>
<td>Not Costitution effect</td>
<td>262.86</td>
<td>429.39</td>
</tr>
<tr>
<td>&quot;New&quot; Catastropic Reserve</td>
<td>577.83</td>
<td>822.99</td>
</tr>
<tr>
<td>&quot;New&quot; Net Result before taxes</td>
<td>725.88</td>
<td>388.44</td>
</tr>
</tbody>
</table>

|                          | 2006  | 2007  | 2008  | 2009  | 2010  | 2011  |
| **Catastrophic Reserve** | -519.52 | -969.55 | -286.79 | -409.76 | 597.71 | 1,152.63 |
| Net Result before taxes  | 1,823.23 | 2,180.98 | 1,834.55 | 1,762.24 | 581.66 | -90.08 |
| Taxes                    | 128.41 | 114.06 | -40.45 | 69.04 | 168.19 | 102.27 |
| Net Result               | 1,785.96 | 2,231.71 | 1,976.63 | 1,796.19 | 501.69 | -89.63 |
III. Mexican Industry Concerns

Recognition of Risk Transfer from Non-proportional Reinsurance Contracts

(1) Cesions to Not Registered Reinsurers

=>

Non proportional Risk Transfer

Retained PML

 Requirement
II. Summary of the Mexican Catastrophic Insurance Regulation

Decision:

Requirement | Option a) | Option b) | Option c)
IV. Ruin Probability.
IV. Ruin Probability, considerations

These systems model the random variable of the "loss values annual exceedance rate".

For a specific loss level (P):

$$\nu(P) = v_0 \left[ 1 - F(P) \right]$$

Where \( F(P) \) is the distribution function of the loss and \( v(0) \) is the number of events in one year.

The inverse of this function is the return period:

$$TR(P) = \frac{1}{\nu(P)}$$
IV. Ruin Probability, considerations

The Mexican regulator establishes that the PML (Probable Maximum Loss) is such value that \( TR(P) = 1,500 \).

Considering that there are multiple events, using the total probability theorem, \( v(P) \) can be rewritten as:

\[
v(P) = \sum_{i=1}^{N} Pa_i P(P \geq p \mid \text{event}_i)
\]

where
- \( Pa_i = \) Annual occurrence probability for event \( i \)
- \( P(P \geq p \mid \text{event}_i) \) is the probability that the loss exceeds the threshold \( p \), given that event \( i \) occurred.
- \( N = \) Total number of events
To model the losses, it is assumed that it is a Poisson process and that the number of events exceeding p value in time $T$ has the following distribution:

$$P(N = n) = \frac{(\nu(P)T)^n e^{-\nu(P)T}}{n!}$$

For a specific event $i$, its return period would be:

$$\frac{1}{Pa_i}$$

If we knew that the period of time is 150 years and that the p threshold is 0.1, then:

$$\nu_i(p) = Pa_i \Pr(P > p | i) = \frac{1}{150} \times 0.1 = 0.000667 = 1/1500$$

IV. Ruin Probability, considerations
If we suppose that the PML to consider should correspond to a return period of 1,500 years, then we are looking for a value of PML such that:

\[ v(PML) = \frac{1}{1500} \]

If we want to estimate the probability that the reserve is sufficient for the next T years, assuming that \( v(.) \) is a Poisson process, that probability would be:

\[ P_Q = 1 - e^{-v(PML)T} \]

For T=1:

\[ P_Q(T = 1) = 1 - e^{-v(PML)} \approx 1 - (1 - v(PML)) = v(PML) \]
IV. Ruin Probability, considerations

For T=20 years

- Return period of 1,500 years:

\[ P_Q(T = 20\,\text{years}) = 1 - e^{-\frac{20}{1500}} = 1 - e^{-0.013} \approx 0.01324 = 1.32\% \]

- Return period of 200 years:

\[ P_Q(T = 20\,\text{years}) = 1 - e^{-\frac{20}{200}} = 1 - e^{-0.1} \approx 0.095 = 9.5\% \]

Due to these calculations, ERN suggested the supervisor to use 1,500 years as a return period on the desirable coverage level for catastrophic risks.
V. Ruin Probability, the Model.
V. Ruin Probability, the Model

**Event Loss Table “ELT”**

These tables are generated by ERN systems, *RH-MEX* y *RS-MEX* and they include the different scenarios with their correspondent occurrence probability data, mean, variances and exposed values for each of the perils.

The system *RH-MEX* also generates the following files:

- **ERNFTST.SQM** (Total values, net from deductibles, coinsurance, and limits)
- **ERNFTSR.SQM** (Retained values, net from deductibles, coinsurance, limits, and proportional reinsurance contracts)
- **ERNFTSE.SQM** (Summary of the worst scenarios for the specific portfolio)

The system *RS-MEX* generates a unique output file:

- **Fuentes.dat** (Retained values, net from deductibles, coinsurance, limits, and proportional reinsurance contracts)
V. Ruin Probability, the Model

Event Loss Table “ELT”
V. Ruin Probability, the Model

To verify the behavior of the catastrophic events on one year, we used the Collective Risk Model, simulating for each event:

- Frequency \(\xrightarrow{}\) Poisson
- Severity \(\xrightarrow{}\) Beta

**Beta Distribution**

\[
f(x) = \frac{\Gamma(a + b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1 - x)^{b-1}\]

para \(a > 0, b > 0\) y \(0 \leq x \leq 1\)
V. Ruin Probability, the Model

BETA Parameter Estimation

For each of the events in the table, we considered the mean and standard deviation of the event.

