



AFIR/ERM

Investment Risk Taking Policy in the Context of ERM

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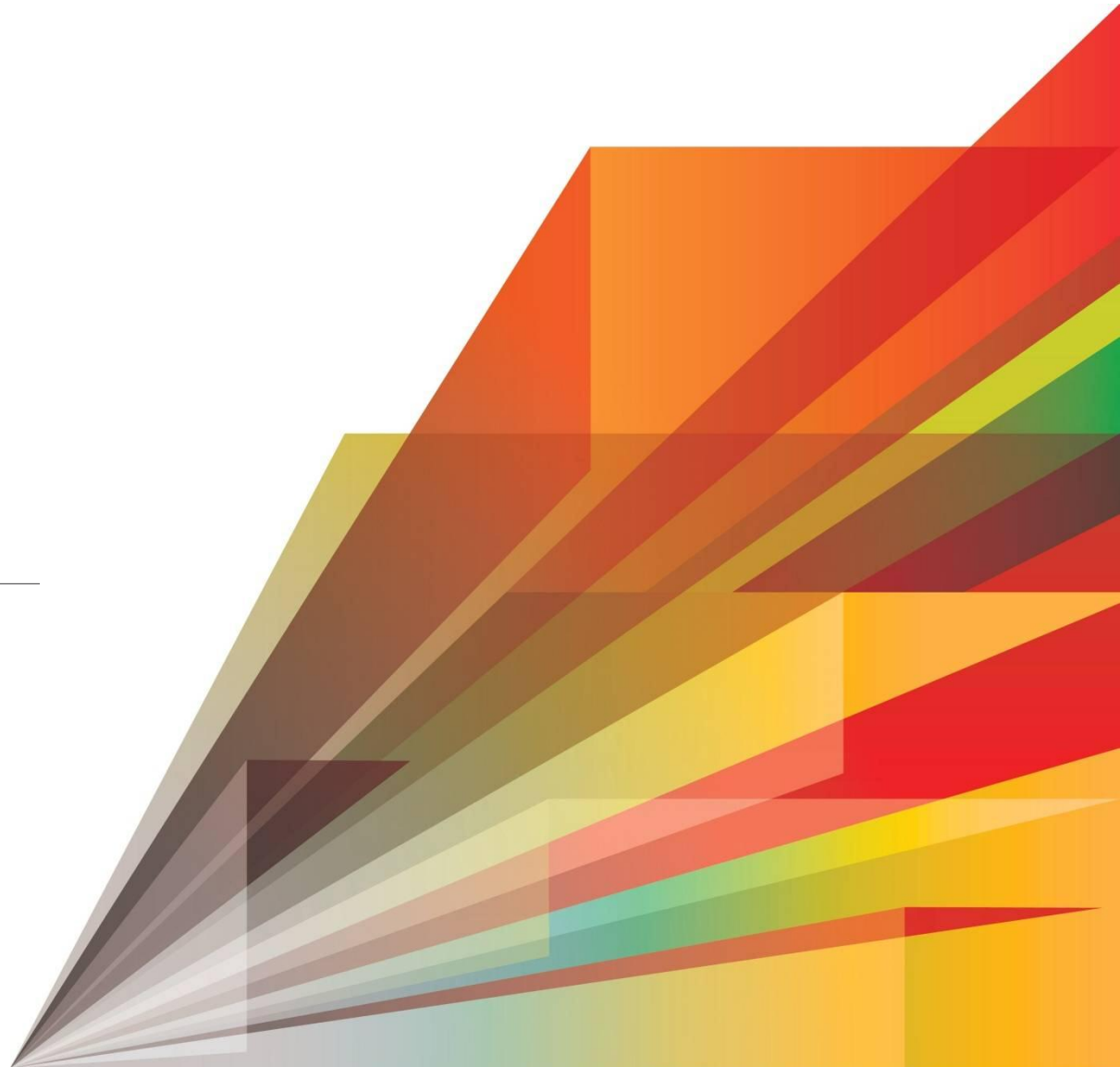


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I. Introduction

● Pension ERM Framework

Risks for Pensions

Pension Liability: **Longevity Risk**

Pension Investment Asset: Market Risk

Pension Management: **Liquidity Risk**
Operational Risk

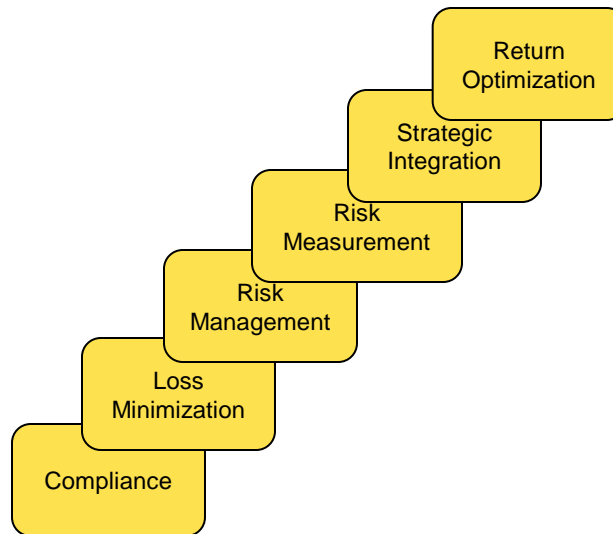
Sponsor: **Default Risk**

Regime: **Legal/Regulation/Politics**

etc.,

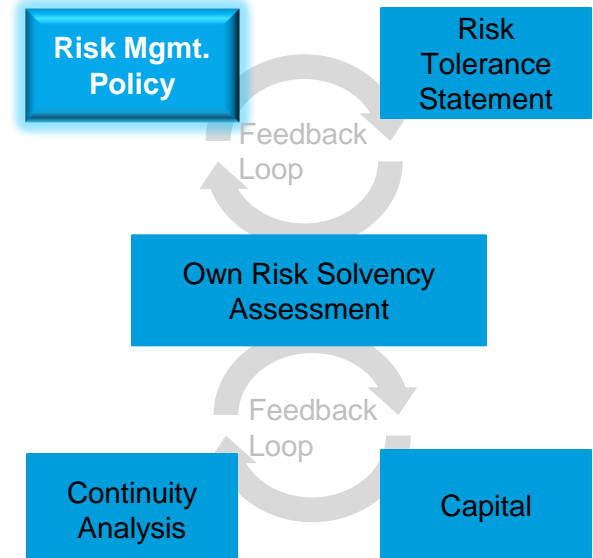
For Pensioners
For Pension Trustees
For Pension Sponsors

