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The fundamental definition of the Solvency Capital Requirement in Solvency II

Marcus C. Christiansen, Andreas Niemeyer | October 2, 2012 |
Institute of Insurance Science, University of Ulm, Germany

Agenda

Definition of the Solvency Capital Requirement (SCR)

Directive of the European Parliament and the Council

- Article 101: the SCR “shall correspond to the Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period”
- Citation 64: “the Solvency Capital Requirement should be determined as the economic capital to be held by insurance ... undertakings in order to ensure ... that those undertakings will still be in a position with a probability of at least 99.5%, to meet their obligations to policy holders and beneficiaries over the following 12 months.”

Definition of the SCR

Possible interpretations of the Solvency II framework:

Article 101

- $SCR_0^{a1} := \mathrm{VaR}_{0.995}\Big(N_0 - v^{rl}(0,1)N_1\Big)$
- $SCR_0^{a2} := \mathrm{VaR}_{0.995}\Big(N_0 - v^{co}(0,1)N_1\Big)$
- $SCR_0^{b} := \mathrm{VaR}_{0.995}\Big(\mathbb{E}_{\mathrm{Q}}\big(v^{rl}(0,1)N_1\big) - v^{rl}(0,1)N_1\Big)$

Citation 64

- $SCR_0^{c} := \inf\big\{N_0 \in \mathbb{R} : \mathrm{P}\left(N_1 \geq 0\right) \geq 0.995\big\}$

where v^{rl} is the riskless discount factor and v^{co} the company specific discount factor

Questions

(1) Are (some of) the definitions equivalent?

(2) Are Article 101 and Citation 64 consistent?

(3) If the different definitions cannot be harmonized, are there other arguments that support or disqualify some versions?

(4) Is it possible to solve the circular reference in the definition of the Risk Margin?

Example

$$\begin{tikzcd} & 105 \\ K^1: \; 100 \arrow[ur, "\omega_1"] \arrow[dr, "\omega_2"'] & \\ & 105 \end{tikzcd} \qquad \begin{tikzcd} & 120 \\ K^2: \; 100 \arrow[ur, "\omega_1"] \arrow[dr, "\omega_2"'] & \\ & 90 \end{tikzcd}$$

- the insurance company holds two shares of stock K^2
- the company has to pay after one year exactly one stock to a policy holder
- $SCR_0^{a1} = SCR_0^b = \frac{100}{7}$
- $SCR_0^{a2} = SCR_0^c = 0$

Which SCR is adequate here, zero or greater than zero?

Discussion of definition c

Theorem 1

Given that $\widetilde{N}_0$ is deterministic, we have

$$\begin{aligned} SCR_0^c &= \inf\left\{\widetilde{N}_0 \in \mathbb{R} : \mathrm{P}(\widetilde{N}_1 \geq 0) \geq 0.995\right\} \\ &= \mathrm{VaR}_{0.995}\left(N_0 - v^{ad}(0,1)N_1\right) \end{aligned}$$

- $v^{ad}(0,1)$ is the discount factor of the additional assets
- the mathematical structure of Article 101 and Citation 64 are similar

Relation between definition a1 and b

Theorem 2

We assume a risk-neutral measure Q and a discount factor v^{rl} such that $K_t = \mathbb{E}_{\mathrm{Q}}(v^{rl}(t,u)\,K_u|\mathcal{F}_t)$ for all $u \geq t \geq 0$ and such that $L_t = \int_{(t,\infty)} \mathbb{E}_{\mathrm{Q}}(v^{rl}(t,u)\,\mathrm{d}(-Z_u - Y_u)|\mathcal{F}_t)$ for all $t \geq 0$.

Then we have $N_s = \mathbb{E}_{\mathrm{Q}}(v^{rl}(s,s+1)\,N_{s+1}|\mathcal{F}_s)$ for all $s \geq 0$ and

$$\begin{aligned}
SCR_s^{a1} &= \mathrm{VaR}_{0.995}\Big(N_s - v^{rl}(s,s+1)\,N_{s+1}\Big|\mathcal{F}_s\Big) \\
&= \mathrm{VaR}_{0.995}\Big(\mathbb{E}_{\mathrm{Q}}(v^{rl}(s,s+1)\,N_{s+1}|\mathcal{F}_s) - v^{rl}(s,s+1)\,N_{s+1}\Big|\mathcal{F}_s\Big) \\
&= SCR_s^b\,.
\end{aligned}$$

Relation between the SCR definitions

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
SCR^{a1} \arrow[d, shift right, "\mathbb{Q}\text{ exists}"'] & SCR^{a2} \arrow[d, leftrightarrow, "v^{ad} := v^{co}"] \\
SCR^{b} \arrow[u, shift right] \arrow[r, shift left, "v^{ad} := v^{rl}"] & SCR^{c} \arrow[l, shift left, "v^{ad} := v^{rl}\ \&\ \mathbb{Q}\text{ exists}"] \arrow[ul, leftrightarrow, "v^{ad} := v^{rl}"']
\end{tikzcd}
$$

Example

- insurance company has assets of 200 and has to fulfill insurance liabilities of 105 in one year
- the share ratio of the additional assets is 0.4
- free surplus today: 100 and $y^{(1)} = SCR_0^{a1} = \frac{100}{7} \approx 14.3$
- assets can be reduced to $100 + \frac{100}{7}$ (additional assets are paid out according to the strategy that corresponds to v^{ad})
- new SCR $y^{(2)} = \frac{340}{49} \approx 6.9$
- and $y^{(3)} \approx 6.4$
- ... (repeating this procedure infinitely often)
- converges to $6.25 = SCR_0^c$

Is this always the case?

