VALUATION AND SOLVENCY OF LONG TERM INSURANCE LIABILITIES

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Motivation

- Long term aspect of pension provisions
- Short term vision of Solvency 2
- Decrease of the investment in risky assets for life insurers in Europe

- Form of short term “myopia” ?
- How to take into account the time horizon of liabilities in a solvency measurement ?

OUTLINE

- 1. The liability side
- 2. The financial risks
- 3. The longevity risk
- 4. Probability of default
- 5. Solvency capital
- 6. Extension to regime switching model

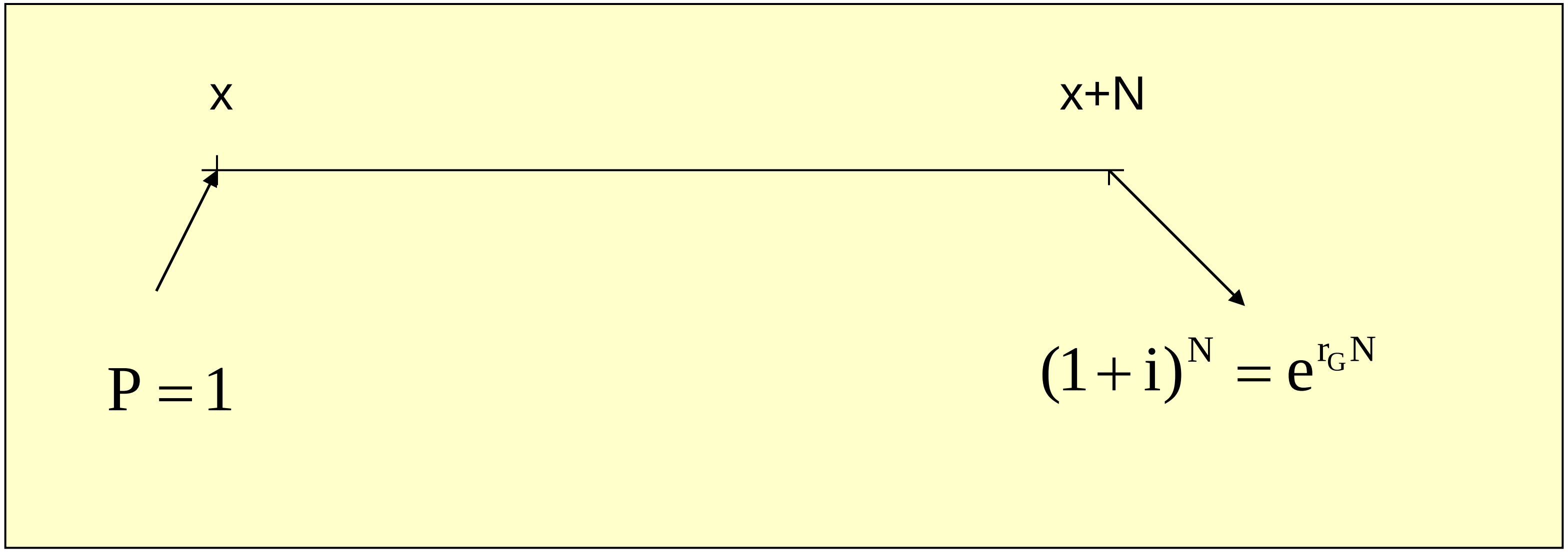
1. The liability side

We consider a ***pension investment product***
with a fixed minimum guaranteed return given at retirement
(cash balance scheme – pure endowment)

Notations :

P=1 : initial premium paid at time t=0 at age x
N : maturity of the pension account
i : fixed guaranteed return

L(N) : liability at retirement age



The lump sum at retirement age x+N is only paid in case of survival.

First order basis for mortality :

$$ {}_t p_x = \text{survival probability between age } x \text{ and age } x+t $$

Assumption : GOMPERTZ life table

$$ {}_t p_x = \exp(-\int_0^t \mu_{x+s}\, ds) $$

$$ \text{with: } \quad \mu_{x+s} = \mu_x . e^{\beta s} $$

Then :

$$ {}_t p_x = \exp(-\mu_x . \frac{e^{\beta t} - 1}{\beta}) $$

Taking into account the mortality credit , the liability to pay is :

$$ L(N) = \frac{e^{r_G N}}{{}_N p_x} = \exp(r_G . N + \mu_x . \frac{e^{\beta N} - 1}{\beta}) $$

