New Directions
in the Modelling of Longevity Risk

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Plan

- Motivation
- Genealogy
- New directions in modelling
- Numerical illustrations – single population models
- Remarks on multiple populations
Motivation

- Application focus:
  - risk measurement and management of longevity risk
  - multiple populations
  - life insurance diversification benefits
  - basis risk in standardised longevity contracts
- industry requires robust models
Development of New Models

- Many new stochastic mortality models since Lee-Carter
- Are they fit for purpose?
- Are they robust?
GENEALOGY: 1st GENERATION MODELS

Lee-Carter (M1)
1992

Currie/Richards (M4)
2-D P-splines
2002, ...

Eilers/Marx
P-splines

CBD-1 (M5)
2006

Time
Improvements + more complexity

- Currie/Richards (M4)
  - 2-D P-splines
- Eilers/Marx
  - P-splines
- APC model (M3)
  - Booth et al.
- Hyndman et al.
- APC model (M3)
- Lee-Carter (M1)
  - Renshaw-Haberman (M2)
  - CBD-1 (M5)
  - CBD-2 (M6)
  - CBD-3 (M7)
  - CBD-4 (M8)
- DDE
More improvements + even more complexity

- APC model (M3)
- Eilers/Marx
- 2-D P-splines
- Booth et al.
- Hyndman et al.
- Lee-Carter (M1)
- Renshaw-Haberman (M2)
- Currie/Richards (M4)
- 2-D P-splines
- DDE
- Plat
- CBD-1 (M5)
- CBD-2 (M6)
- CBD-3 (M7)
- CBD-4 (M8)

Time
Multiple population modelling

- APC model (M3)
- Lee-Carter (M1)
- Renshaw-Haberman (M2)
- CBD-1 (M5)
- CBD-2 (M6)
- CBD-3 (M7)
- CBD-4 (M8)

- Currie/Richards (M4)
- 2-D P-splines
- DDE
- Hyndman et al.
- Booth et al.
- Plat

- Eilers/Marx
- P-splines

- Multi-population

Time
Why do we need complexity?

Lee–Carter Model

CBD Model + Cohort Effect

Black $\Rightarrow$ model *over*-estimates $m(x, t)$ death rate

Gray $\Rightarrow$ model *under*-estimates $m(x, t)$ death rate

LC: non-random clusters $+$ errors are too big
Issues on complexity

- More complex $\Rightarrow$ More random processes
- More random processes $\Rightarrow$
  MUCH more difficult to model multiple populations
- Excessive complexity $\Rightarrow$
  potentially less robust forecasts
A Possible Way Forward

**Single-population models**

- Focus on **small number of key drivers**
  => much easier to extend to multi-populations

- Focus on greater robustness of forecasts
Case Study: CBD/Plat Revisited

\[ \log m(x, t) = \beta(x) + \kappa_1(t) + \kappa_2(t)(x - \bar{x}) \]

Red ⇒ actual deaths > expected deaths
CBD/Plat Revisited: **Key Idea: Possible responses**

\[
\log m(x, t) = \beta(x) + \kappa_1(t) + \kappa_2(t)(x - \bar{x})
\]

**Add:**

- Cohort effect, \(\gamma(t - x)\)
- Extra age-period effects
- Do something new ......
Key Idea: CBD/Plat Revisited

Underlying \( \log m(x, t) = \)

- \( \beta(x) + \kappa_1(t) + \kappa_2(t)(x - \bar{x}) \): two key drivers

PLUS

\( R(x, t) \) Residuals

- Assume: vector \( R(t) \rightarrow R(t + 1) \) mean reverting process

\( \Rightarrow \) long term risk depends on two key drivers
Specific Model

\[ \log m(x, t) = \beta(x) + \kappa_1(t) + \kappa_2(t)(x - \bar{x}) + R(x, t) \]

- \((\kappa_1(t), \kappa_2(t))\): bivariate random walk

- \(R(t) = (n_x \times 1\ \text{vector})\ \text{VAR(2)},\ \text{reverting to 0}\)
  \[ R(t) = AR(t - 1) + BR(t - 2) + Z(t) \]

- \(Z(x, t) \sim \text{i.i.d.} \mathcal{N}(0, \sigma_Z^2)\)

