Long-Term Insurance Products and Volatility under the Solvency II Framework

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Outline

Three parts:

i **Capital volatility under Solvency II for long-term products**

ii A capital consumption model driven solely by defaults

iii An option trading strategy protecting against extra volatility

Part I: The insurance contract

Consider a simple setup with the following pure savings liability L:

- Policyholder pays a single premium π_0 at time $t = 0$.
- At maturity (retirement), the policyholder (or the heir) receives a single payout 1.
- The policyholder cannot withdraw funds before maturity.

Part I: The backing asset

The liability L is backed by a defaultable bond A. The value of this bond with maturity m at time $t \in \mathbb{N}$ is given by:

$$B(t,m) = e^{-(m-t)s_t}\, q^{m-t}\, \Gamma_t\, P(t,m)$$

Three components:

i The price of the defaultable bond, $B(t,m)$, is lower than the price of a corresponding **risk free bond**, $P(t,m)$.

ii Γ_t, the **stochastic default process**. $\Gamma_t = 1$ if no default has happened up to time t, otherwise $\Gamma_t = 0$.
$q = 1 - p$ is the real world probability of survival of Γ_t during any time step.

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Three components:

iii Since we are interested in real world volatilities, we work with the real world deflator formalism.
Historically, we know that:

$$p_{\text{implied}}^{\text{risk neutral}} \gg p$$

with $p_{\text{implied}}^{\text{risk neutral}}$, the risk-neutral implied probability of default.
The **illiquidity spread process**, s_t, allows to accounts for this effect in the deflator formalism.

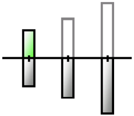
Part I: The consumption stream

The stakeholders:

- **The policyholder** pays for the product, and waives the right to receive early payment.
- **The insurer** promises to pay the policyholder at maturity without any possibility of default.
  The insurer invests the received premium in a bond.
- **The bond issuer** promises to pay the insurer at bond maturity. However, the bond issuer has the possibility to default.

The (capital) **consumption stream** is the cash flow between insurer and product, to maintain $A_t = L_t$.
The **liability value** is *defined* as a risk free bond $L_t = P(t, m)$.

Part I: Full Consumption Stream Model



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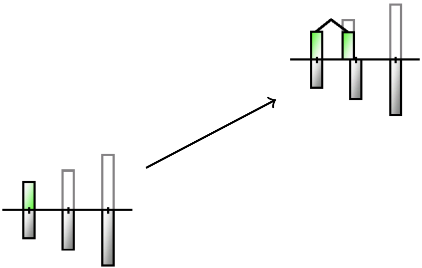
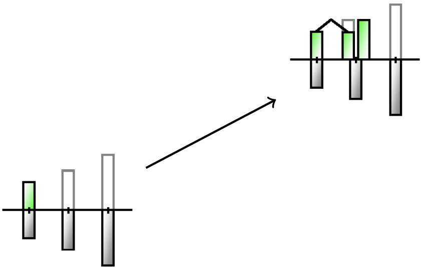


Part I: Full Consumption Stream Model

$C_1 < 0$, or $C_1 \geq 0$



Capital flow induced by spread fluctuations.

Part I: Full Consumption Stream Model

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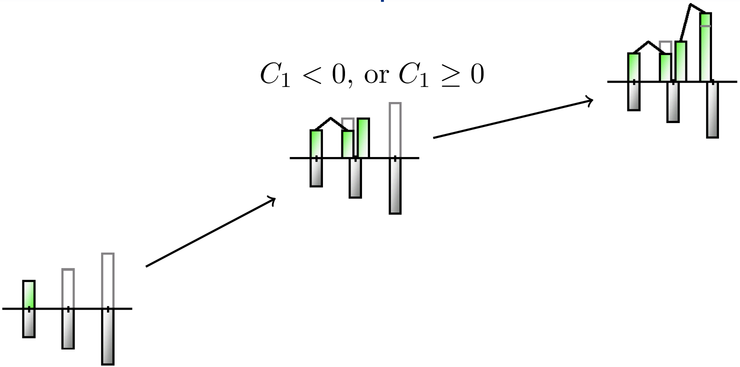

