Beyond Boundaries

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AFIR/ERM | IAA FINANCIAL RISKS AND ERM SECTION
SECTION DES RISQUES FINANCIERS ET GRE DE L'AAI

Parametrization of CIR interest rate model of zero coupon yield and its application for selecting the discount rate and using floorlets and caplets for postretirement and pension plans

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Research goal

To show an efficient procedure to determine the discount rate for pension liabilities in a long duration (immature) pension fund

Motivation

- As life expectancy increases, so does the tenor and duration of pension fund liabilities
- Since duration summarizes the interest rate sensitivity of the fund, M2M practice makes P&L more volatile
- Difficult to set up immunization strategies, since there is not enough liquidity in listed securities

Motivation

- Interest rate options might be found OTC
- Option valuation requires volatility parameter
- Volatility along the yield curve is not constant
- Interest rates usually converge to a long-run stationary level

Alternative solutions

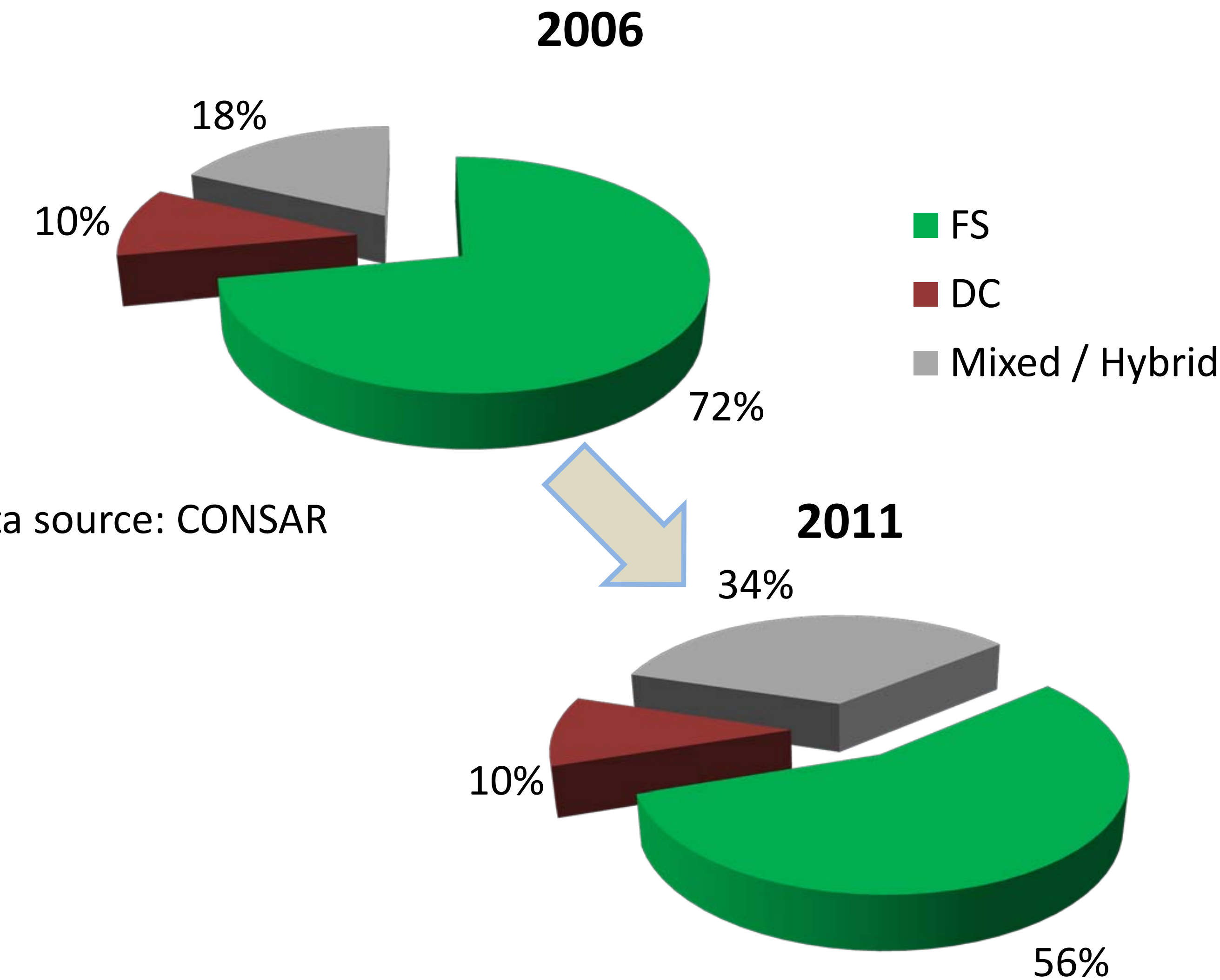
- To determine the most efficient hedge through floorlets/caplets over the unmatched liabilities on each node, or