- \(A = A_1 + A_2\) and \(B = -A_2A_1\)
VAR matrices $A_1$ and $A_2$

$$A_i = \begin{pmatrix}
a_i & 0 & 0 & \cdots \\
c_i & d_i & 0 & 0 & \cdots \\
d_i/2 & c_i & d_i/2 & 0 & 0 & \cdots \\
0 & d_i/2 & c_i & d_i/2 & 0 & 0 & \cdots \\
0 & 0 & d_i/2 & c_i & d_i/2 & 0 & 0 & \cdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots
\end{pmatrix}$$

$a_i =$AR terms for new members;

$c_i =$ cohort persistence;

$d_i =$ diffusion coeff.
Further details

- Deaths: $D(x, t) \sim \text{Poisson} \left( m(x, t)E(x, t) \right)$

- Bayesian approach:
  
  posterior density $= \text{likelihood} \times \text{prior}$

- Upcoming results: mode of posterior density

- Further work: Bayesian parameter uncertainty
England and Wales, Males 1971–2008

Death Rate (log scale)

Age 85
Age 75
Age 65
Age 55

Underlying trend, R(x,t)=0

R(x,t) = 0

FULL

Year

1980 2000 2020 2040 2060
Cohort-type effects

England and Wales, Males 1971–2008

Death Rate (log scale)

Year

1980 2000 2020 2040 2060

0.002 0.005 0.010 0.020 0.050 0.100 0.200

FULL

R(x,t) = 0
Red ⇒ $Z(x, t) > 0$
D(x,t): Actual vs Expected Deaths

Red ⇒ actual > expected
Comparison with related models

\[ \log m(x, t) = \beta(x) + \kappa_1(t) + \kappa_2(t)(x - \bar{x}) \]

\[ \log m(x, t) = \beta(x) + \kappa_1(t) + \kappa_2(t)(x - \bar{x}) + \gamma(t - x) \]

\[ \log m(x, t) = \beta(x) + \kappa_1(t) + \kappa_2(t)(x - \bar{x}) + R(x, t) \]

\[ R(t) = AR(t - 1) + BR(t - 2) + Z(t) \] (\(A, B\) as specified earlier)

\[ \log m(x, t) = \beta(x) + \kappa_1(t) + \kappa_2(t)(x - \bar{x}) + R(x, t) \]

\[ R(t) = AR(t - 1) + BR(t - 2) + Z(t) \] (simplified \(A, B\))
Conclusions: Model Comparisons

- Long term underlying trends ($\kappa(t)$) are reasonably consistent

- Model risk more evident in the mean reverting $R(x, t)$

Further work

- Bayesian parameter uncertainty

- Multiple populations: focus on underlying $\kappa(t)$
  $\Rightarrow$ less complexity
Multipopulations

Mortality – version 1:

- Population, $P$, specific $\kappa^{(P)}_i(t)$ correlated
- $R^{(P)}(x, t)$: assume independent

Mortality – version 2:

- All populations have the same $\kappa_i(t)$
- $R^{(P)}(x, t)$: assume independent
- Greater role for $R(x, t)$ as country specific effect
Questions

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Multipopulations

Borrow from multifactor asset models: e.g.

- Asset $i$ return: $R_i = \alpha_i + \beta_{i1} F_1 + \beta_{i2} F_2 + \epsilon_i$

- $F_1, F_2$ are systematic risk factors

- $\epsilon_i =$ idiosyncratic risks
Other models for $R(x, t)$

1. $R(x, t) = \phi R(x - 1, t - 1) + Z_R(x, t)$

2. $R(x, t) = \phi R(x - 1, t - 1) + \text{diffusion} + Z_R(x, t)$

3. Smooth underlying period effects, $\kappa_1(t), \kappa_2(t)$

   plus annual shocks

   e.g. $R(1), R(2), \ldots$ are i.i.d. vectors, correlated across ages
Issues on complexity

- Lee-Carter, CBD-1: simple and robust
  BUT underlying assumptions are violated:
  A: Deaths, $D(x, t)$ are cond. Poisson($m(x, t)E(x, t)$)
  B: Death counts in neighbouring $(x, t)$ cells are independent

- More complexity e.g. CBD-1 $\rightarrow$ CBD-3 $\rightarrow$ Plat ...
  - Underlying assumptions now okay
  - But excessive complexity $\Rightarrow$ less robust forecasts???

- Dowd et al. (2010a,b): out-of-sample backtesting

  Models that fit much better in sample
  are not obviously better at out-of-sample forecasting