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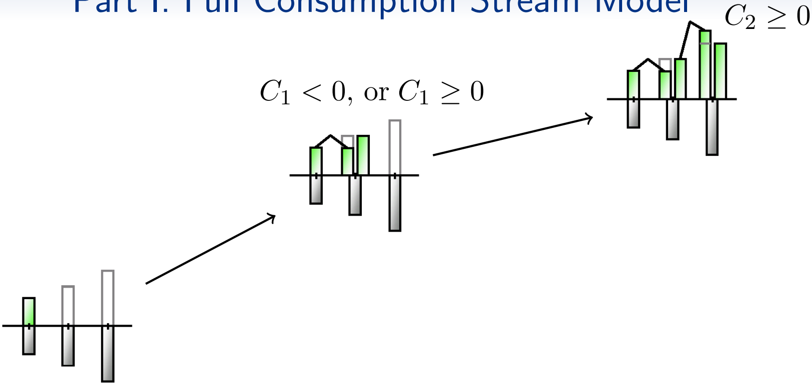

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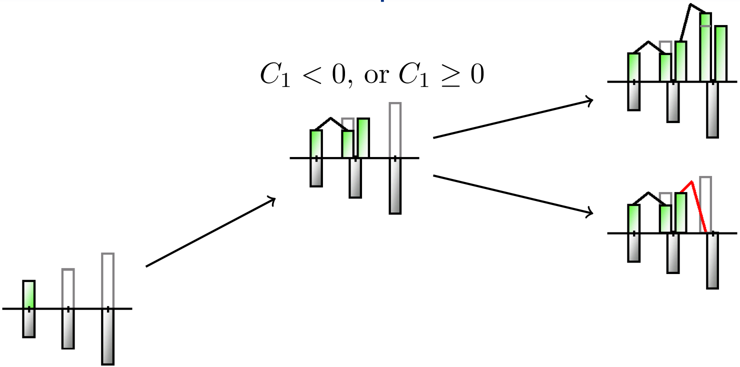

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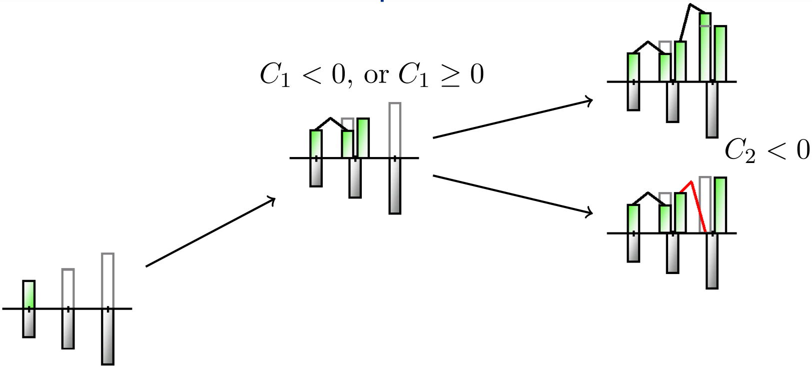

Capital flow induced by default.

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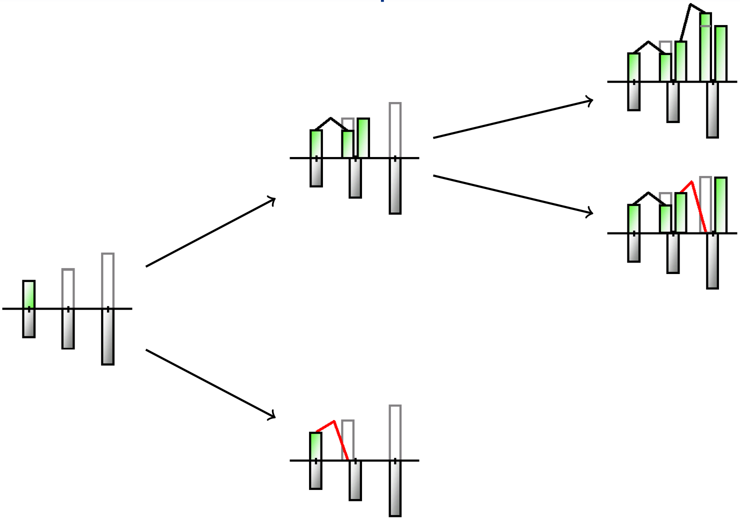