Evolution of ERM



Source: S&P

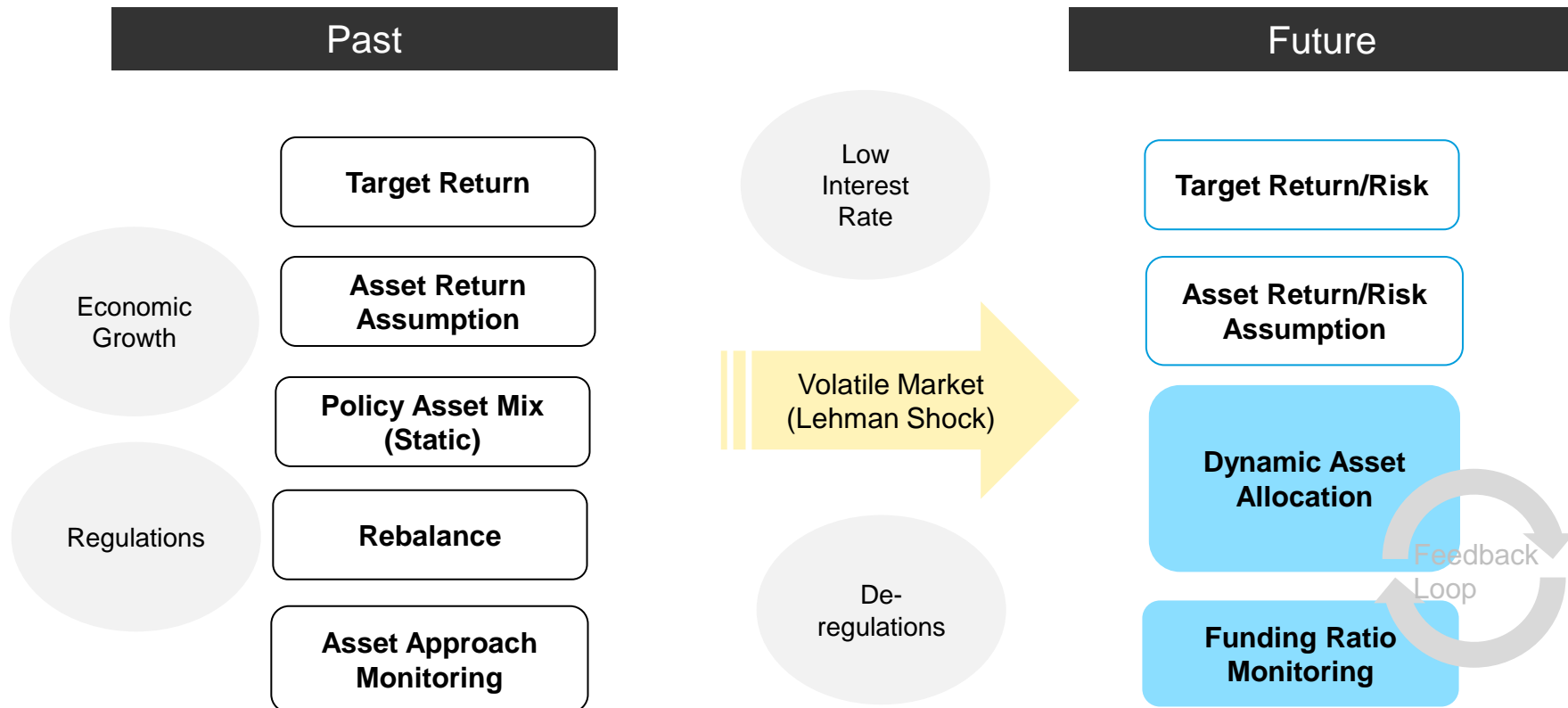
ERM Framework



Source: AAI/IAA

I. Introduction

- Past and Future of **Investment aspect** of Pension ERM



II. Research Questions

- Feedback Loop: When start to work?
 How to work?

Funding Ratio and Risk Averseness

- ❑ How pension fund's risk averseness changes according to Funding Ratio?
- ❑ Hypothesis:
Under-Funding may force pension fund re-risking for aiming to covering funding ratio.

However, Under-Funding may also induce de-risking relying on sponsor's additional contribution.

Over-Funding encourage de-risking as "Give me a break."

However, Over-Funding make pensions re-risking for better benefit.

Downside Risk Averseness

- ❑ How pension fund's downside risk averseness changes according to Funding Ratio?
- ❑ Hypothesis:
Lesser Funding-Ratio, more Downside Risk Averse.

III. Research Methods

Funding Ratio and Risk Averseness

- See pension fund's asset mix for each Funding Ratio group.
- Measure Risk Averseness (γ) using the below formula with past 5 year financial market return data.

$$U(w_t) : \text{utility function. } \frac{w_t^{1-\gamma} - 1}{1-\gamma} \text{ (CRRA type)}$$
$$w_{t+dt} - w_t = \left[\varphi_t (\mu^S dt + \sigma^S b_t \sqrt{dt}) + (1 - \varphi_t) r^f \right] w_t$$
$$E_{dt}(U(w_{t+dt})) \approx U(w_t) + U'(w_t) w_t dt \left[\varphi_t \mu^S + (1 - \varphi_t) r^f \right] + \frac{1}{2} U''(w_t) w_t^2 (\sigma^S)^2 dt$$
$$\approx U(w_t) + w_t dt U'(w_t) \left[\left[r^f + \frac{1}{2} \frac{(\mu^S - r^f)^2}{\sigma^S} \right] - \frac{1}{2} \gamma (\sigma^S)^2 \left[\varphi_t - \frac{(\mu^S - r^f)}{\gamma \sigma^S} \right]^2 \right]$$

We can describe the risk averseness as follows by the maximization condition.

$$\gamma = \frac{\left(\frac{\mu^S - r^f}{\sigma^S} \right)}{\varphi_t \sigma^S}$$

III. Research Methods

Downside Risk Averseness

Utility Assumption:

$$\sum_m \left[p \frac{(\varphi_m y_m)^{v_+}}{v_+} - \lambda \frac{(-(\varphi_m y_m))^{v_-}}{v_-} (1-p) \right]$$