2. The financial risks

Investment of the initial single premium in 3 possible kinds of assets :

- cash
- bonds → Interest rate risk
- equities → Market risk

Constant proportion portfolio :

$x_B = \text{part in bonds}$

$x_S = \text{part in equities}$

$1 - x_B - x_S = \text{part in cash}$

Cash asset :

Short term risk free rate (under P) :

$$dr(t) = a(b - r(t))\,dt - \sigma_r dw_r(t)$$

$$r(t) = r(0).e^{-at} + b(1 - e^{-at}) - \sigma_r e^{-at} \int_0^t e^{as} dw_r(s)$$

Saving account :

$$Z(t) = \exp\left(\int_0^t r(s)\,ds\right)$$

Bonds :

Investment in fixed duration bonds (***rolling bonds*** of maturity K)

Zéro coupon bonds :

$$P(t,s) = \text{price at time } t \text{ of a ZC maturity } s$$

$$\frac{dP(t,s)}{P(t,s)} = (r(t) + \sigma_B(t-s)\lambda_r)dt + \sigma_B(t-s)dw_r(t)$$

with :

$$\sigma_B(t-s) = \sigma_r \frac{1-e^{-a(t-s)}}{a} = \text{volatility}$$

$$\lambda_r = \text{bond risk premium}$$

Bonds (2) :

Investment in fixed duration bonds (rolling bonds of maturity K)

Rolling bonds :

$$P_K(t) = P(t, t+K)$$

$$\frac{dP_K(t)}{P_K(t)} = (r(t) + \sigma_K \lambda_r)dt + \sigma_K dw_r(t)$$

With :

$$\sigma_K = \sigma_r \frac{1 - e^{-aK}}{a}$$

$$\lambda_r = \text{bond risk premium}$$

Equity :

Investment in an equity index :

X(t)

$$\frac{dX(t)}{X(t)} = (r(t) + \sigma_S \lambda_S)dt + \sigma_S dw_S(t)$$

With :

$$\lambda_S = \text{equity risk premium}$$

$$\rho = \text{correlation equity} - \text{bonds}$$

independent

$$dw_S(t) = \rho . dw_r(t) + \sqrt{1-\rho^2} . dw(t)$$

Mixed portfolio :

Y(t) = mixed portfolio

$$dY(t) = Y(t).(r(t) + x_B \sigma_K \lambda_r + x_S \sigma_S \lambda)\, dt$$
$$+ Y(t).((x_B \sigma_K + x_S \sigma_S \rho)\, dw_r(t) + x_S \sigma_S \sqrt{1-\rho^2} dw(t))$$

or :

$$dY(t) = Y(t).(r(t) + m)\, dt + Y(t)(\sigma_1 dw_r(t) + \sigma_2 dw(t))$$

Risk premium of the portfolio (m)

Interest rate volatility (σ_1)

Equity vol (σ_2)

Mixed portfolio (2) :

Distribution of the mixed portfolio :

$$Y(t) = \exp(B(t))$$

where B is a Gaussian process :

$$B(t) = (m - (\sigma_1^2 + \sigma_2^2)/2)\,t + \int_0^t r(u)\,du + \sigma_1 w_r(t) + \sigma_2 w(t)$$

$$Y(t) \sim \exp(N(\delta(t), \eta^2(t))$$

Mixed portfolio (3) :

$$B(t) = (m - (\sigma_1^2 + \sigma_2^2)/2)\,t + \int_0^t r(u)\,du + \sigma_1 w_r(t) + \sigma_2 w(t)$$

↓

$$EB(t) = \delta(t) = (m + b - (\sigma_1^2 + \sigma_2^2)/2)\,t + \left(\frac{r(0) - b}{a}\right).(1 - e^{-at})$$

$$\mathrm{Var}\ B(t) = \eta^2(t) = \eta_1.t + \eta_2.\frac{1 - e^{-at}}{a} + \eta_3.\frac{1 - e^{-2at}}{2a}$$

with :

$$\eta_1 = \frac{\sigma_r^2}{a^2} + \sigma_1^2 + \sigma_2^2 - 2\frac{\sigma_1\sigma_r}{a}$$

$$\eta_2 = -2\left(\frac{\sigma_r^2}{a^2} + \frac{\sigma_1\sigma_r}{a}\right)$$

$$\eta_3 = \frac{\sigma_r^2}{a^2}$$

3. The longevity risk

Stochastic model for the spot mortality intensity of the generation initially aged x.

