The Leveled Chain Ladder Model for Stochastic Loss Reserving

Appears in CAS E-Forum with R/JAGS Code
http://casact.org/pubs/forum/12sumforum/

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Presented at the ASTIN Colloquium
October 2, 2012
S&P Report, November 2003
Insurance Actuaries – A Crisis in Credibility

“Actuaries are signing off on reserves that turn out to be wildly inaccurate.”
How to Approach the Problem

• There are a LOT of different approaches to stochastic loss reserving derived from any number of “reasonable” assumptions.

• First step – Decide on how to evaluate stochastic loss reserve models.

• Appeal to the “Gold Standard” for predictive modeling – Performance on holdout data.
Criteria for a “Good” Stochastic Loss Reserve Model

• Using the upper triangle “training” data, predict the distribution of the outcomes in the lower triangle

• Using the predictive distributions, find the percentiles of the outcome data.

• The percentiles should be uniformly distributed.
  
  – Histograms
  
  – Test with PP Plots/KS tests
    
    • Plot Expected vs Predicted Percentiles
Illustrative Tests of Uniformity
The CAS Loss Reserve Database

• Schedule P (Data from Parts 1-4) for several US Insurers
  – Private Passenger Auto
  – Commercial Auto
  – Workers’ Compensation
  – General Liability
  – Product Liability
  – Medical Malpractice (Claims Made)

• Available on CAS Website
  
  http://www.casact.org/research/index.cfm?fa=loss_reserves_data
The CAS Loss Reserve Database

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<thead>
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<th>Accident Year</th>
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<td>1997</td>
<td>xxx</td>
<td>xxx xxx xxx</td>
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</table>

Training Data from 1997 Schedule P

Outcome Data from Later Schedule Ps

- Can we predict the distribution of outcomes? Or sums of outcomes?
History

  – Coauthored with Peng Shi
  – Described the CAS Loss Reserve Database and (1) demonstrated two models that don’t validate on incremental paid data, and (2) Found that reported loss reserve estimates were more accurate than the models above.
Meyers – Shi 2011
Tested Two Models on Incremental Paid Data for Commercial Auto

**BAT Model**
Bayesian Autoregressive Tweedie

**BCL Model**
Bootstrap Chain Ladder
Lessons from History

• Subsequent to Meyers-Shi, I investigated what might have gone wrong.
  – Concluded that the incurred information available to company actuaries could explain the increased accuracy.
  – Decided to start over armed with the lessons learned

• Features of the start over
  – Work with both paid and incurred data and compare results
    • Use models based on cumulative rather than incremental data.
  – Use Bayesian models implemented with the latest MCMC algorithms.
  – Aggressive retrospective testing of the predictive distribution on actual outcomes
    • Use the CAS Loss Reserve database
Design of Retrospective Test
For 50 Insurers in CA, PA, WC and OL

- Estimate the predictive distribution of the cumulative reported claims, $C_{w,10}$, at accident year $w$ and development year 10, for each insurer using both models.

$$\sum_{w=2}^{10} C_{w,10}$$

- Calculate the percentile of the reported sum for each insurer using a model.

- Test the uniformity of the calculated percentiles for both models.

- Goal is to find a model that passes this uniformity test.

- It may not be possible. Consider an environment where losses are repeatedly influenced by different “black swan” events that we, the modelers, and the “data” do not see coming.
Analysis of the Mack Model

• Example on illustrative triangle
  – On real data from CAS Loss Reserve Database
  – Use the R ChainLadder Package

• Fit with both incurred and paid triangles for all 50 insurers in each line of insurance

• Calculate percentiles of outcomes

• Test for the uniformity of percentiles
  – For each line
  – For all lines combined
The Illustrative Triangle

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<th>3</th>
<th>4</th>
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### Results for Mack on the Illustrative Triangle

<table>
<thead>
<tr>
<th>$w$</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>CV</th>
<th>Actual</th>
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<td>Total $w=2-10$</td>
<td>34,997</td>
<td>1,057</td>
<td>0.0302</td>
<td>36,144</td>
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</tbody>
</table>

The table above shows the results for Mack on the Illustrative Triangle. Each row represents a different value of $w$, with columns for the estimate, standard error, coefficient of variation (CV), and actual value. The total for $w=2-10$ is also provided at the bottom.
Uniformity Tests with 50 Insurers
Uniformity Tests with 50 Insurers

Workers Comp - Mack Model

Predicted Percentile of Incurred Claims

Other Liability - Mack Model

Predicted Percentile of Incurred Claims

Workers Comp - Mack Model

Oops!

