Bayesian Sensitivity Analysis for VaR and TVaR

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ABSTRACT

It is known that $VaR$ and $TVaR$ are useful to measure huge quantities at low probabilities i.e. measures of extreme scenarios, nonetheless these values are difficult to estimate precisely because they have high sensitivity for low probabilities and sample size, also these measures inherit the error of estimation of $\theta$, where $\theta$ is usually estimated by the maximum likelihood method, say $\hat{\theta}$, therefore $\hat{\theta}$ is the most probable value but not the real one. For the above, to create new perspectives is relevant to analyze and understand the variations of the estimates of the extreme scenarios measures.

KEYWORDS

$VaR$; $TVaR$, Extreme Values, Maximum Likelihood, Monte Carlo Simulation, Delta Method, Prior Distribution, Posterior Distribution; Likelihood Ratio Test.
1. INTRODUCTION

This paper presents a methodology which explains the variability of estimates from a function of \( \theta \) and answers the next questions from a Bayesian perspective: What is the expected value?, What is the most probable value?, What is the variance?, What is the most probable maximum value?, Do the estimators have a probability distribution?.

The methodology is the following: Firstly, a parametric space of \( \Theta \) from \( \theta \) will be built, secondly the prior distribution \( P[\theta] \) and posterior distribution \( P[\theta|Z_n] \) will be generated, immediately the simulation process of random values \( \theta_i \sim P[\theta|Z_n] \) will be held, subsequently the VaR and TVaR will be calculated from the random values of \( \{\theta_i\}_{i=1}^r \). Hence, the expected value, mode, variance, most probable maximum value, and confidence intervals from VaR and TVaR will be obtained. Finally, this paper is showing whether the values of \( \{VaR_i\}_{i=1}^m \) and \( \{TVaR_i\}_{i=1}^m \) follow a probability distribution.

Two numerical examples will be shown, one to show our methodology, and another one to compare the proposal methodology with the delta method for the estimation of the variance of VaR and compare under different sample sizes the stressed VaR with itself.

2. HYPOTHESIS

This methodology assumes the following

- The sample \( \{x_i\}_{i=1}^n \) comes from the random variable \( X|\theta=x \).
- It is \( \text{Cov}(X_i, X_j|\theta)=0 \) \( \forall \ i \neq j \).
- The distribution of \( X|\theta \) with \( \theta \in \mathbb{R}^d \) is known and continuous.
3. **DEFINITION OF THE PARAMETRIC SPACE**

We refer to the vector of parameters \((\theta_1, \theta_2, \theta_3, \cdots, \theta_d)\) as \(\theta\) from the distribution \(X|\theta\), to the estimation of \(\theta\) with the maximum Likelihood method MVL as \(\hat{\theta}\) and to the likelihood of the random sample as \(L(\theta)\).

The parametric space \(\Theta\) of \(\theta\) at confidence level \(1-\alpha\) will be the space \(\Theta_{\alpha} \subset \mathbb{R}^d\) such that:

\[
\Theta_{\alpha} := \left\{ \theta \in \mathbb{R}^d \mid -2 \ln \left[ \frac{L(\theta)}{L(\hat{\theta})} \right] \leq \chi^2_{(df, \alpha)} \right\}
\]

This space contains all the points \(\theta \in \mathbb{R}^d\) such that they are statically equal to \(\hat{\theta}\). In fact this space is constructed with the notion of likelihood ratio test. The test is \(H_0: \hat{\theta} = \theta\) Vs \(H_1: \hat{\theta} \neq \theta\) with this test statistics \(T = -2 \ln \left( \frac{L(\theta)}{L(\hat{\theta})} \right)\), where \(T\) is approximately chi-square with a degree of freedom equal to the number of free parameters under alternative hypothesis minus the number of free parameters under null hypothesis. This test can be consulted at [3].

![Behavior of \(\Theta_{\alpha}\) at different confidence levels with \(\theta \in \mathbb{R}^2\).](image)

**Fig. 3.1.** Behavior of \(\Theta_{\alpha}\) at different confidence levels with \(\theta \in \mathbb{R}^2\).
4. **DEFINITION OF** $P[\theta]$ & $P[\theta|Z_n]$