φ_m : ratio of risky asset vs. total asset value at month m

p: probability of positive return

$$u(x(m)) = \begin{cases} \frac{x^{v_+}}{v_+} & (=u+(x)) \text{ if } x \geq 0, \\ -\lambda \frac{(-x)^{v_-}}{v_-} & (=u-(x)) \text{ if } x < 0, \end{cases}$$

$$\text{Utility } U = \sum_m [u(x(m))]$$

v_+ CRRA Utility, Risk Averseness for positive returns

v_- CRRA Utility, Risk Averseness for negative returns

λ Parameter for positive returns vs. negative returns

Source: Based on Hwang et al. (2010)

III. Research Methods

● Data and Assumptions

- Data for Asset breakdown and Funding ratio (at risk base) of pension funds: PFA surveys
- It researched Japanese pension funds' investment status for the end of fiscal year 2006, 2007, 2008, 2009 and 2010.
- The survey participants are two type of pension fund and number of employees' pension scheme participated is 595 and number of defined benefit scheme participated is 807 in 2010. Those total asset will be more than 90% of total pension funds in Japan. The employee's pension scheme tends to be used by industrial segment coalitions and still that is defined benefit.
- The breakdown items consist of domestic bond, domestic equity, foreign bond, foreign equity, Guaranteed Insurance Contract (GIC), hedge fund, and others other than cash. Funding ratio grouping ranges are 10% isobar groups and they are for instance, under 80%, from 80% to 90%, from 90% to 100%, from 100% to 110%, and 110% more.
- Regarding financial market data, I prepared monthly returns for past 5 years. For each of the asset breakdown items, we set benchmark indices as proxy of the followings.
 - Nomura BP index: domestic bond and GIC (Only the non-risky asset)
 - TOPIX: domestic equity
 - Citi WGBI ex Japan: foreign bond
 - MSCI Kokusai: foreign equity
 - CS / Tremont Hedge Fund index: hedge fund
 - S&P Global REIT index: others
- Pension funds decide its Asset breakdown according to past 5 years market results.
- Pension funds with the same level Funding Ratio have the same risk averse tendency.

IV. Results and Conclusion

Funding Ratio and Risk Averseness

● In 2008, both employees' pension scheme and defined benefit scheme show that;

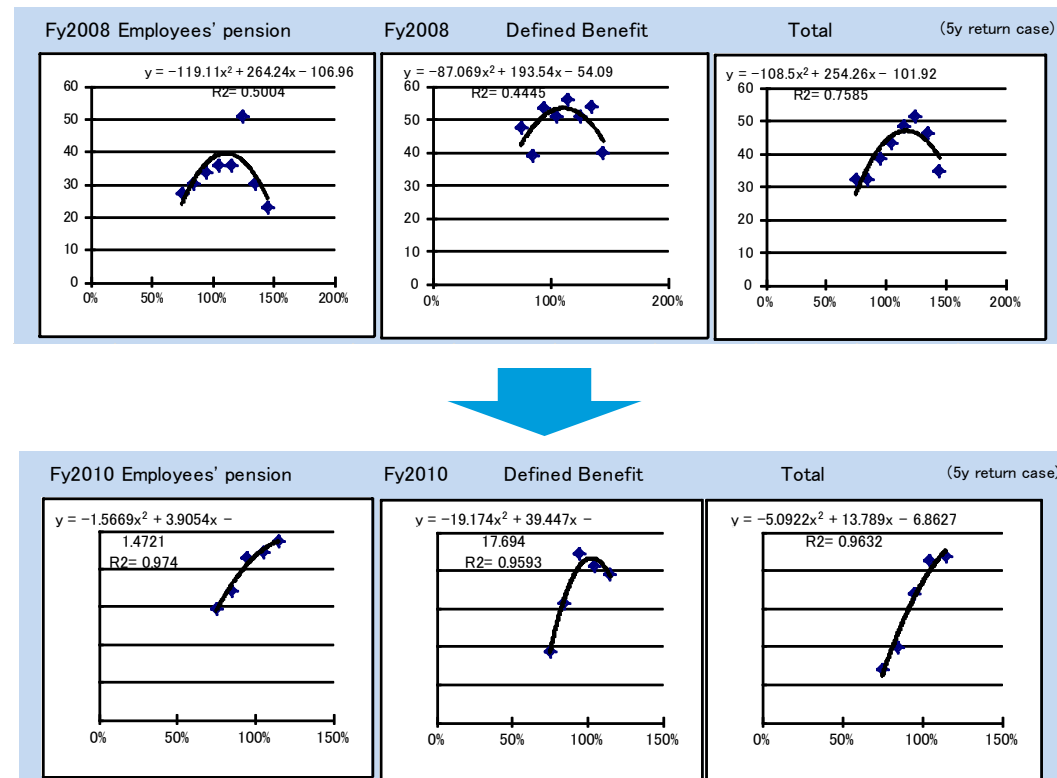
- #1) If the funding ratio is low, the risk averseness is low. (taking more risk)
- #2) If the funding ratio is near to full funding, the risk averseness increase. (less risk taking)
- #3) If the funding ratio is larger and overfunding, the risk averseness again decreased. (again risk taking)

● However, in 2010, #3) tendency disappeared for employees' pension scheme.

This is thought that even overfunding status, they think they do not have to take risk and just to maintain funding ratio near full funding is important. To take the risk even at overfunding status is none sense.

Of course, for defined benefit scheme, which consists of more larger pension scheme, still has tendency to increase risk taking after overfunding. But the magnitude decreased.

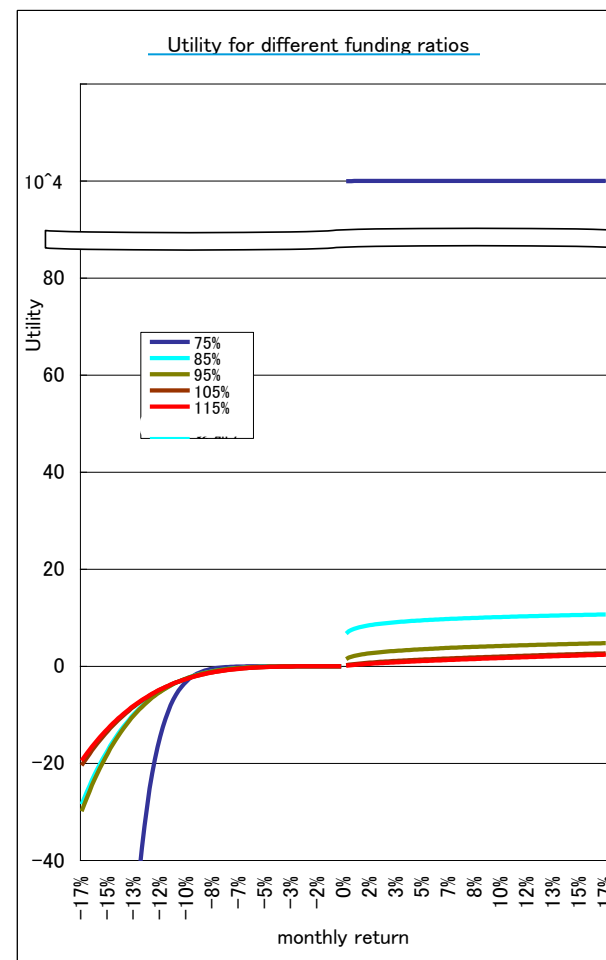
Risk Averseness Dependency for Funding Ratio