By the moment methodology, beta parameters are estimated by:

\[
\hat{\alpha} = \left( \frac{\text{BetaMean}^2}{\text{BetaStd}^2} \right) \times (1 - \text{BetaMean}) - \text{BetaMean}
\]

\[
\hat{\beta} = \frac{\hat{\alpha}}{\text{BetaMean}} - \hat{\alpha}
\]
V. Ruin Probability, the Model

Collective Risk Model

Through the collective risk model we estimated the event frequency (number of claims) and the event severity (claims amount).

Procedure:

1. For each of the scenarios on the loss table, a Poisson distribution with lambda equal to the event frequency is used to simulate the number of claims.

2. Then a loss severity is simulated for each event. If the specific event has 5 claims, then 5 severities are simulated. The result of applying the Beta distribution is considered as the affected percentage of the sum insured, of damage to the building/contents/BI.
V. Ruin Probability, the Model

To find the **loss amount on a specific year**, all claims of specific simulated claims are summed up.

To estimate the **maximum event in one year**, we take the **maximum value** of the claims or simulated events.

This procedure is applied for each of the scenarios on the loss event table.

To obtain the **catastrophic claims amount for one year**, we sum all the claims for all the scenarios, and to obtain the maximum event in one year, the highest claim of the set of scenarios that integrate the portfolio is considered.

To obtain the **catastrophic claims curve** for one year and the **maximum event curve** for one year, this process is made at least 15,000 times. (To obtain the needed number of simulations to be made, we used the Hellinger distance index)
V. Ruin Probability, the Model

Aggregated Annual PML (AEP) and Maximum PML per Event (OEP)

\[ PML_{AEP}(n \text{ years of return}) = \text{Percentile}_{\frac{1-\frac{1}{n}}{}} (\text{Total Claims Curve}) \]

\[ PML_{OEP}(n \text{ years of return}) = \text{Percentile}_{\frac{1-\frac{1}{n}}{}} (\text{Maximum Event Curve}) \]

Risk Premium (Pure Premium) Calculation

\[ PR = \frac{\sum_{i=1}^{n} \text{TotalClaims}_i}{n} \]

where:
N = number of simulations
TotalClaims = Total catastrophic claims of i-simulation
V. Ruin Probability, the Model

Discrete Ruin Probability

Taking each of these results, the resources efficiency is evaluated to verify if they cover the diverse claims scenarios.

These evaluations can be made twofold:

- **Gross**.
- **Net** from Reinsurance.
V. Ruin Probability, the Model

**GROSS Discrete Ruin Probability**

Eligible Assets:
- ☑ Catastrophic Reserve
- ☑ **Gross** Unearned premium reserve

To the sum of these concepts, the claims of each simulation should be deducted.

The *gross discrete ruin probability* is obtained by:

\[
PRBR = \frac{n(-)}{N}
\]

The number of times the resources are insufficient to cover the responsibilities is counted and finally is divided into the number of total simulations to obtain an estimator of the ruin probability in one year.
V. Ruin Probability, the Model

Retained Discrete Ruin Probability (NET of Reinsurance)

For each of the claims simulations, we must consider the following amounts:

- The effect of proportional and non-proportional reinsurance contracts
- Non-proportional reinsurance costs.
- Reinstatement costs (if applicable).

These results should be compared to the sum of the following eligible assets:

- Catastrophic reserve
- Retained unearned premium reserve

The number of times the resources are insufficient to cover the responsibilities is counted and finally is divided into the number of total simulations to obtain an estimator of the ruin probability in one year.
V. Ruin Probability, the Model

We estimated the ruin probability for 25 companies on Hydrometeotrological Perils and for 24 companies for Earthquake with information on the third quarter 2010.

Insurance Companies in the Study

We have individual information for each company, but, due to confidentiality, we grouped them on big, medium, and small companies for presentation results.
V. Ruin Probability, the Model

Results on the Probable Maximum Loss (PML)

The **Retained Probable Maximum Loss** for Hydrometeorological perils estimated with a **1 in 1,500** years return period, was, of **$878 million pesos** on average for the market. This is **1.06 more times (factor of 2.06)** the PML estimated for a return period of **1 in 200 years**.