The prior distribution of $\theta$ will be $P[\theta]=\pi_0(\theta)=1$, because does not exist previous information available about $\theta$. Then, with the sample $Z_n=(x_1,x_2,x_3,\ldots,x_n)$ the posterior distribution of $\theta$ is such that $P[\theta|Z_n] \propto \pi_0(\theta)P(Z_n|\theta)$, for the Bayes's Theorem, $\pi_0(\theta)=1$ and $P[Z_n|\theta]=L(\theta)$, finally we conclude that $P[\theta|Z_n] \propto L(\theta)$. For our analysis over $\Theta_{\alpha_1}$, the exact posterior distribution is:

$$P[\theta|Z_n]=\frac{L(\theta)}{\int_{\theta \in \Theta_{\alpha_1}} L(\theta) d\theta} \quad \forall \theta \in \Theta_{\alpha_1}$$

5. **SIMULATION OF** $\theta|Z_n$

Given $P[\theta|Z_n]$ the simulation of $\theta$ is as follows:

Simulation of $\{U_i\}_{i=1}^m$, where each one has the form $U_i=\{u_{i1},u_{i2},u_{i3},\ldots,u_{id}\}$ with $u_{ij} \sim U(a_j,b_j)$, where $W_j$ has the form $W_j=[t \in \mathbb{R} \mid a_j \leq t \leq b_j]$, then, let $W_{\Omega}=W_1 \times W_2 \times W_3 \times \cdots \times W_d$, such that: $\Theta_{\alpha_1} \subset W_{\Omega}$.

![Diagram](image)

**Fig. 5.1.** $W_{\Omega}$ contains completely the space $\Theta_{\alpha_1}$.
With the values \( \{U_i\}_{i=1}^m \) select only the values that \( U_i \in \Theta_{\alpha_i} \) and let us denote by \( M_2 \) the set of subscripts such that \( M_2 = \{i \in 1, 2, 3, \ldots, m | U_i \in \Theta_{\alpha_i}\} \). Then the values to simulate will be \( \theta_i = U_i \quad \forall i \in M_2 \) with the following density function:

\[
P^*[\theta = \theta_i | Z_n] = \frac{P[Z_n | \theta_i]}{\sum_{j \in M_2} P[Z_n | \theta_j]} \quad \forall i \in M_2
\]

The expression above is equivalent to:

\[
P^*[\theta = \theta_i | Z_n] = \frac{L(\theta_i)}{\sum_{j \in M_2} L(\theta_j)} \quad \forall i \in M_2
\]

Computationally is better to work with log-likelihood \( l(\theta) \), then, the expression above is equivalent to:

\[
P^*[\theta = \theta_i | Z_n] = \frac{e^{l(\theta_i)}}{\sum_{j \in M_2} e^{l(\theta_j)}} \quad \forall i \in M_2
\]

This expression may need a numeric correction in order to be calculated, this correction is needed because the expression \( e^x \) does not make sense in \( \forall x \in (-\infty, -745) \cup (709, \infty) \), at least in the Statistical Software \textbf{R}, hence, is necessary to find a number \( C \) such that \(-745 < l(\theta_i) + C < 709 \quad \forall i \in M_2 \), one possible solution for \( C \) is the next:

\[
C = -\frac{1}{2} \left( \min_{j \in M_2} \left( l(\theta_j) \right) + \max_{j \in M_2} \left( l(\theta_j) \right) \right)
\]
Then, the approximated density function of $\theta | Z_n$ is:

$$P^*[\theta = \theta_i | Z_n] = \frac{e^{l(\theta_i) + C}}{\sum_{j \in M_2} e^{l(\theta_j) + C}} \forall i \in M_2$$

Denote $P_i = \sum_{j \leq i} P^*[\theta = \theta_j | Z_n]$, the simulation of $\theta$ is:

- Simulate $m_3$ random numbers $r_i \sim U(0,1)$
- $\theta_k$ belongs $r_i$, when $P_{k-1} < r_i < P_k$

Doing the above we get the random sample $\{\theta_{\{i\}_{i=1}^{m_3}} - \theta | Z_n\}$.

Part of this method is similar to 5.1 in [1]

6. SENSITIVITY OF VaR & TVaR

With $\{\theta_{\{i\}_{i=1}^{m_3}} - \theta | Z_n\}$ is possible to observe the behavior of any function of $\theta$, in our case, the functions are $VaR$ and $TVaR$

For a continuous random variable $X$ is known that:

- $VaR_X(p) = F_X^{-1}(p)$
- $TVaR_X(p) = E[X | X > VaR_X(p)]$

Hence, it is possible to calculate the observations $\{VaR_X[\theta](p)_{i=1}^{m_3}\}$ and $\{TVaR_X[\theta](p)_{i=1}^{m_3}\}$, from which we can obtain statistics for understanding the sensitivity of these measures over changes of $\theta$. Let $g(\theta)$ any function of $\theta$ and $g_{(i)}$ the i-th order statistic from the values $\{g(\theta)\}_{i=1}^{m_3}$. And finally we say that $g(\theta)$ have a distribution $f_{g(\theta)}$ if it is tested with a goodness of fit and the test statistic is favorable.