IV. Results and Conclusion

Downside Risk Averseness

- Each funding ratio group has different risk averseness parameters and the Figure right shows the Asymmetric Utility Function type shape using the analysis results.
- Lower funding ratio group has more risk averseness for negative returns and less risk averseness for positive returns. This tendency is very natural.



IV. Results and Conclusion

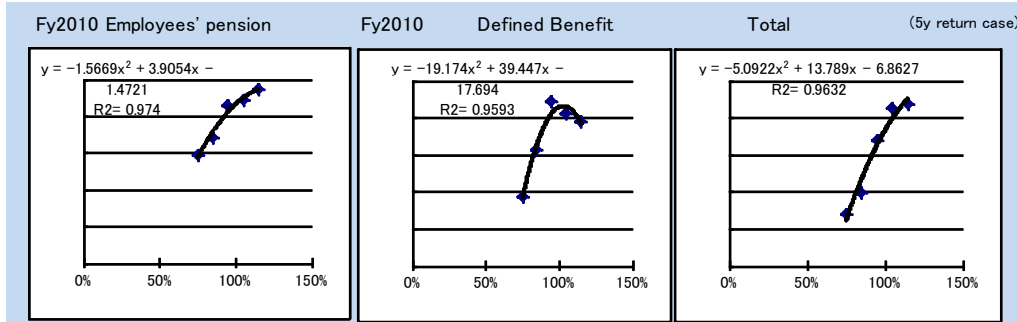
- ✓ Our findings are the followings.
 - (a) If under-funding, risk averseness is small (to take risk aggressively) and increases as funding ratio increases (less risk taking), and;
 - (b) If at over-funding, risk averseness is decreases as funding ratio increases.
- ✓ Result (a) is on line with necessity of return seeking but lucking downside risk that affects sponsor contribution and result (b) seems to be showing natural increase of risk taking as over-funding status is strengthened but is contradicting against liability hedging concept. For the last point, there is the fact that all pension scheme is alive (on going) and need to beat expected discount rate.
- ✓ However, recently the tendency decreased. It seems that recent capital market debacles forced pensions more conservative risk management. Once full funding or over funding, after that, there is no reason to take excess risk. ALM type management should be more important. This may be the beginning of ERM management for those pensions in Japan.
- ✓ The “Risk Management Policy” could be;

Under-Funding: Re-Risking

Over-Funding: De-Risking

Apendix

FY2010 Risk Averseness for each Funding Ratio



Employees' pension

| Range Median | -80% | 80%-90% | 90%-100% | 100%-110% | 110%-115% |
|-----------------------|----------|---------|----------|-----------|-----------|
| Range Median | 75% | 85% | 95% | 105% | 115% |
| Domestic Bond | 23.8% | 24.5% | 26.0% | 28.0% | 25.6% |
| Domestic Equity | 25.6% | 25.2% | 23.1% | 24.8% | 22.2% |
| Foreign Bond | 8.6% | 9.1% | 9.0% | 9.8% | 13.1% |
| Foreign Equity | 21.5% | 20.2% | 19.0% | 18.1% | 17.6% |
| GIC | 4.8% | 8.7% | 10.1% | 9.4% | 11.7% |
| Hedge Fund | 5.9% | 4.8% | 4.9% | 3.9% | 1.8% |
| Others (J-REIT etc.) | 5.3% | 4.2% | 5.3% | 3.0% | 3.4% |
| Cash | 4.5% | 3.4% | 2.5% | 3.0% | 4.5% |
| Total | 100% | 100% | 100% | 100% | 100% |
| Historical Return | 0.5% | 0.5% | 0.6% | 0.5% | 0.5% |
| Historical Volatility | 11.3% | 10.8% | 10.3% | 10.0% | 9.6% |
| Lammda | 0.39 | 0.43 | 0.53 | 0.53 | 0.55 |
| Risk Aburese | 0.585162 | 0.68484 | 0.857785 | 0.886325 | 0.94852 |