<table>
<thead>
<tr>
<th>Companies</th>
<th>PML (1 in 1,500)</th>
<th>PML (1 in 200)</th>
<th>No. of times more PML (1 in 1,500) vs PML (1 in 200)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>Maximum</td>
<td>Minimum</td>
</tr>
<tr>
<td>Grandes</td>
<td>2,299</td>
<td>5,239</td>
<td>189</td>
</tr>
<tr>
<td>Medianas</td>
<td>860</td>
<td>2,331</td>
<td>9</td>
</tr>
<tr>
<td>Chicas</td>
<td>298</td>
<td>2,107</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>878</td>
<td>5,239</td>
<td>2</td>
</tr>
</tbody>
</table>

*Figures in millions of Mexican pesos (approximate exchange rate $13 pesos per dollar)*
V. Ruin Probability, the Model

Results on the Probable Maximum Loss (PML)

The Retained Probable Maximum Loss for Earthquake estimated with a 1 in 1,500 years return period, was, of $1,040 million pesos on average for the market. This is 1.36 more times (factor of 2.36) the PML estimated for a return period of 1 in 200 years.

<table>
<thead>
<tr>
<th>Companies</th>
<th>Retained Probable Maximum Loss</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PML (1 in 1,500)</td>
<td>PML (1 in 200)</td>
<td>No. of times more PML (1 in 1,500) vs PML (1 in 200)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>Maximum</td>
<td>Minimum</td>
<td>Average</td>
</tr>
<tr>
<td>Grandes</td>
<td>2,242</td>
<td>3,449</td>
<td>290</td>
<td>920</td>
</tr>
<tr>
<td>Medianas</td>
<td>1,750</td>
<td>5,636</td>
<td>47</td>
<td>625</td>
</tr>
<tr>
<td>Chicas</td>
<td>179</td>
<td>727</td>
<td>2</td>
<td>76</td>
</tr>
<tr>
<td>Total</td>
<td>1,040</td>
<td>5,636</td>
<td>2</td>
<td>396</td>
</tr>
</tbody>
</table>

* Figures in millions of Mexican pesos (approximate exchange rate $13 pesos per dollar)
V. Ruin Probability, the Model

a) Hydrometeorological Perils

The results show that 6 companies are above the maximum permitted probability under Solvency II:

<table>
<thead>
<tr>
<th>Companies</th>
<th>Ruin Probability (AEP)</th>
<th>Ruin Probability (OEP)</th>
<th>No. of Companies where pr(ruina) &gt; 0.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Promedio</td>
<td>Máximo</td>
<td>Mínimo</td>
</tr>
<tr>
<td>Grandes</td>
<td>0.002773</td>
<td>0.012333</td>
<td>0.000000</td>
</tr>
<tr>
<td>Medianas</td>
<td>0.000233</td>
<td>0.001000</td>
<td>0.000000</td>
</tr>
<tr>
<td>Chicas</td>
<td>0.028250</td>
<td>0.155000</td>
<td>0.000000</td>
</tr>
<tr>
<td>Total</td>
<td>0.014189</td>
<td>0.155000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>
V. Ruin Probability, the Model

\textit{b) Earthquake}

The results show that 5 companies are above the maximum permitted probability under Solvency II:

\begin{table}
\centering
\begin{tabular}{|c|ccc|ccc|c|}
\hline
\textbf{Companies} & \textbf{Ruin Probability (AEP)} & & \textbf{Ruin Probability (OEP)} & & \textbf{No. of Companies where \( pr(\text{ruina}) > 0.5\% \)} \\
& \textbf{Promedio} & \textbf{Máximo} & \textbf{Mínimo} & \textbf{Promedio} & \textbf{Máximo} & \textbf{Mínimo} & \\
\hline
Grandes & 0.000367 & 0.000667 & 0.000000 & 0.000367 & 0.000667 & 0.000000 & 0 \\
Medianas & 0.002919 & 0.012733 & 0.000000 & 0.002533 & 0.011800 & 0.000000 & 2 \\
Chicas & 0.017779 & 0.095733 & 0.000000 & 0.016574 & 0.085000 & 0.000000 & 3 \\
Total & 0.009956 & 0.095733 & 0.000000 & 0.009221 & 0.085000 & 0.000000 & 5 \\
\hline
\end{tabular}
\end{table}
VI. Conclusions.
VI. Conclusions

- This study shows the practical application of ruin theory to the Mexican insurance market.

- Actual catastrophic reserves regulation rules privilege the use of proportional reinsurance contracts over non-proportional ones, and distorts the market by incentivizing the outflow of premiums and reserves to countries with less prudential regulation (to countries that do not require catastrophic reserves).

- Companies that are still constituting catastrophic reserves without reaching their technical limit, have an additional implicit cost in comparison to those with capped reserves.

- The extensive use of proportional contracts is a limitation to price increases.
VI. Conclusions

• Under actual circumstances, there is a crossed subsidy with other lines of business, by transferring utilities to compensate the losses in catastrophic perils.

• The accumulated amount in the catastrophic reserves does not recognize the particular risk of the portfolios and does not measure the ruin exposition for each company.

• If the criteria used to establish the prudential margins to determine the PML on Catastrophic Risks was to use the approximate ruin probability of 1.32%, this is not been accomplished, due to the fact that there is great diversity between ruin probability between companies.

• It is necessary to review the models and evaluate the convenience to look for new prudential criteria that allow a more efficient supervision, based on risk management.