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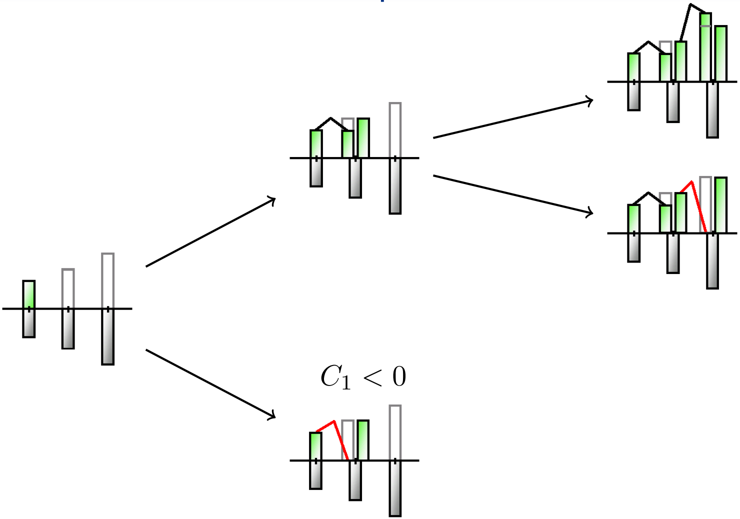

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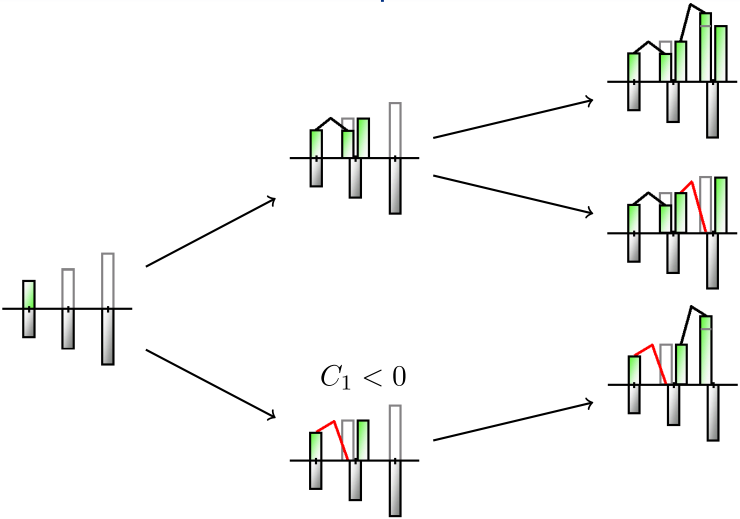

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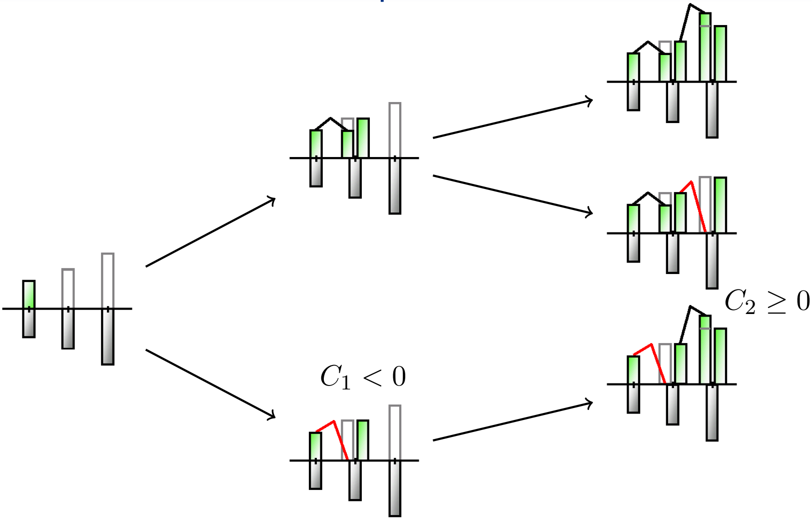

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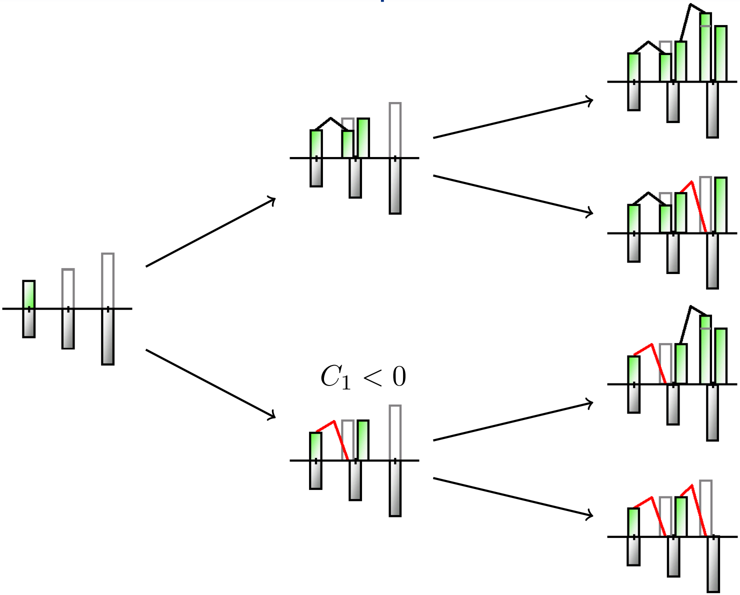