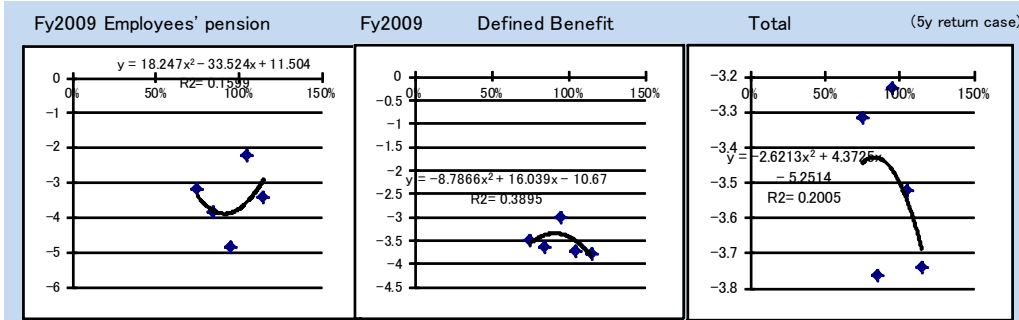
defined benefit corporate plans

| Range Median | -80% | 80%-90% | 90%-100% | 100%-110% | 110%-115% |
|-----------------------|----------|----------|----------|-----------|-----------|
| Range Median | 75% | 85% | 95% | 105% | 115% |
| Domestic Bond | 28.3% | 27.6% | 31.5% | 30.6% | 27.5% |
| Domestic Equity | 20.8% | 19.2% | 17.7% | 18.2% | 17.1% |
| Foreign Bond | 9.8% | 9.6% | 9.9% | 9.6% | 9.8% |
| Foreign Equity | 17.3% | 16.3% | 14.5% | 14.8% | 14.1% |
| GIC | 13.1% | 17.2% | 16.4% | 17.6% | 20.5% |
| Hedge Fund | 2.7% | 4.5% | 4.4% | 4.0% | 2.9% |
| Others (J-REIT etc.) | 4.9% | 3.1% | 2.7% | 2.6% | 4.0% |
| Cash | 3.1% | 2.6% | 2.9% | 2.7% | 4.1% |
| Total | 100% | 100% | 100% | 100% | 100% |
| Historical Return | 0.5% | 0.7% | 0.8% | 0.8% | 0.7% |
| Historical Volatility | 9.3% | 8.6% | 7.9% | 8.0% | 7.8% |
| Lammda | 0.63 | 0.99 | 1.30 | 1.21 | 1.12 |
| Risk Aburese | 1.129961 | 1.876285 | 2.6523 | 2.458238 | 2.347443 |

Total

| Range Median | -80% | 80%-90% | 90%-100% | 100%-110% | 110%-115% |
|-----------------------|----------|----------|----------|-----------|-----------|
| Range Median | 75% | 85% | 95% | 105% | 115% |
| Domestic Bond | 25.2% | 25.6% | 29.3% | 30.2% | 27.4% |
| Domestic Equity | 24.2% | 23.0% | 19.8% | 19.1% | 17.5% |
| Foreign Bond | 9.0% | 9.3% | 9.5% | 9.6% | 10.0% |
| Foreign Equity | 20.2% | 18.8% | 16.3% | 15.2% | 14.4% |
| GIC | 7.3% | 11.7% | 13.9% | 16.5% | 19.8% |
| Hedge Fund | 4.9% | 4.7% | 4.6% | 3.9% | 2.8% |
| Others (J-REIT etc.) | 5.2% | 3.8% | 3.7% | 2.7% | 3.9% |
| Cash | 4.0% | 3.1% | 2.7% | 2.7% | 4.1% |
| Total | 100% | 100% | 100% | 100% | 100% |
| Historical Return | 0.5% | 0.6% | 0.7% | 0.7% | 0.7% |
| Historical Volatility | 10.7% | 10.0% | 8.8% | 8.2% | 7.9% |
| Lammda | 0.45 | 0.59 | 0.91 | 1.07 | 1.06 |
| Risk Aburese | 0.701288 | 0.988685 | 1.692128 | 2.123829 | 2.190644 |

●FY2009 Risk Averseness for each Funding Ratio



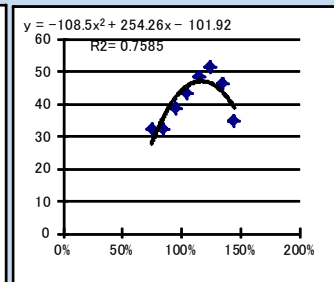
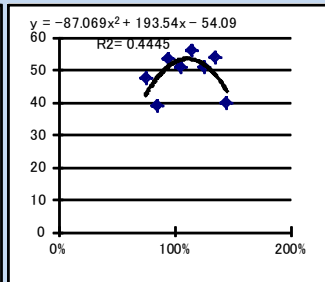
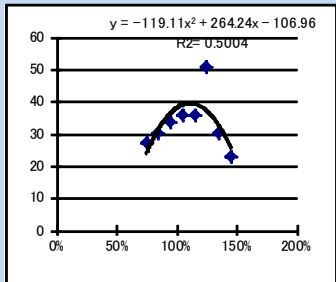
| employees' pension funds MV of Asset/PBO | | | | | |
|--|-----------------|-----------------|-----------------|-----------------|-----------------|
| Range Median | -80% | 80%-90% | 90%-100% | 100%-110% | 110%- |
| Range Median | 75% | 85% | 95% | 105% | 115% |
| Domestic Bond | 25.1% | 24.3% | 19.4% | 17.2% | 36.4% |
| Domestic Equity | 27.8% | 25.2% | 23.3% | 15.4% | 19.5% |
| Foreign Bond | 9.4% | 10.1% | 8.2% | 10.8% | 12.2% |
| Foreign Equity | 18.7% | 14.6% | 13.6% | 13.2% | 13.3% |
| GIC | 5.8% | 9.6% | 21.0% | 15.6% | 11.8% |
| Hedge Fund | 4.7% | 2.9% | 1.2% | 8.1% | 1.0% |
| Others (J-REIT etc.) | 4.2% | 4.0% | 4.4% | 6.9% | 0.2% |
| Cash | 4.3% | 9.3% | 8.3% | 12.8% | 5.6% |
| Total | 100% | 100% | 100% | 100% | 100% |
| Historical Return | -1.7% | -1.3% | -1.3% | -0.5% | -0.6% |
| Historical Volatility | 9.0% | 7.8% | 7.3% | 6.7% | 6.1% |
| Lammda | -2.06 | -2.20 | -2.49 | -1.20 | -1.59 |
| Risk Aburese | -3.18142 | -3.86913 | -4.85489 | -2.21313 | -3.43733 |
| defined benefit corporate plans | | | | | |
| Range Median | -80% | 80%-90% | 90%-100% | 100%-110% | 110%- |
| Range Median | 75% | 85% | 95% | 105% | 115% |
| Domestic Bond | 27.4% | 29.8% | 31.1% | 30.4% | 26.9% |
| Domestic Equity | 23.7% | 21.0% | 19.1% | 18.9% | 20.3% |
| Foreign Bond | 10.5% | 9.9% | 10.2% | 10.7% | 9.8% |
| Foreign Equity | 16.3% | 14.1% | 12.5% | 13.1% | 12.5% |
| GIC | 11.6% | 15.1% | 15.5% | 19.4% | 18.8% |
| Hedge Fund | 3.3% | 3.3% | 5.3% | 1.4% | 3.6% |
| Others (J-REIT etc.) | 3.1% | 3.5% | 2.6% | 2.9% | 3.1% |
| Cash | 4.0% | 3.4% | 3.6% | 3.3% | 4.9% |
| Total | 100% | 100% | 100% | 100% | 100% |
| Historical Return | -1.2% | -0.9% | -0.6% | -0.7% | -0.8% |
| Historical Volatility | 7.7% | 6.9% | 6.3% | 6.3% | 6.5% |
| Lammda | -2.00 | -1.88 | -1.50 | -1.75 | -1.87 |
| Risk Aburese | -3.51445 | -3.63778 | -3.0144 | -3.72713 | -3.79747 |
| Total | | | | | |
| Range Median | -80% | 80%-90% | 90%-100% | 100%-110% | 110%- |
| Range Median | 75% | 85% | 95% | 105% | 115% |
| Domestic Bond | 25.8% | 28.4% | 30.0% | 28.8% | 27.5% |
| Domestic Equity | 26.5% | 22.1% | 19.6% | 18.5% | 20.2% |
| Foreign Bond | 9.7% | 9.9% | 10.1% | 10.7% | 10.0% |
| Foreign Equity | 17.9% | 14.2% | 12.6% | 13.1% | 12.6% |
| GIC | 7.7% | 13.6% | 16.0% | 18.9% | 18.4% |
| Hedge Fund | 4.2% | 3.2% | 4.9% | 2.2% | 3.5% |
| Others (J-REIT etc.) | 3.9% | 3.6% | 2.8% | 3.4% | 2.9% |
| Cash | 4.2% | 4.9% | 4.1% | 4.4% | 4.9% |
| Total | 100% | 100% | 100% | 100% | 100% |
| Historical Return | -1.5% | -1.0% | -0.7% | -0.7% | -0.8% |
| Historical Volatility | 8.6% | 7.1% | 6.4% | 6.3% | 6.5% |
| Lammda | -2.06 | -1.99 | -1.62 | -1.69 | -1.84 |
| Risk Aburese | -3.31728 | -3.76383 | -3.23278 | -3.52222 | -3.74201 |