- To take advantage of the long-run convergence of IR to a stationary level

Types of pension funds in México

- Defined Benefit –DB (Finally salary) → The benefit is well defined
  - Used to be funded by the population bond
- Defined contribution-DC → The outcome is uncertain due to capital P&L
  - The case of the current SS – RPFs
- Mixed funds → DB & DC funds coexist independently
- Hybrid funds → DC & DC are correlated
  - Floor plans

Migration of private funds in México over time

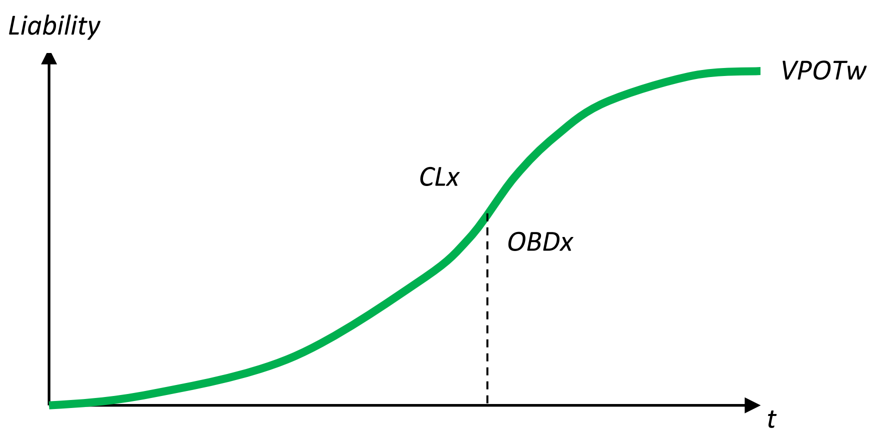


Data source: CONSAR

Population as of May 2011: 1,946 private funds

Valuation of DB pension liabilities

- There are 3 key related elements in DB pension liabilities valuation:
  - Total Liability Present Value (VPOT, in spanish)
  - Defined Benefit Liability (OBD, in spanish)
  - Service Cost (CL, in spanish)



$$\lim_{x \to w} OBD_x A = VPOT_w$$

Net cost nature

- The net cost (CNP) to be charged at P&L over a period is:

  CNP=CL + (CF + REAP)+APPA

  - Where CL is the service cost, CF is the financial cost, REAP is expected return on plan assets and APPA are amortizations of pending liabilities

- CL & CF are both sensitive to the discount rate

Main Issue for Discounting Liabilities

How should it be considered or calculated the discount rate??

Accounting issues to be considered

- Under IAS #19
  - Discount rate (high quality corporate bonds in deep market)
- Under FAS #87
  - As IAS or constructing a synthetic rate for maturity terms as the pension payments
- Under Mexican NIF D-3
  - As FAS #87 before 2006 that computations were held on a real basis (net of inflation), rather than nominal as today.

Warning: Emerging Economies still face high inflation risks!