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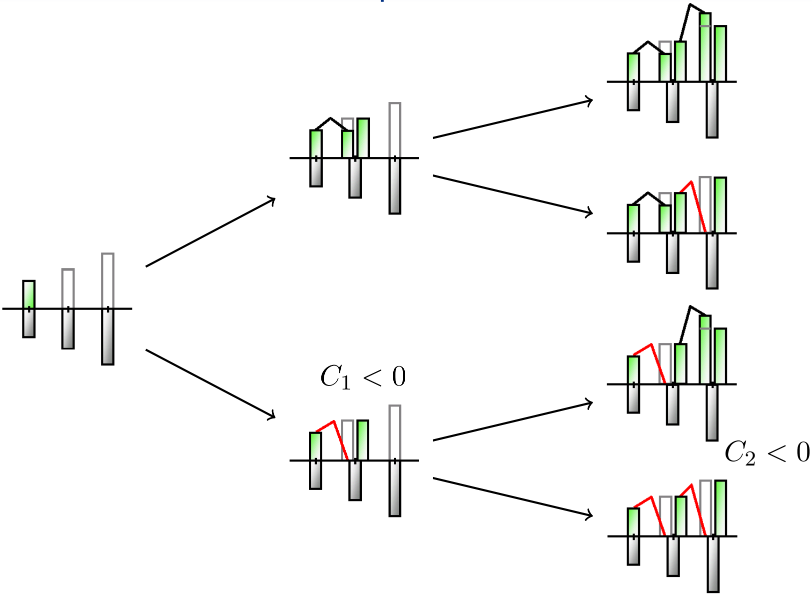

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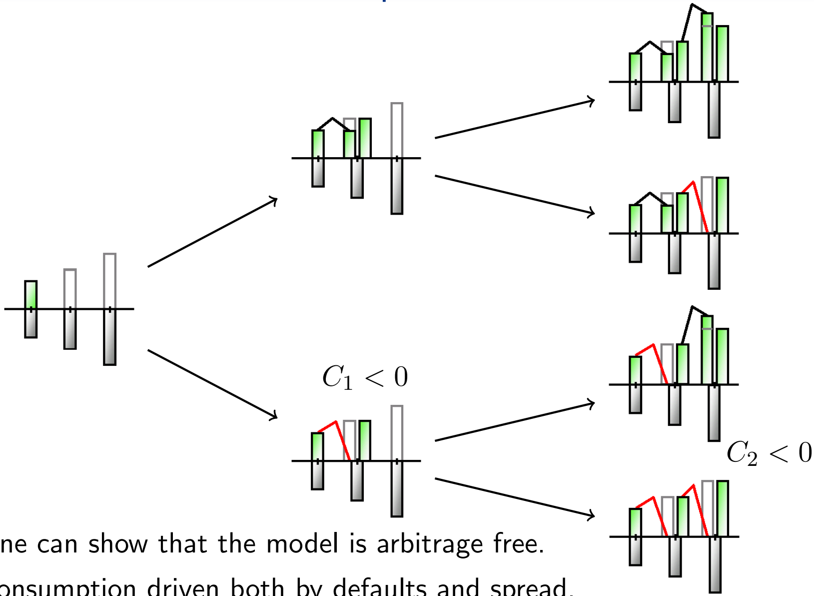

One can show that the model is arbitrage free.

Consumption driven both by defaults and spread.

