Asset and Liability Composition in Participating Life Insurance: The Impact on Shortfall Risk and Shareholder Value

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Introduction: Motivation

- Participating life insurance along with annuity contracts:
  - Important product design in German life insurance market
  - Include interest rate guarantees and bonus mechanisms through which profits are distributed and appropriated to the policyholders

- Furthermore:
  - Management decisions regarding the asset and liability composition have an impact and should be accounted for when evaluating the risk situation of a life insurer
  - And they also affect the fair risk-adjusted compensation offered to equityholders for providing safety capital
Introduction: Aim of paper

• Examine a life insurer’s risk situation which provides
  • Traditional participating life insurance policies and annuities with different surplus appropriation schemes:
    • Bonus system (for both products): surplus increases death and survival benefit, and the annual annuity payment, respectively
    • Interest-bearing accumulation (endowment insurance): accumulates surplus on a separate account, death benefit is kept constant
    • Direct payment (annuity): surplus is directly paid out on top of the annuity
  • And which allows for management decisions regarding assets and liabilities:
    • Path-dependent and dynamic adjustment of the portion invested in high-risk assets
    • Composition of the product portfolio
    • Type of surplus appropriation scheme

• Analysis accounts for mortality risk and ensures a fair situation for shareholders
Model framework: Insurance contracts

- Pool of temporary annuities (payments in arrear):
  - Actuarially priced based on the mortality table with safety loading: DAV 2004 R
  - Single premium is given by the equivalence principle:
    \[ P_{\text{single}}^R = R_1 \cdot a_{x:n} \]

- Pool of traditional participating life insurance products:
  - Actuarially priced based on the mortality table with safety loading: DAV 2008 T
  - Single premium is given by the equivalence principle:
    \[ P_{\text{single}}^S = S_1 \cdot A_{x:n} \]
  - Constant annual premium is given by annuitizing the single premium:
    \[ P^S = P_{\text{single}}^S / \ddot{a}_{x:n} \]

- It holds:
  - \[ A_{x:n} = \sum_{k=0}^{n-1} v^{k+1} \cdot k p_x \cdot q_{x+k} + v^n \cdot n p_x \]
  - \[ \ddot{a}_{x:n} = \sum_{k=0}^{n-1} v^k \cdot k p_x \text{ and } a_{x:n} = \sum_{k=1}^{n} v^k \cdot k p_x \]
Model framework: Policy reserves and modeling mortality

- Actuarial reserve for an individual annuity contract is given by
  \[ tV_x^R = R_{t+1} \cdot a_{x+t:n-t} \]

- Actuarial reserve for an individual endowment contract is given by
  \[ tV_x^S = S_{t+1} \cdot A_{x+t:n-t} - P^S \cdot \bar{a}_{x+t:n-t} \]

- Corresponding portfolio policy reserve is determined by
  \[ PR_{t-}^j = \left( N^j - \sum_{i=1}^{t} d_i^j \right) \cdot tV_x^j, j = R, S \]
  - \( N = \) total number of contracts sold
  - \( N^R = \varphi \cdot N \) annuity policies, \( N^S = (1 - \varphi) \cdot N \) endowment contracts
  - \( \sum_{i=1}^{t} d_i^j = \) number of deaths of the pool \( j \) until year \( t \)

- Actual mortality rates are based on the best estimates of the corresponding mortality tables (without safety loading)
Model framework: Development of the asset base

- Asset portfolio follows a geometric Brownian motion
  \[ dA(t) = \mu \cdot A(t) \cdot dt + \sigma \cdot A(t) \cdot dW^P(t) \]
- Portfolio is composed of bonds and stocks, with a continuous one-period return of the portfolio, given by
  \[ r_t = a_t \cdot r_S + (1 - a_t) \cdot r_B, \text{ with } E(r_t) = \mu - \sigma^2/2 \]
- Assets at the end of year \( t \), after accounting for death and annuity payments, result to
  \[ A_t^- = A_{(t-1)^+} \cdot \exp(r_t) - R_t \cdot \left( N^R - \sum_{i=1}^{t} d_i^R \right) - S_t \cdot d_t^S \]
  \[ A_0^- = 0, A_0^+ = P_{\text{single}}^R \cdot N^R + P_{(\text{single})}^S \cdot N^S + E_0 \]
Model framework: Surplus appropriation schemes