●FY2008 Risk Averseness for each Funding Ratio

Fy2008 Employees' pension

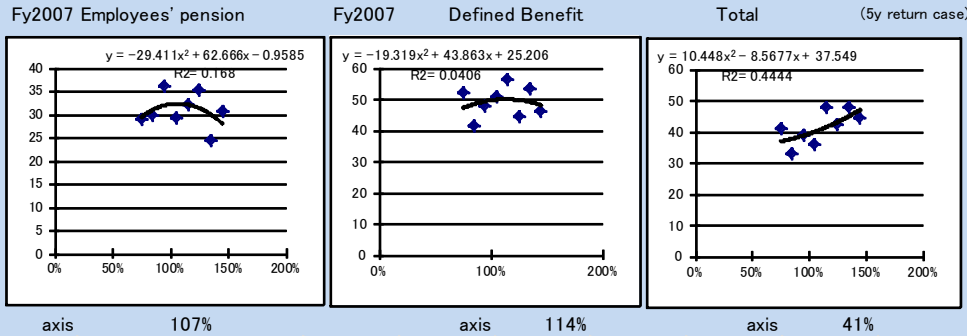
Fy2008 Defined Benefit

Total (5y return case)



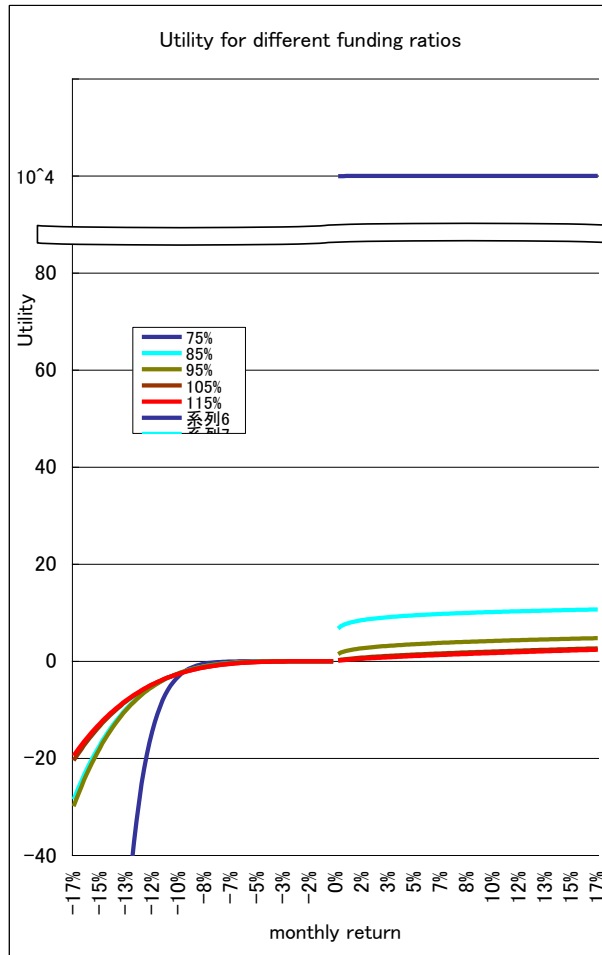
| employees' pension funds | MV of Asset/PBO | | | | | | | | |
|--|-----------------|----------|----------|-----------|-----------|-----------|-----------|----------|--|
| | -80% | 80%-90% | 90%-100% | 100%-110% | 110%-120% | 120%-130% | 130%-140% | 140%- | |
| Range Median | 75% | 85% | 95% | 105% | 115% | 125% | 135% | 145% | |
| Domestic Bond | 20.7% | 23.9% | 26.5% | 26.5% | 28.5% | 32.8% | 18.7% | 20.0% | |
| Domestic Equity | 28.5% | 27.5% | 25.5% | 25.8% | 25.4% | 21.0% | 27.0% | 27.7% | |
| Foreign Bond | 12.2% | 11.0% | 11.8% | 11.3% | 12.1% | 8.8% | 8.3% | 20.7% | |
| Foreign Equity | 16.6% | 15.0% | 15.2% | 14.6% | 15.1% | 14.4% | 14.2% | 21.6% | |
| GIC | 2.7% | 3.7% | 6.5% | 8.9% | 7.1% | 10.2% | 10.2% | 3.3% | |
| Hedge Fund | 7.3% | 8.8% | 6.3% | 4.8% | 3.5% | 3.2% | 3.3% | 2.5% | |
| Others (J-REIT etc.) | 4.8% | 4.3% | 4.3% | 3.4% | 3.1% | 1.8% | 11.9% | 3.1% | |
| Cash | 7.3% | 5.7% | 3.9% | 4.7% | 5.1% | 7.7% | 6.4% | 1.0% | |
| Total | 100% | 100% | 100% | 100% | 100% | 100% | 100% | 100% | |
| Historical Return | 7.6% | 7.3% | 6.9% | 6.6% | 6.5% | 5.5% | 7.6% | 7.8% | |
| Historical Volatility | 6.3% | 6.0% | 5.7% | 5.6% | 5.5% | 4.7% | 6.3% | 6.7% | |
| Lambda | 18.92 | 20.23 | 21.34 | 21.31 | 21.25 | 24.94 | 19.49 | 17.23 | |
| Risk Aburese | 27.25516 | 30.38027 | 33.82591 | 35.58074 | 35.89704 | 50.69439 | 30.12819 | 22.79262 | |
| defined benefit corporate plans | | | | | | | | | |
| Range Median | -80% | 80%-90% | 90%-100% | 100%-110% | 110%-120% | 120%-130% | 130%-140% | 140%- | |
| Range Median | 75% | 85% | 95% | 105% | 115% | 125% | 135% | 145% | |
| Domestic Bond | 18.7% | 29.7% | 30.1% | 29.1% | 30.5% | 28.1% | 27.5% | 29.0% | |
| Domestic Equity | 19.7% | 24.9% | 20.2% | 20.7% | 20.0% | 21.1% | 21.0% | 24.0% | |
| Foreign Bond | 9.4% | 12.5% | 10.8% | 12.8% | 10.7% | 11.3% | 12.0% | 12.9% | |