$$\lambda_x(t) = \text{mortality intensity at age } x + t$$

Non mean reverting model :

(see LUCIANO,REGIS VIGNA (2012)):

$$d\lambda_x(t) = \gamma_x . \lambda_x(t)\, dt + \upsilon_x . dw_1(t)$$

Longevity risk :

Survival process (*stochastic process*!) :

$$ {}_t p_x = \exp\left(-\int_0^t \mu_{x+s} \, ds\right) $$

↓

$$ S_x(t) = \exp\left(-\int_0^t \lambda_x(s) \, ds\right) $$

Longevity risk (2):

Solution for the survival process :

$$S_x(t) = \exp -\left(\lambda_x(0).\frac{e^{\gamma_x t} - 1}{\gamma_x} + \upsilon_x . R_x(t)\right)$$

With :

$$R_x(t) = \int_0^t \int_0^s e^{\gamma_x (s-u)} \, dw_1(u) \, ds$$

$$= \int_0^t \left(\frac{e^{\gamma_x (t-u)} - 1}{\gamma_x}\right) . dw_1(u)$$

(*Gaussian Process*)

Longevity risk (3):

Longevity variance :

$$\theta^2(t) = \mathrm{Var}\,(\upsilon_x . R_x(t))$$

$$= \upsilon_x{}^2 . E((\int_0^t (\frac{e^{\gamma_x(t-u)} - 1}{\gamma_x}) dw_1(u))^2).$$

$$= \upsilon_x{}^2 \int_0^N \left(\frac{e^{\gamma_x(t-u)} - 1}{\gamma_x} \right)^2 du$$

$$= \frac{\upsilon_x{}^2}{2\gamma_x{}^3} (2\gamma_x N - 4e^{\gamma_x N} + 2e^{2\gamma_x N} + 3)$$

Longevity risk (4):

Remarks :

1°) We only take into account the non diversifiable part of the longevity risk (large portfolios)

2°) Possibility to introduce a correlation between longevity risk and financial risks !! (not the case here)

$$w_r = \text{interest rate risk}$$
$$w_S = \text{equity risk}$$
$$w_l = \text{longevity risk}$$

ρ

Financial and longevity risks :

Liability side ***Asset side***

$$L(N) = \frac{e^{r_G N}}{{}_N p_x}$$

First order basis

$$A(N) = \frac{Y(N)}{S_x(N)}$$

Financial risks

Longevity risk

Financial and longevity risks (2) :

Global asset (real available asset for each survivor) :

$$A(N) = \frac{Y(N)}{S_x(N)}$$

$$= \exp(B(N) + \lambda_x(0)\frac{e^{\gamma_x N} - 1}{\gamma_x} + \upsilon_x R_x(N))$$

$$= \exp(B(N) + C(N))$$

Finance

Longevity

Financial and longevity risks (3) :

Distribution of the global asset :

$$A(N) = \exp(A^*(N))$$

where :

$$A^*(N) \sim N(e(N), \sigma_G^2(N))$$

With :

$$e(N) = \delta(N) + \lambda_x(0) \frac{e^{\gamma_x N} - 1}{\gamma_x}$$

$$\sigma_G^2(N) = \eta^2(N) + \theta^2(N)$$

(assumption of no correlation between finance and longevity)

Financial and longevity risks (4): particular cases:

Pure Financial risk

$$\upsilon_x = 0$$
$$\gamma_x = \beta$$
$$\lambda_x(0) = \mu_x$$

(*real mortality*
=
first order)

Pure longevity risk

$$x_B = x_S = 0$$
$$r(0) = b = r_G$$
$$\sigma_r = 0$$

(*cash asset without risk*)

4. Probability of default

Probability of default at maturity seen as a function of the time horizon N

$$\varphi(N) = P(A(N) < L(N))$$

$$= P\left(A^*(N) < r_G N + \mu_x \frac{e^{\beta N} - 1}{\beta}\right)$$

$$= \Phi\left(\frac{r_G N - \delta(N) + \mu_x \frac{e^{\beta N} - 1}{\beta} - \lambda_x \frac{e^{\gamma_x N} - 1}{\gamma_x}}{\sigma_G(N)}\right)$$

$$\Phi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{-u^2/2}\, du$$

With :

$$\delta(N) = (m + b - (\sigma_1^2 + \sigma_2^2)/2)\,N + \left(\frac{r(0) - b}{a}\right).(1 - e^{-aN})$$