Expected Percentile for Incurred Claims

Other Liability - Mack Model

Expected Percentile for Incurred Claims
Uniformity Tests with 4 x 50 Insurers

PA+CA+WC+OL - Mack Model

Predicted Percentile of Incurred Claims

Frequency
Uniformity Tests with 4 x 50 Insurers

PA+CA+WC+OL - Mack Model

Oops!
Conclusion - Mack Model

• Incurred data
  – Mack model underestimates the thickness of the tails

• A better model will have greater variability in the tails
  – Ways to increase variability
    • Bayesian Estimation
    • Correlations
The Case for Bayesian Estimation

- Mack gives a point estimation of variance
- Bayesian estimate of variance with posterior $P$:

$$\text{Var} \left[ C_{w,d} \right] = E_P \left[ \text{Var} \left[ C_{w,d} \mid P \right] \right] + \text{Var}_P \left[ E \left[ C_{w,d} \mid P \right] \right]$$

- The last term: $\text{Var}_P \left[ E \left[ C_{w,d} \mid P \right] \right]$
  - Is equal to ZERO for a point estimate
  - Increases as we increase the number of parameters
  - Decreases as we increase the number of data points

- Loss Reserve Models – Many parameters, few data points
- See the “Thinking Outside the Triangle” paper.

http://www.actuaries.org/ASTIN/Colloquia/Orlando/Papers/Meyers.pdf
Calculating the Bayesian Posterior Using MCMC Sampling

• Generates samples from the posterior distribution
• MCMC sampling can be used to quantify the variability for many Bayesian models with few restrictions on the model specification.
  – There are freely available software packages to help with this. I use JAGS.
• JAGS generates a list (or distribution) of parameters, R then takes over to simulate a distribution of quantities of interest, such as loss reserve outcomes.
• See my AR column

  http://casact.org/newsletter/index.cfm?fa=viewart&id=6352
The Leveled Chain Ladder Model
Version 2

- $C_{1,d} \sim \text{lognormal}(\alpha_w + \beta_d, \sigma_d)$
- $C_{w,d} \sim \text{lognormal}(\alpha_w + \beta_d + z \cdot (\log(C_{w-1,d}) - \alpha_{w-1} - \beta_d), \sigma_d)$
- $z \sim U(-1,1)$
- $\alpha_w$ and $\beta_d$ are uniformly distributed ($\beta_1 = 0$).
- $\sigma_d \equiv \sum_{i=d}^{10} a_i \quad a_i \sim U(0,1)$
  Guarantees $\sigma_d$ decreases as $d$ increases
- Estimate distribution of $\sum_{w=2}^{10} C_{w,10}$
- If we set $z \equiv 0$ we get LCL Version 1
Using MCMC Models

1. Get \( \{\alpha_w\}, \{\beta_d\}, \{\sigma_d\} \) and \{z\} from JAGS output

2. Randomly select \( C_{w,10} \) from a lognormal distribution with

   - log mean = \( \alpha_w + \beta_{10} + z \cdot (\log(C_{w-1,10}) - \alpha_{w-1} - \beta_{10}) \)
   - log standard deviation = \( \sigma_{10} \)

3. Form the sum \( \sum_{w=2}^{10} C_{w,10} \)

4. Repeat the above 10,000 times.
LCL v2 on Illustrative Triangle

Histogram of z

The z’s tend to be positive indicating positive correlation between accident years.
# Results for LCL v2 and Mack on the Illustrative Triangle

<table>
<thead>
<tr>
<th>$w$</th>
<th>Leveled Chain Ladder V2</th>
<th>Chain Ladder/Mack</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>Std. Error</td>
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<tr>
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<tr>
<td>Total $w=2-10$</td>
<td>34,918</td>
<td>2,192</td>
</tr>
</tbody>
</table>
What Have We Accomplished?

• Found a model that validates on holdout data consisting of 200 separate triangles!
  – Bayesian
  – Correlations across development year
    • Consequence of random level parameters
  – Correlations across accident years
    • Autoregressive model linking prior accident years
  – Works with incurred claim data
Paid Claims Data

• Differences between Paid and Incurred Data in Schedule P
• Incurred = Paid + Case Reserves
• Case Reserves take into account information known to claims adjusters.
• Frequent decreases in Case Reserves resulting in negative incremental claims amounts.
Meyers – Shi 2011
Tested Two Models on Incremental Paid Data for Commercial Auto

**BAT Model**
Bayesian Autoregressive Tweedie

**BCL Model**
Bootstrap Chain Ladder
Suggested Conclusions Subject to Debate

• Incurred data contain information that is relevant for stochastic loss reserving that paid data do not.
• Mack model understates variability.
• Models (e.g. ODP/Bootstrap) that depend on incremental paid data will fail because they look at the wrong data.
"Aggressive" Retrospective Testing

"If you want something done right, you have to live in the past."

But will it work in the future??