| Foreign Equity | 11.9% | 13.5% | 12.3% | 12.5% | 11.8% | 11.7% | 11.7% | 13.3% | |
| GIC | 12.6% | 8.5% | 14.3% | 15.0% | 15.0% | 15.0% | 9.3% | 7.4% | |
| Hedge Fund | 0.0% | 2.7% | 4.2% | 4.1% | 5.5% | 5.8% | 3.7% | 5.3% | |
| Others (J-REIT etc.) | 11.9% | 3.8% | 3.7% | 2.5% | 2.6% | 3.3% | 1.4% | 3.5% | |
| Cash | 15.9% | 4.5% | 4.4% | 3.2% | 4.0% | 3.7% | 13.3% | 3.8% | |
| Total | 100% | 100% | 100% | 100% | 100% | 100% | 100% | 99% | |
| Historical Return | 6.3% | 6.3% | 5.7% | 5.7% | 5.6% | 5.9% | 5.3% | 6.4% | |
| Historical Volatility | 5.0% | 5.3% | 4.6% | 4.6% | 4.5% | 4.7% | 4.5% | 5.2% | |
| Lambda | 25.06 | 22.35 | 27.44 | 26.78 | 28.27 | 27.15 | 26.75 | 23.37 | |
| Risk Aburese | 47.37405 | 38.94253 | 53.60092 | 50.91031 | 55.87105 | 51.028 | 53.71005 | 39.60673 | |
| Total | | | | | | | | | |
| Range Median | -80% | 80%-90% | 90%-100% | 100%-110% | 110%-120% | 120%-130% | 130%-140% | 140%- | |
| Range Median | 75% | 85% | 95% | 105% | 115% | 125% | 135% | 145% | |
| Domestic Bond | 20.1% | 25.4% | 27.6% | 27.9% | 29.9% | 29.0% | 25.3% | 27.5% | |
| Domestic Equity | 25.9% | 26.8% | 23.9% | 23.0% | 21.6% | 21.0% | 22.5% | 25.3% | |
| Foreign Bond | 11.4% | 11.4% | 11.5% | 12.1% | 11.1% | 10.8% | 11.1% | 14.2% | |
| Foreign Equity | 15.2% | 14.7% | 14.3% | 13.4% | 12.8% | 12.3% | 12.4% | 14.8% | |
| GIC | 5.6% | 4.9% | 8.9% | 12.2% | 12.6% | 14.1% | 9.6% | 6.7% | |
| Hedge Fund | 5.2% | 7.3% | 5.7% | 4.4% | 4.9% | 5.3% | 3.6% | 4.8% | |
| Others (J-REIT etc.) | 6.9% | 4.2% | 4.1% | 2.9% | 2.8% | 3.0% | 4.1% | 3.5% | |
| Cash | 9.8% | 5.4% | 4.0% | 3.9% | 4.3% | 4.5% | 11.6% | 3.3% | |
| Total | 100% | 100% | 100% | 100% | 100% | 100% | 100% | 100% | |
| Historical Return | 7.2% | 7.1% | 6.6% | 6.1% | 5.9% | 5.8% | 5.9% | 6.7% | |
| Historical Volatility | 5.9% | 5.8% | 5.4% | 5.0% | 4.8% | 4.7% | 4.9% | 5.6% | |
| Lambda | 20.70 | 20.75 | 22.91 | 24.07 | 25.77 | 26.76 | 24.87 | 21.60 | |
| Risk Aburese | 32.04086 | 32.22269 | 38.5115 | 43.13921 | 48.44822 | 51.0645 | 46.3098 | 34.50983 | |