Part I: Long-Term Consumption Stream Volatility

Extending the model from two time steps to longer maturities:

- A small variation in spread, has a big impact on the bond value. Thus, the spread becomes the dominant driver behind the consumption stream (not defaults).
- Indeed, the volatility of the consumption stream contains three contributions (under certain assumptions):

$$\begin{aligned} \mathbb{V}\mathrm{ar}[C_1] &= (\dots)\,\mathbb{V}\mathrm{ar}\left[e^{-(m-1)\,(s_1-s_0)}\right] \\ &\quad + (\dots)\,\mathbb{V}\mathrm{ar}[\Gamma_1] + (\dots)\,\mathbb{V}\mathrm{ar}\left[P(1,m)\right] \end{aligned}$$

A spread term, a term driven by the default process and a contribution from the term structure volatility.
In the paper, we show that the **spread term becomes dominant for longer maturities**.

Part I: Conclusions

- We conclude that under Solvency II: it will become more expensive to offer the insurance, as the maturity of the product increases.
- For example: If we compare a defined benefit insurance with maturities up to 40y with a savings product of 8y and a unit linked product, then the defined benefit product will relatively become much more expensive due to the much higher volatility of the consumption stream.
- For the unit linked product, it is the policyholder that is exposed to the market risk. A shift to such products in the market can be expected.

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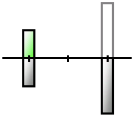
Part II: Reduced Consumption Stream Model



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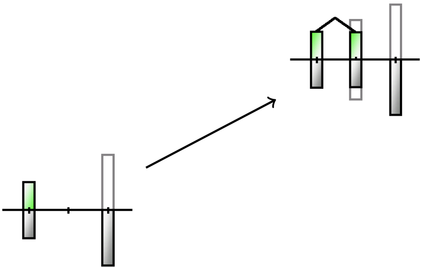

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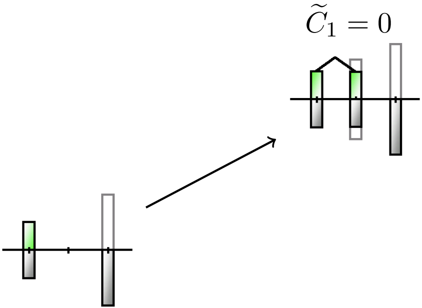
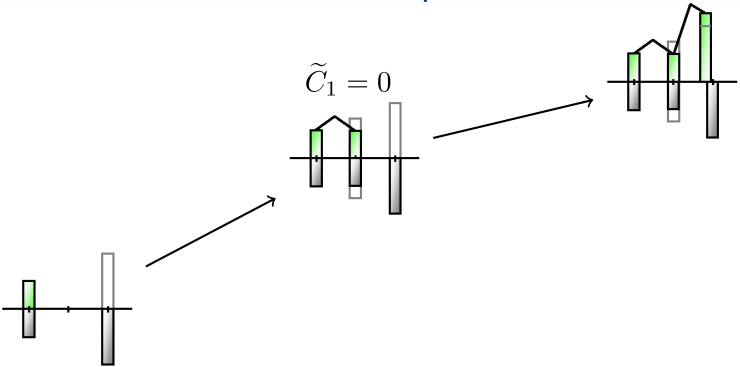


No default, thus no consumption stream.

Part II: Reduced Consumption Stream Model



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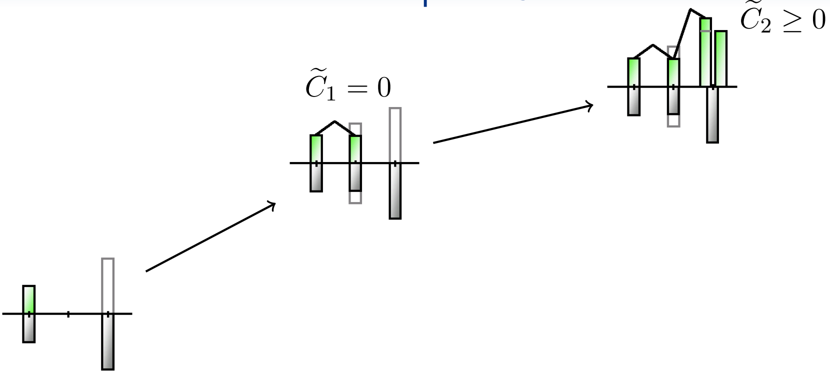