- Actual policy interest rate credited to the policyholders for period \( t - 1 \) until \( t \), based on a smoothing scheme by Grosen and Jørgensen (2000), is given by

\[
r_t^P = \max \left\{ r_G, \alpha \cdot \left( \frac{B_{(t-1)^+}}{PR_{(t-1)^-}^R + PR_{(t-1)^-}^S + IA_{(t-1)^-}} - \gamma \right) \right\}
\]

- \( \alpha \) = surplus distribution ratio
- \( \gamma \) = target buffer ratio
- \( r_G \) = guaranteed interest rate

- Surplus for the \( t \)-th year results to \( PR_j^{(t-1)^-} \cdot (r_t^P - r_G) \), \( j = R, S \)

\( \Rightarrow \) Amount is used differently depending on the appropriation scheme

- Endowment insurance: bonus system, interest-bearing accumulation
- Annuity insurance: bonus system, direct payment
Model framework: Bonus system (endowment policy)

- Surplus is used to increase the initially guaranteed sum insured $S_1$ (death and survival benefit)
- Done by using the surplus as a single premium for an additional contract of the same type with the same maturity:

$$\Delta - \text{values}$$

original contract

\[0\quad t\quad n\quad \text{time}\]

- Surplus per insured results in an additional sum insured of

$$\Delta S_t \cdot A_{x+t:n-t} = PR_{(t-1)}^S \cdot (r_t^P - r_t^G) \left/ \left( N^S - \sum_{i=1}^{n} d_i^S \right) \right. \]

$\Rightarrow$ Increased sum insured is given by

$$S_{t+1} = S_t + \Delta S_t$$
Model framework: Interest-bearing accumulation

- Sum insured is kept constant, i.e.
  \[ S_t = S_1, \forall t = 1, \ldots, T \]

- Surplus is accumulated on a separate account, \( IA_t \)

- Forward projection of this account is given by

\[
IA_t = IA_{(t-1)} \cdot (1 + r^{IA}) \cdot \left(1 - \frac{d_t^S}{N^S - \sum_{i=1}^{t-1} d_i^S}\right) + PR_{(t-1)}^S \cdot (r_t^P - r^G)
\]

* Adjustment for death: funds that belonged to policyholders that died within the \( t \)-th year, are passed to the collectivity of policyholders
Model framework: Annuities’ appropriation schemes

- **Bonus system:**
  - Analogously to the endowment policy’s bonus system: surplus is used to increase the initially guaranteed annuity $R_1$ as follows
  \[
  \Delta R_t \cdot a_{x+t:n-t} = PR^R_{(t-1)} \cdot (r^P_t - r^G) / \left( N^R - \sum_{i=1}^{t} d_i^R \right)
  \]
  - Increased annuity is given by
  \[
  R_{t+1} = R_t + \Delta R_t
  \]

- **Direct payment:**
  - Surplus amount (per insured) is paid in each year on top of the annuity payment to the policyholder:
  \[
  R_{t+1} = R_1 + PR^R_{(t-1)} \cdot (r^P_t - r^G) / \left( N^R - \sum_{i=1}^{t} d_i^R \right)
  \]
Model framework: Management’s discretion

- **Asset side:**
  - Dynamic CPPI-based feedback mechanism, where the stock portion at time $t$ is given by
  
  $$a_t = \min \left( \max \left( \frac{A_t^+ - PR^R_{(t-1)^-} - PR^S_{(t-1)^-} - IA_{(t-1)^-}}{A_t^+} \cdot m, 0 \right), a_{max} \right)$$

  - $a_0 = a =$ initial stock portion
  - $a_{max} =$ maximum stock portion allowed
  - $m =$ multiplier, controls the extent to which assets are shifted towards bonds

- **Liability side:**
  - Company’s risk profile can be altered by means of product portfolio composition:
    - Varying fraction $\varphi$ of annuity policies
    - And endowment contracts $1 - \varphi$
  - Choosing the type of surplus appropriation scheme
Model framework: Fair valuation (shareholder’s viewpoint)