●FY2007 Risk Averseness for each Funding Ratio



| employees' pension funds | MV of Asset/PBO | | | | | | | | |
|---------------------------------|-----------------|----------|----------|-----------|-----------|-----------|-----------|----------|--|
| Range Median | -80% | 80%-90% | 90%-100% | 100%-110% | 110%-120% | 120%-130% | 130%-140% | 140%- | |
| Range Median | 75% | 85% | 95% | 105% | 115% | 125% | 135% | 145% | |
| Domestic Bond | 24.1% | 23.7% | 22.1% | 22.2% | 18.4% | 27.6% | 19.8% | 24.6% | |
| Domestic Equity | 29.8% | 27.7% | 24.8% | 28.4% | 28.7% | 25.4% | 31.4% | 21.6% | |
| Foreign Bond | 12.5% | 12.3% | 12.5% | 12.3% | 11.8% | 18.2% | 11.8% | 14.8% | |
| Foreign Equity | 17.9% | 18.8% | 17.0% | 19.1% | 17.3% | 16.6% | 20.5% | 22.5% | |
| GIC | 2.8% | 3.4% | 12.2% | 4.5% | 7.7% | 2.3% | 1.4% | 7.3% | |
| Hedge Fund | 5.5% | 7.6% | 5.8% | 6.5% | 6.2% | 3.5% | 6.5% | 5.6% | |
| Others (J-REIT etc.) | 3.7% | 4.0% | 2.1% | 4.6% | 6.2% | 2.6% | 3.8% | 2.0% | |
| Cash | 3.6% | 2.4% | 3.6% | 2.3% | 3.6% | 3.9% | 4.8% | 1.6% | |
| Total | 100% | 100% | 100% | 100% | 100% | 100% | 100% | 100% | |
| Historical Return | 7.9% | 8.1% | 6.9% | 8.2% | 8.5% | 7.3% | 8.3% | 7.1% | |
| Historical Volatility | 6.3% | 6.2% | 5.5% | 6.3% | 6.1% | 5.6% | 6.8% | 5.9% | |
| Lambda | 20.18 | 21.04 | 22.48 | 20.79 | 22.60 | 23.30 | 18.02 | 20.35 | |
| Risk Aburese | 29.08203 | 29.88663 | 36.14462 | 29.32552 | 32.19365 | 35.14382 | 24.35696 | 30.60471 | |
| defined benefit corporate plans | | | | | | | | | |
| Range Median | -80% | 80%-90% | 90%-100% | 100%-110% | 110%-120% | 120%-130% | 130%-140% | 140%- | |
| Range Median | 75% | 85% | 95% | 105% | 115% | 125% | 135% | 145% | |
| Domestic Bond | 27.5% | 19.5% | 23.1% | 23.8% | 37.8% | 20.0% | 35.2% | 24.3% | |
| Domestic Equity | 19.9% | 19.9% | 21.8% | 21.5% | 19.2% | 21.5% | 20.8% | 22.3% | |
| Foreign Bond | 16.3% | 15.9% | 16.2% | 14.8% | 11.1% | 13.7% | 11.6% | 12.6% | |
| Foreign Equity | 14.5% | 17.5% | 13.6% | 13.4% | 15.1% | 15.0% | 13.6% | 15.2% | |
| GIC | 11.3% | 8.5% | 6.8% | 15.5% | 9.1% | 19.0% | 11.2% | 17.6% | |
| Hedge Fund | 4.7% | 10.4% | 4.3% | 5.8% | 3.3% | 7.6% | 4.9% | 3.6% | |
| Others (J-REIT etc.) | 3.7% | 5.2% | 1.6% | 3.0% | 2.7% | 1.7% | 1.2% | 2.0% | |
| Cash | 2.2% | 3.1% | 12.5% | 2.2% | 1.7% | 1.6% | 1.5% | 2.3% | |
| Total | 100% | 100% | 100% | 100% | 100% | 100% | 100% | 100% | |
| Historical Return | 6.9% | 8.0% | 6.2% | 6.7% | 6.0% | 6.5% | 5.8% | 6.2% | |
| Historical Volatility | 4.7% | 5.3% | 4.8% | 4.8% | 4.6% | 4.9% | 4.6% | 4.9% | |
| Lambda | 30.80 | 28.62 | 27.46 | 29.74 | 29.06 | 26.51 | 27.72 | 25.61 | |
| Risk Aburese | 52.11598 | 41.53418 | 47.76483 | 50.83045 | 56.53479 | 44.56259 | 53.20221 | 45.97203 | |
| Total | | | | | | | | | |
| Range Median | -80% | 80%-90% | 90%-100% | 100%-110% | 110%-120% | 120%-130% | 130%-140% | 140%- | |
| Range Median | 75% | 85% | 95% | 105% | 115% | 125% | 135% | 145% | |
| Domestic Bond | 26.2% | 22.4% | 22.3% | 22.8% | 32.2% | 21.6% | 33.4% | 24.3% | |
| Domestic Equity | 23.7% | 25.3% | 24.0% | 25.7% | 21.9% | 22.4% | 22.1% | 22.2% | |
| Foreign Bond | 14.8% | 13.5% | 13.5% | 13.3% | 11.3% | 14.6% | 11.6% | 12.8% | |
| Foreign Equity | 15.9% | 18.4% | 16.1% | 16.9% | 15.7% | 15.3% | 14.4% | 15.8% | |
| GIC | 8.0% | 5.1% | 10.8% | 8.8% | 8.7% | 15.4% | 10.0% | 16.7% | |
| Hedge Fund | 5.0% | 8.5% | 5.4% | 6.2% | 4.2% | 6.7% | 5.1% | 3.8% | |
| Others (J-REIT etc.) | 3.7% | 4.3% | 1.9% | 4.0% | 3.7% | 1.9% | 1.5% | 2.0% | |
| Cash | 2.7% | 2.6% | 6.0% | 2.3% | 2.3% | 2.1% | 1.9% | 2.3% | |
| Total | 100% | 100% | 100% | 100% | 100% | 100% | 100% | 100% | |
| Historical Return | 7.3% | 8.0% | 6.7% | 7.6% | 6.7% | 6.7% | 6.1% | 6.3% | |
| Historical Volatility | 5.3% | 5.9% | 5.3% | 5.7% | 5.0% | 5.1% | 4.8% | 5.0% | |
| Lambda | 25.82 | 23.06 | 23.60 | 23.63 | 27.11 | 25.76 | 26.09 | 25.15 | |
| Risk Aburese | 40.9154 | 32.93633 | 38.74863 | 35.7543 | 47.73054 | 42.29545 | 47.70335 | 44.4373 | |

● Utility Function for each Funding Ratio level



Utility Assumption:

$$\sum_m \left[p \frac{(\varphi_m y_m)^{v_+}}{v_+} - \lambda \frac{(-\varphi_m y_m)^{v_-}}{v_-} (1-p) \right]$$

φ_m : ratio of risky asset vs. total asset value at month m

p: probability of positive return

$$u(x(m)) = \begin{cases} \frac{x^{v_+}}{v_+} & (=u_+(x)) \text{ if } x \geq 0, \\ -\lambda \frac{(-x)^{v_-}}{v_-} & (=u_-(x)) \text{ if } x < 0, \end{cases}$$

$$\text{Utility } U = \sum_m [u(x(m))]$$

v_+ CRRA Utility, Risk Averseness for positive returns

v_- CRRA Utility, Risk Averseness for negative returns

λ Parameter for positive returns vs. negative returns

| | | | | | |
|-----------------|--------|-------|-------|-------|-------|
| p | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| lambda | 5 | 5 | 5 | 5 | 5 |
| Funding Ratio | 75% | 85% | 95% | 105% | 115% |
| my ⁺ | 0.000 | 0.100 | 0.250 | 0.550 | 0.666 |
| my ⁻ | 10.000 | 4.761 | 4.860 | 4.051 | 3.934 |

Thank you for your kind attention.