Convergence

We consider the following framework:

- $y^{(1)} := SCR_s^a = \mathrm{VaR}_{0.995}\left(N_s - v(s,s+1)\, N_{s+1} \middle| \mathcal{F}_s\right)$
- $y^{(n)} = \mathrm{VaR}_{0.995}\left(y^{(n-1)} - v(s,s+1)\left(N_{s+1} + (y^{(n-1)} - N_s)v^{ad}(s,s+1)^{-1}\right) \middle| \mathcal{F}_s\right)$

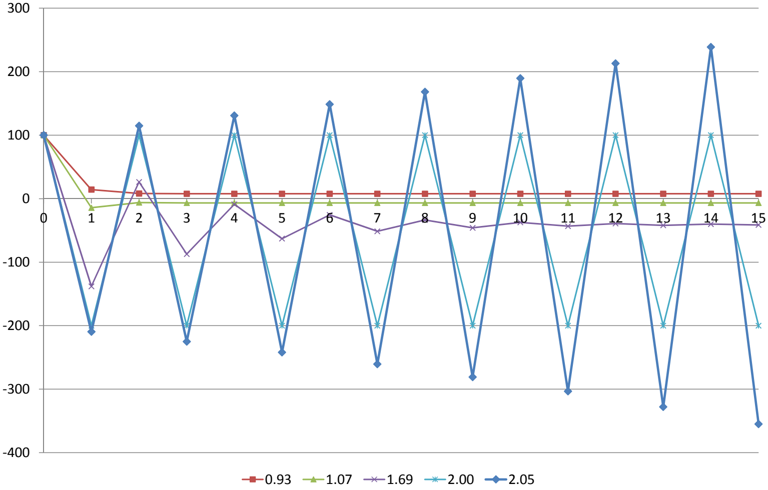
Theorem 3

If there exits an $\epsilon \in (0,1)$ such that

$$\epsilon < \frac{v(s,s+1)}{v^{ad}(s,s+1)} < 2 - \epsilon\,,$$

then $\lim_{n\to\infty} y^{(n)} = SCR_s^c$.

Speed of convergence



Risk margin

- "The value of technical provisions shall be equal to the sum of a best estimate and a risk margin." (Paragraph 1 of article 77)
- "The risk margin shall be calculated by determining the cost of providing an amount of eligible own funds equal to the Solvency Capital Requirement necessary to support the insurance and reinsurance obligations over the lifetime thereof." (Paragraph 5 of article 77)
- definition of the risk margin in QIS 5

$$RM = c \sum_{k \geq 0} \frac{SCR_k^{RU}}{(1 + r_{k+1})^{k+1}},$$

Risk margin definition

Definition 1

The RM at time s is defined as

$$RM_s := \sum_{k \geq s} \mathbb{E}_{\mathrm{Q}/\mathrm{P}}\left(c(k)\, v^{rl}(s, k+1)\, SCR_k^{RU} \middle| \mathcal{F}_s\right),$$

where $c(k)$ is the cost-of-capital rate at time k. Since the choice of the measure is unclear, we write Q/P to indicate that a mixture of both measures is likely.

Risk margin circular reference

Theorem 4

The SCR of a reference undertaking at time s can be calculated without a circular reference, more precisely

$$SCR_s^{RU} = \frac{1}{1 + \mathbb{E}_{\mathrm{Q/P}}\left(c(k)\, v^{rl}(s, s+1)\middle|\mathcal{F}_s\right)} \mathrm{VaR}_{0.995}\left(\Lambda_s|\mathcal{F}_s\right) \; ,$$

where

$$\begin{aligned} \Lambda_s :=& (A_s - B_s) - v(s, s+1)(A_{s+1} - B_{s+1}) \\ &+ v(s, s+1) \sum_{t \geq s+1} \left(E_{s+1,t} - E_{s,t}\right) \text{ and} \\ E_{s,t} :=& \mathbb{E}_{\mathrm{Q/P}}\left(c(t)\, v^{rl}(s, t+1)\, SCR_t^{RU}\middle|\mathcal{F}_s\right), \quad t \geq s \, . \end{aligned}$$

Conclusion

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(2) Are Article 101 and Citation 64 consistent?

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(4) Is it possible to solve the circular reference in the definition of the Risk Margin?

Contact

Institute of Insurance Science

University of Ulm

Andreas Niemeyer

andreas.niemeyer@uni-ulm.de

Thank you very much for your attention!