Global volatility :

$$\sigma_G{}^2(N) = \eta_1.N + \eta_2.\frac{1 - e^{-aN}}{a} + \eta_3.\frac{1 - e^{-2aN}}{2a}$$

$$+ \frac{\upsilon_x{}^2}{2\gamma_x{}^3}(2\gamma_x N - 4e^{\gamma_x N} + 2e^{\gamma_x N} + 3)$$

5. Solvency capital

Initial level of capital SC(N) at time t=0 assumed to be invested in the mixed portfolio in such a way that the probability of default at maturity is below a fixed level (*value at risk approach*).

Remarks :

1°) A Tail value at risk computation can also be considered.

2°) the solvency capital could be invested in a different way (for instance in the cash asset) .

Equation of the Solvency Capital :

$$P(A(N).(1+SC(N)) < L(N)) = 1 - \alpha_N$$

?

With :

$$\alpha_N = \text{safety level for a duration } N$$

for instance : (???)

$$\alpha_N = (0.995)^N$$

Equation of the Solvency Capital (2) :

$$P(A(N).(1+SC(N))<L(N))$$

$$=P(\exp(A^*(N).(1+SC(N))<\exp(r_G N+\mu_x \frac{e^{\beta N}-1}{\beta}))$$

$$=P(A^*(N)<r_G N+\mu_x \frac{e^{\beta N}-1}{\beta}-\ln(1+SC(N))$$

$$N(e(N),\sigma_G{}^2(N))$$

Equation of the Solvency Capital (3) :

$$SC(N) = \exp(r_G N - \delta(N) + \mu_x \frac{e^{\beta N} - 1}{\beta} - \lambda_x \frac{e^{\gamma_x N} - 1}{\gamma_x} - \sigma_G(N) z_{1-\alpha_N}) - 1$$

- $r_G N$: Financial guarantee
- $\delta(N)$: Best Estimate Return
- $\mu_x \frac{e^{\beta N} - 1}{\beta}$: Longevity guarantee
- $\lambda_x \frac{e^{\gamma_x N} - 1}{\gamma_x}$: Best estimate longevity
- $\sigma_G(N)$: Global vol
- $z_{1-\alpha_N}$: Quantile

Tail value at risk approach :

The solvency capital (invested in the mixed portfolio) SC*(N) is then solution of the equation :

$$0 = E(\{L(N) - A(N).(1 + SC^*(N)\} \mid (L(N) - A(N)) \geq VaR_{\alpha_N})$$

?

$$VaR_{\alpha_N} = \exp\{r_G N + \mu_x \frac{e^{\beta N} - 1}{\beta}\} - \exp\{\delta(N) + \lambda_x \frac{e^{\gamma_x} - 1}{\gamma_x} + \sigma_G(N).z_{1-\alpha_N}\}$$

6. Regime switching model

How to find a good model for long term investment ?

<u>Model for equity :</u>

Geometric Brownian motion

LEVY
Processes
(NIG , variance gamma)

Regime switching
Brownian motion

Regime switching model : Motivations

- Traditional models for stock returns fail to capture extreme movements in the long period
- The market may switch from a stable low volatility state to a high volatility regime
- The solvency of long term insurance product can be negatively affected by these movements in financial markets.

Financial risk under Regime switching model

Liability side ***Asset side***

$$L(N) = e^{r_G N}$$

$$A(N)$$

The value of liability is fixed

The value of the assets depends on the switching between different states

Assumption : no mixed portfolio, no mortality, constant risk free rate

The model:

Markow Regime Switching Lognormal model (MRSL) with two states (Hardy, 2001), adapted to insurance long term products.

Considering two states E1 and E2, the evolution of the assets is:

$$dA(s)\big|E_1 = \delta_1 A(s)ds + \sigma_1 A(s)dw(s)$$
$$dA(s)\big|E_2 = \delta_2 A(s)ds + \sigma_2 A(s)dw(s)$$

The switching from E_1 to E_2 and vice versa is regulated by a Markov chain with the transition matrix $P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$

The model:

The final value of assets $\tilde{A}(N)$ depends on the occupation time spent in each state.