- Annual dividend payments to equityholders: \( D_t = \beta \cdot E_0 \)

- Constant dividend rate \( \beta \) is calibrated such that the expected value of the payments to the shareholders equals their initial contribution \( E_0 \):

\[
E_0 = E^Q \left( e^{-r_f \cdot T} \cdot E_T + \sum_{t=1}^{T} e^{-r_f \cdot t} \cdot D_t \right) \\
= E^Q \left( e^{-r_f \cdot T} \cdot \min \{ E_0, E_0 + B_{T-} \} \cdot 1 \{ T_S > T \} \right) \\
+ E^Q \left( \sum_{t=1}^{T} e^{-r_f \cdot t} \cdot \beta \cdot E_0 \cdot 1 \{ T_S > t \} \right)
\]

- \( T_S = \inf \{ t : A_{t^-} < PR_{t^-}^R + PR_{t^-}^S + IA_{t^-} \} \), \( t = 1, \ldots, T \) (time of shortfall)
- \( D_t = \) dividend at time \( t \)
- \( E_T = \) final payment
- \( r_f = \) risk-free rate
Model framework: Risk assessment and input parameters

- Shortfall probability (assets not sufficient to cover liabilities):
  \[ SP = P\left( T_s \leq T \right), \text{ with } T_s = \inf \left\{ t : A_t^- < PR_t^R + PR_t^S + IA_t^- \right\}, t = 1, \ldots, T \]

- Input parameters:

<table>
<thead>
<tr>
<th>Total number of contracts sold</th>
<th>100,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contract terms</td>
<td>30</td>
</tr>
<tr>
<td>Guaranteed interest rate</td>
<td>2.25%</td>
</tr>
<tr>
<td>Age of the policyholders at inception</td>
<td></td>
</tr>
<tr>
<td>Temporary annuity</td>
<td>60</td>
</tr>
<tr>
<td>Endowment insurance</td>
<td>35</td>
</tr>
</tbody>
</table>

- Actuarial present values of the benefits for the annuity and the endowment insurance (per insured) are equal
Numerical results: Feedback mechanism (bonus system)

without feedback mechanism
temporary annuity with bonus system

with feedback mechanism (CPPI system)
temporary annuity with bonus system

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Numerical results: Feedback mechanism (direct payment)

without feedback mechanism
temporary annuity with direct payment

with feedback mechanism (CPPI system)
temporary annuity with direct payment

shortfall probability (left axis)
beta (right axis)
Numerical results: Impact of mortality (annuity)

without feedback mechanism, \( a = 10\% \)
temporary annuity with direct payment

with feedback mechanism (CPPI system), \( a_{\text{initial}} = a_{\text{max}} = 10\% \)
temporary annuity with direct payment
Numerical results: Surplus appropriation schemes

without feedback mechanism, $a = 25\%$
endowment: single premium, annuity: single premium

with feedback mechanism (CPPI system), $a_{\text{initial}} = a_{\text{max}} = 25\%$
endowment: single premium, annuity: single premium
Numerical results: Impact of mortality (portfolio)

without feedback mechanism, $a = 10\%$
endowment: annual premium, annuity: single premium

with feedback mechanism (CPPI system), $a_{\text{initial}} = a_{\text{max}} = 10\%$
endowment: annual premium, annuity: single premium
Summary

- Results show: Management’s actions not only have a considerable impact on an insurer’s risk level, but also on the fair risk-adequate position of shareholders.

- For fairly calibrated dividends, the company’s solvency situation is substantially affected by the management:
  - Composition of the product portfolio considerably influences the shortfall risk.
  - Type of surplus appropriation scheme substantially impacts the risk situation.
  - Shortfall risk can be reduced by path dependent management actions.
  - Effectiveness in risk reduction varies substantially depending on the surplus appropriation scheme offered to the customer.

⇒ Management mechanisms and the required compensation by equityholders for providing safety capital should be taken into account when evaluating an insurer’s risk situation.
Thank you very much for your attention!
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