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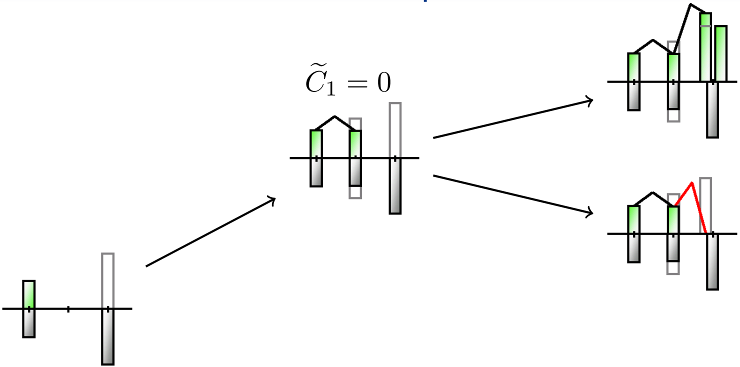

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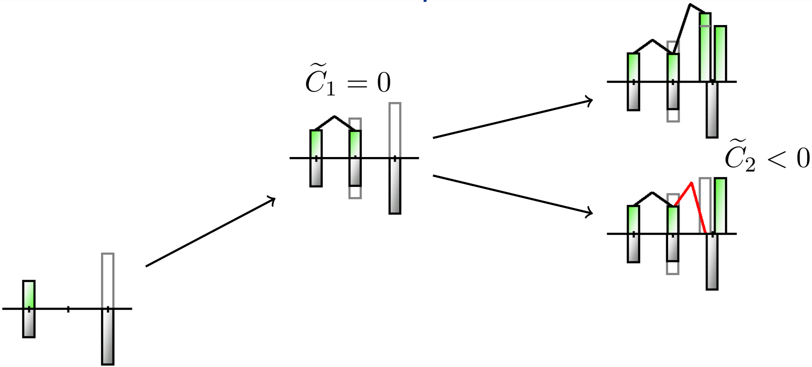

Capital flow induced by default.

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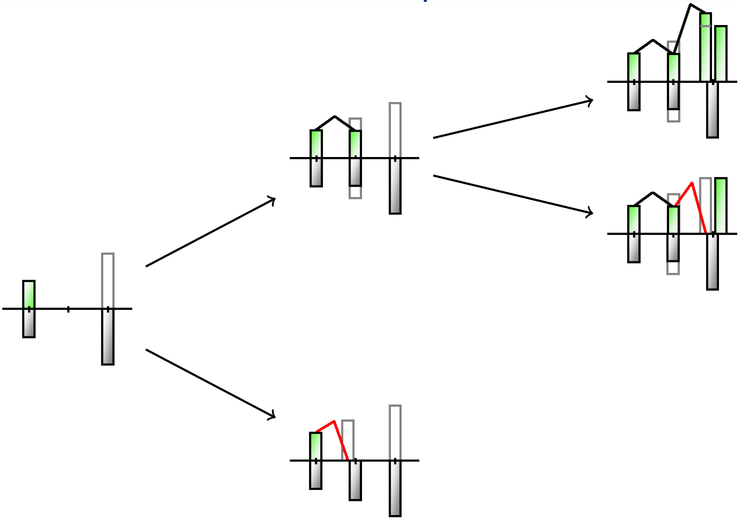

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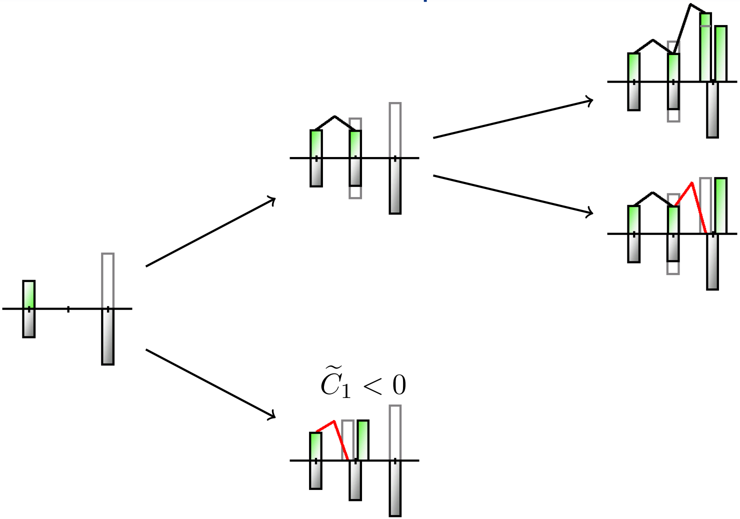