The ***Probability of Default*** and the ***Solvency Capital*** must be calculated according the final value of asset $\tilde{A}(N)$ influenced by the switching between the two financial states :

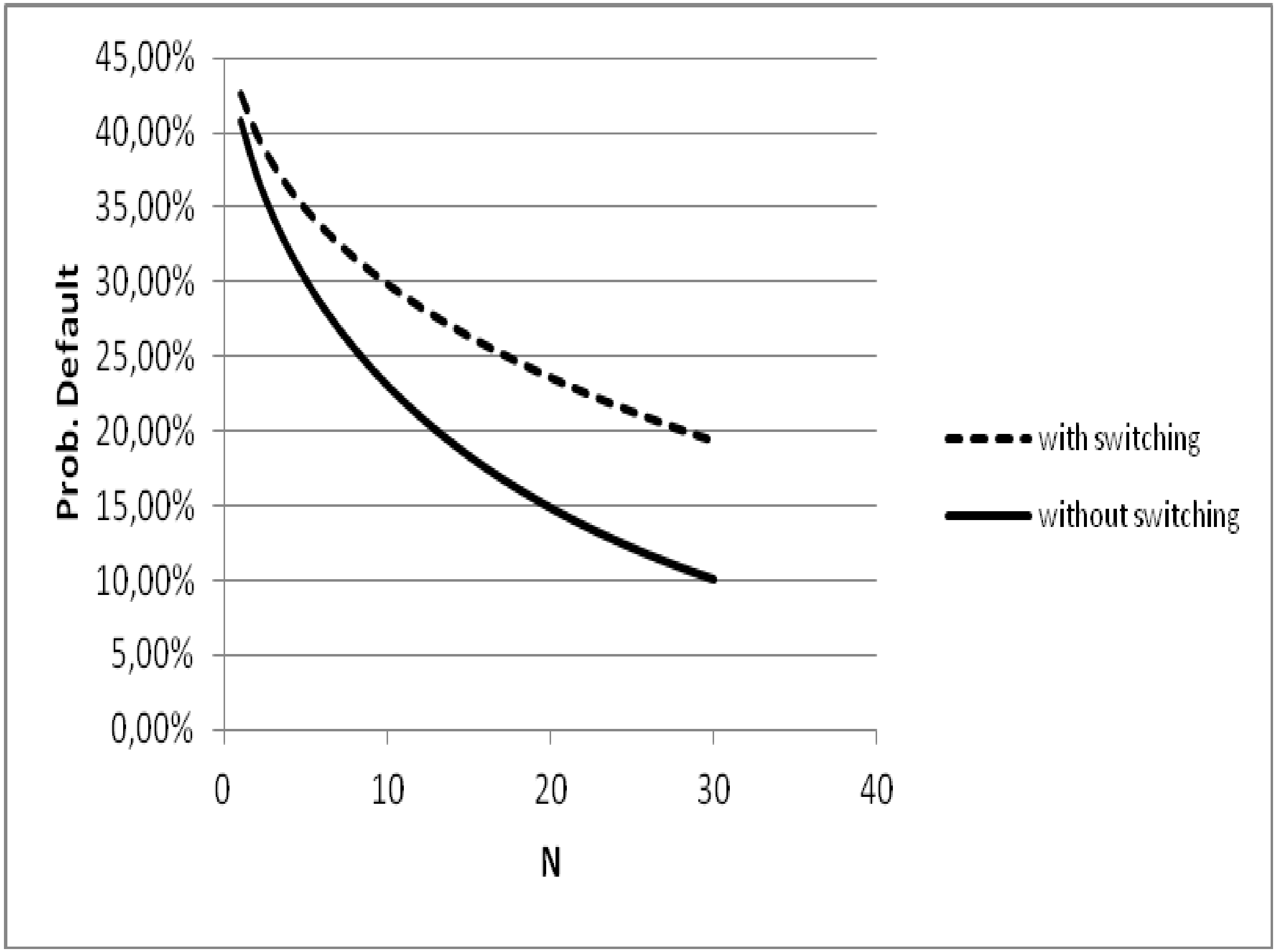
$$\varphi(N) = P(\tilde{A}(N) < L(N))$$

$$P\{\tilde{A}(N)(1 + SC(N)) < L(N)\} = 1 - \alpha_N$$

Example:

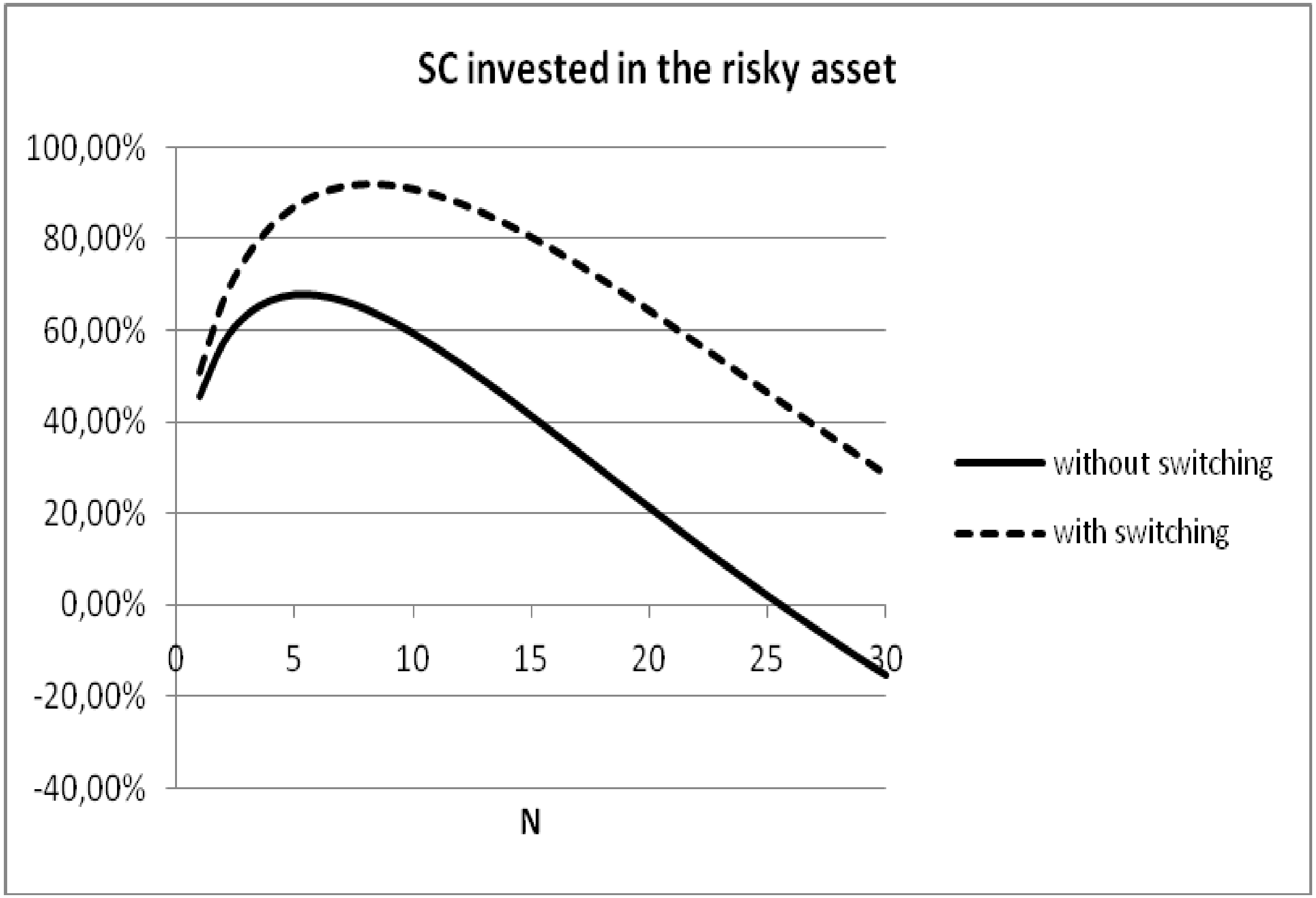
- regime 1: δ_1=0.07, σ_1 =0.16
- regime 2: δ_2 = -0.01, σ_2 =0.25
- transition probabilities: p_{12}= 0.0398, p_{21}=0.3798
- r_G=0.02, r=0.04
- α=0.995

Probability of default



(The hypothesis “without switching” is based just on financial parameters of regime 1)

Solvency capital



Conclusion

- Importance of the time horizon in the solvency measurement of long term life and pension products
- Typical « time decreasing » effect
- Regime switching models can change the magnitude of the conclusions w.r.t. classical Brownian models ; same kind of effect for LEVY models

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THANK YOU

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