Interest rate simulation

- The second simplest one-factor model for IR --> Cox-Ingersoll-Ross

$$dr_t = a(b - r_t) + \sigma r_t^{1/2} dW$$

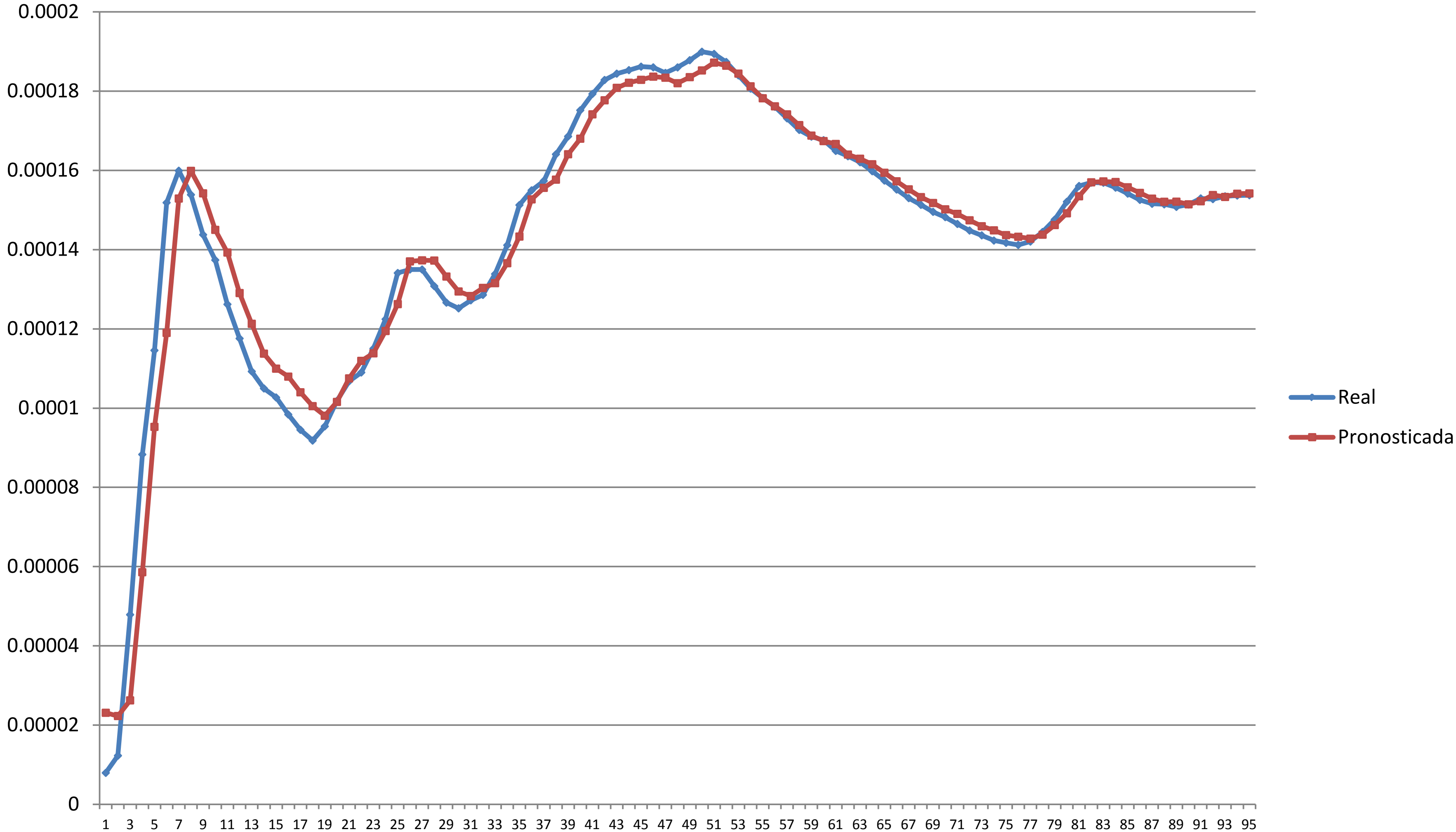
- To overcome the constant volatility, parameter is replaced by a stochastic volatility model

$$dr_t = a(b - r_t) + \sigma_t r_t^{1/2} dW$$

Interest rate simulation in Mexican Case

- GARCH(1,1) model was used to determine parameters for the stochastic volatility
- OLS regression was used to determine parameters for the short rate process
  - Leads to a long-run volatility consistent with stylized inflation, country risk, term risk and real rate
- Data sample: 96 monthly zero coupon curves (Jan'04 – Dec'11) over 15 years (this exercise was also calculated for 10, 20 and 25 years with similar results).
- It is important to determine the duration of pension liabilities and using it as the maturity for the interest rate.