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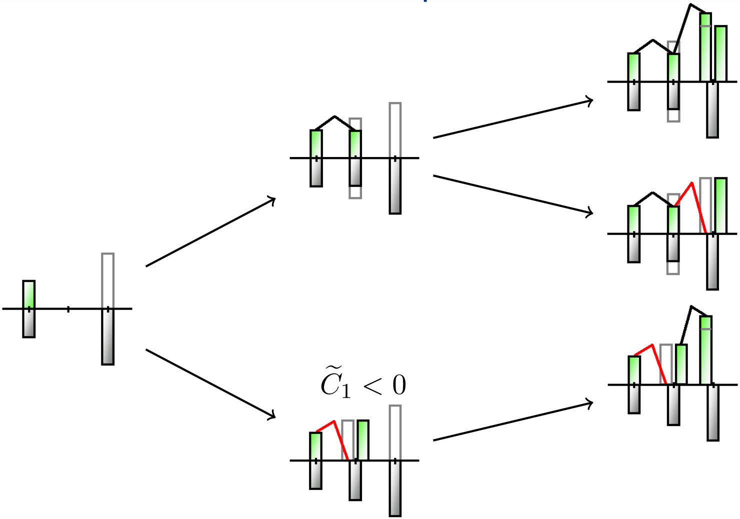

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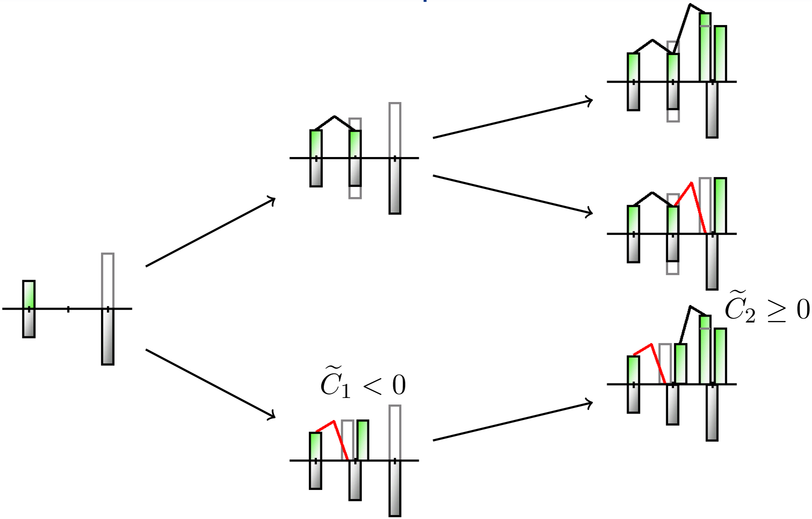

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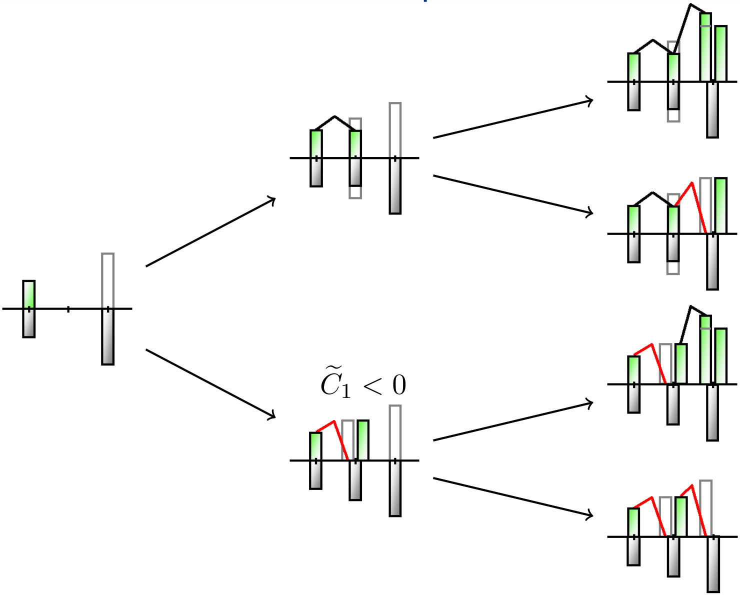