GARCH(1,1) volatility model output



Calibrated model for node to 25 years

- The resulting dynamics were computed as*:

$$dr_t = 0.71104258 * (.08627534 - r_t) + \sigma_t r_t^{1/2} dW$$

and

$$\sigma_t{}^2 = 0.00001511 + 0.00209451\varepsilon_{t-1}^2 + \sigma_{t-1}^2$$

* for the 15-year node

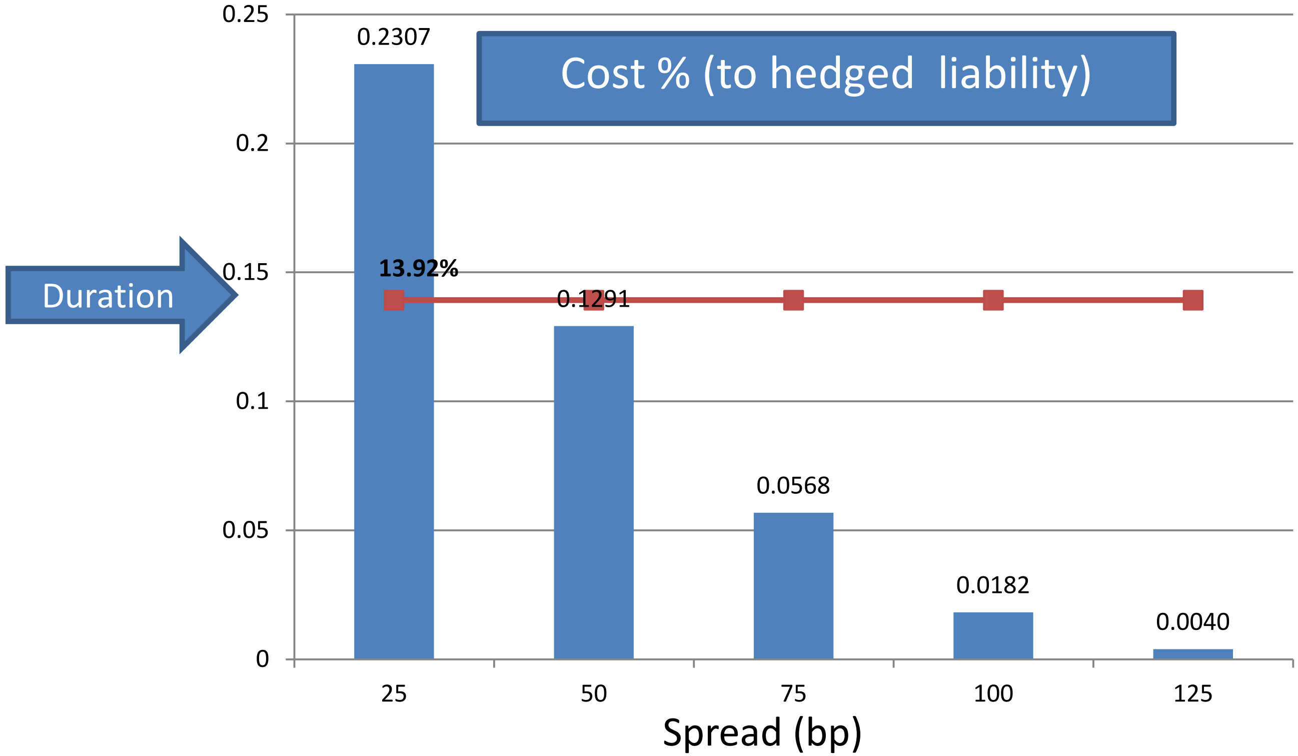
Option valuation

- 10,000 Monte Carlo simulated IR paths were run for every node*
- At certain time points**, the the payoff of option is computed and discounted
- The expected value of these computations represents the cost of hedge
- It was considered for different spreads around the proposed discount rate as an interval
- In the next exercise it was supposed the option in case of decreasing the discount rate under ti-s for exercising the option (i.e. inside [ti-s,ti] is not triggered the option, ti:= discount rate and s:= spread at basis points 25 to 125)

* for the next 8 years

**Time points: 1,2,3,4 and 5 years

Results: One shot cost of hedge



Assumptions: Duration = 15 year / M2M on 7.75% discount rate / Long-run rate 8.62%