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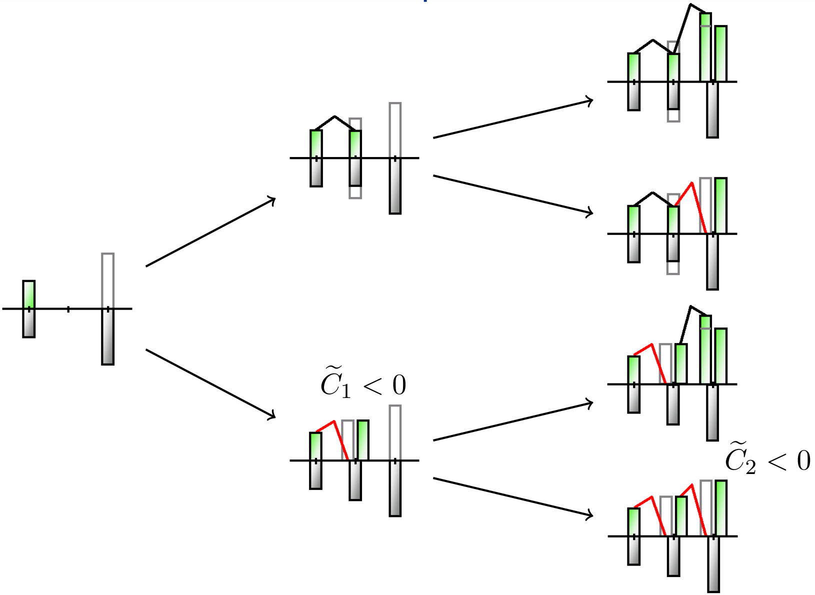

Part II: Reduced Consumption Stream Model

Since insurance liabilities are not traded, a market-consistent model needs to be constructed to derive the price (mark-to-model).
In the reduced consumption stream model:

- $\widetilde{C}_1 = 0$ unless a default occurred.
- Recall that the consumption stream ensures $A_t = L_t$.
- We thus **implicitly defined $\boldsymbol{L_1 = A_1}$**.
- Thus, the liability valuation is asset dependent.
- We have $A_1 = q^{-1}\, e^{2s_0 - s_1}\, P(1,2)$.
  Thus, normally $L_1 = A_1 > P(1,2)$, unless the spread s_1 increased significantly.

Part II: Reduced Consumption Stream Model

Given that we obtained a different liability valuation formula, we check the **no arbitrage** condition. In the paper, we prove that

- $Q_0[\widetilde{\boldsymbol{C}}] = \sum_{u=0}^{2} \mathbb{E}[\varphi_u \, \widetilde{C}_u] = 0$
- $Q_0[\boldsymbol{L}] = \sum_{u=0}^{1} \mathbb{E}[\varphi_{2u} \, L_{2u}] = 0$

  Notice that $Q_0[\boldsymbol{L}] = \sum_{u=0}^{2} \mathbb{E}[\varphi_u \, L_u] \neq 0$, but there is still no arbitrage possible, since a policyholder cannot trade the product at time 1 by design.

Thus, the reduced consumption stream is arbitrage free, for products where the benefits cannot be called in early.

Part II: Conclusions

Comparison full and reduced model:

- In case of default, the consumption stream in both models are equal since then $L_t = P(t, m)$ is imposed. The policyholder thus enjoys the same policyholder protection.
- The price and payout of the product are the same in both models.
- No capital is upstreamed at time 1 in the reduced model.
- The reduced model does not require a calibration of p nor s_t.
- Since by design, the consumption stream is only driven by defaults, the **volatility of the reduced consumption stream is lower**. This also holds if one extends the model to longer maturities.

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Part III: Downside risk

In the full model, a negative consumption stream can be caused by a default or by a spread movement. In the reduced model a negative consumption stream is only caused by a default.
Thus, the probability to have a negative consumption stream is larger in the full model than in the reduced model.

An insurer can protected itself against negative consumption streams either by

- constituting an **additional reserve** to absorb spread volatility,
- or by using an **option trading strategy** as protection.

Part III: An option against downside risk

An insurer willing to support the reduced consumption stream linked with a product, but operating under the constraint that $A_1 \geq P(1, 2)$, could supplement the reduced consumption stream with an option with the following payout:

$$\left[1 - (1 - p)^{-1} e^{2s_0 - s_1}\right]^+ \Gamma_1 P(1, 2)$$

The price of this option is exactly equal to the added cost of the full consumption stream model.
For longer maturities, this price will increase quickly.

Conclusions

i Solvency II valuation leads to high volatility for long-term products, because of the dominant illiquidity spread volatility.

ii An alternative market-consistent liability valuation model exists (for products where benefits cannot be called in early). In this model, the consumption stream is less volatile since it is driven by defaults alone (not spread).

iii An insurer can protect itself against the higher Solvency II volatility either with an additional reserve, or an option trading strategy. The price of the option gives the exact added cost linked with a shift to the Solvency II framework.

Thank you

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