25 spread cost above duration

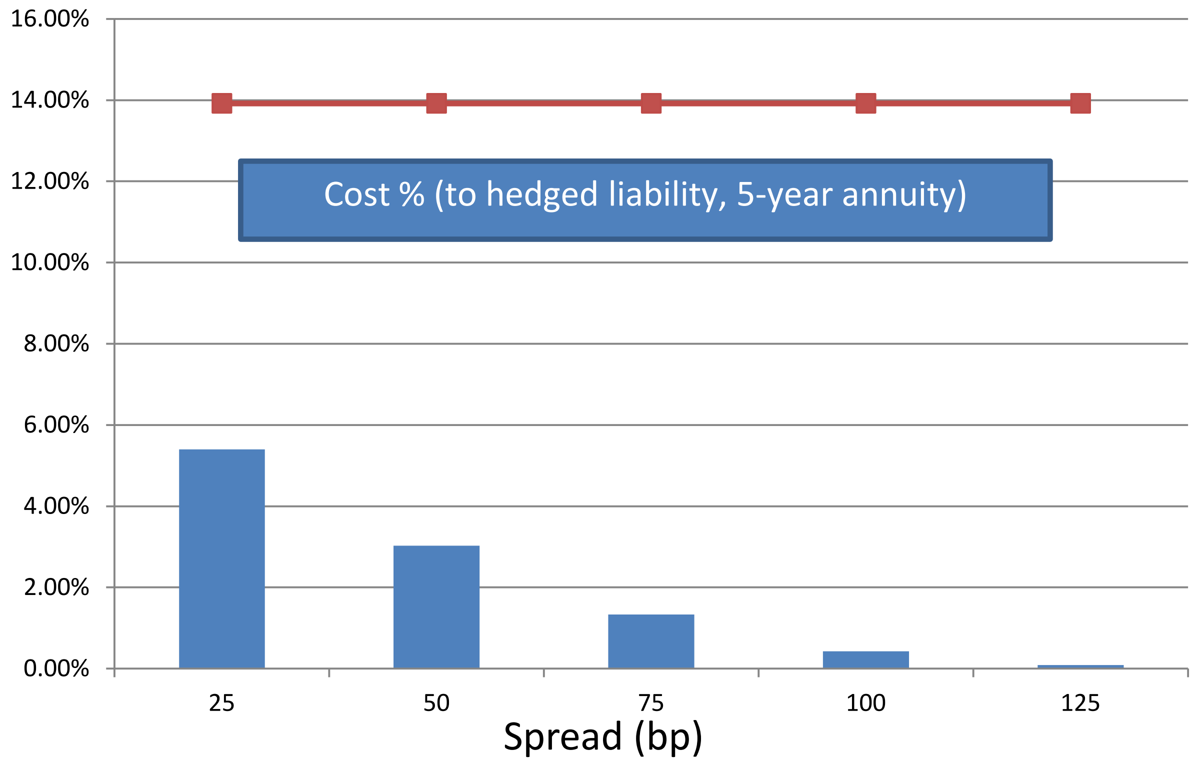
Results: Amortized Collar Cost

Cost estimated for a collar with different spreads during 5-years

Discount rate		**7.75%**	Long-run rate		**8.63%**		
Duration		**15.00x**					
Modified duration		**13.92%**					
Scenario	*OBD*	*Funding*	*Spread (bp)*	*Total*	*Equiv. Rate*	*Anuities*	*Cost (%)*
1	1,000,000,000	80.00%	25	230,736,108	5.50%	54,029,888	5.40%
2	1,000,000,000	80.00%	50	129,138,335	5.52%	30,257,231	3.03%
3	1,000,000,000	80.00%	75	56,793,135	5.54%	13,315,477	1.33%
4	1,000,000,000	80.00%	100	18,174,711	5.57%	4,264,160	0.43%
5	1,000,000,000	80.00%	125	3,957,284	5.59%	929,115	0.09%

As a larger spread is allowed, the cost lowers

Results: Amortized cost of hedge



Assumptions: Duration = 15 year / M2M on 7.75% discount rate / Long-run rate 8.62%

All spread costs below duration

Conclusions

The cost of hedge is feasible at almost every spread, and if amortized, comes to be efficient.

For nodes above 10 years, the long-run rate is was above current rate, which is less volatile reference for discounting.

For shorter term options is efficient to consider the rates form the stylized curves (for mature plans)

Through CIR is possible to model each node for constructing the long term curve and using it for discounting the pension liabilities.

Questions?

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Thanks!

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