6
Empirical Statistic:

- Expected value: \( \hat{\mu}_g = \frac{1}{m_3} \sum_{i=1}^{m_3} g(\theta_i) \)

- Variance: \( \hat{\sigma}_g^2 = \frac{1}{m_3 - 1} \sum_{i=1}^{m_3} (g(\theta_i) - \hat{\mu}_g)^2 \)

- Confidence interval at level \( 1 - \alpha \): \( IC = [g_{(m_3 \alpha/2)}, g_{(m_3 (1 - \alpha/2))}] \)

- Mode: The most repeated over \( \{g(\theta)\}_{i=1}^{m_3} \)

- Maximum possible value: \( VaR_g(p) = g_{(m_3 \ast p)} \)

Parametric Statistics (If exist \( f_g \))

- Expected Value: \( \mu_g = \int_0^{\infty} t f_g(t) dg \)

- Variance: \( \sigma_g^2 = \int_0^{\infty} (t - \mu_g)^2 f_g(t) dg \)

- Confidence interval at level \( 1 - \alpha \): \( IC = [F_g^{-1}(\alpha/2), F_g^{-1}(1 - \alpha/2)] \)

- Mode: \( g_{MAX} = \{t \in \mathbb{R} | \forall w \in \mathbb{R}: f_g(w) \leq f_g(t)\} \)

- Maximum possible value: \( VaR_g(p) = F_g^{-1}(p) \)

Comments:

Given the definition of \( VaR_{X}(p) \) and with the Bayesian context is possible to define the following expressions of risk measures.

- \( VaR_{VaR_{X_{|\theta}(p_1)}}(p_2) \)
- \( VaR \) from \( VaR \) is the maximum possible value of \( VaR_{X|0}(p_1) \) at confidence level \( p_2 \)

- \( VaR_{TVaR_{X_{|\theta}(p_1)}}(p_2) \)
- \( VaR \) from \( TVaR \) is the maximum possible value of \( TVaR_{X|0}(p_1) \) at confidence level \( p_2 \)
7. NUMERICAL EXAMPLES

We show in the section 6.1 and 6.2 numerical examples, one to exemplify our methodology and another one to compare the estimations of the variance of \( \text{VaR} \) with the delta method and the behavior of the MVL and Bayesian methodologies. The calculations were made in \textbf{R i386 2.15.1}, the codes are hosted at https://dl.dropbox.com/u/83706757/CodesEanguianoWEB.rar in order to be able to reproduce all the calculations.

7.1. EXAMPLE ONE (GENERAL METHODOLOGY)

For this example was generated the random sample \( \{ x_i \}_{i=1}^{n=5,000} \), where \( X| \theta \sim \text{Burr} (\alpha, \beta, k) \), where the real parameters of the model are \( \theta = (\alpha, \beta, k) = (4,6,2) \)

For the calculations and a given \( \theta \), we use the following equities :

\[
I. \quad f_X(t) = \frac{\alpha k \left( \frac{t}{\beta} \right)^{\alpha-1}}{\beta \left[ 1 + \left( \frac{t}{\beta} \right)^{k} \right]^{\alpha+1}}
\]
II. \( F_X(t) = 1 - \left[ 1 + \left( \frac{x}{\beta} \right)^a \right]^k \)

III. \( E[X] = \frac{1}{\Gamma(k)} \beta \Gamma(1 + \frac{1}{\alpha}) \Gamma(k - \frac{1}{\alpha}) \)

IV. \( E[X \wedge d] = \frac{1}{\Gamma(k)} \beta \Gamma(1 + \frac{1}{\alpha}) \Gamma(k - \frac{1}{\alpha}) \left[ 1 + \frac{1}{\alpha}, k - \frac{1}{\alpha}, 1-u \right] + du^k \)

V. \( \beta[a,b,t] = \frac{\Gamma(a+b)}{\Gamma(a) \Gamma(b)} \int_0^t x^{a-1} (1-x)^{b-1} dx \)

VI. \( u = \left[ 1 + \left( \frac{d}{\beta} \right)^a \right]^{-1} \)

VII. \( \text{VaR}_{X|\theta}(p) = F^{-1}_{X|\theta}(p) = \beta \left[ (1-p)^{-\frac{1}{k}} - 1 \right] \)

VIII. \( \text{TVaR}_{X|\theta}(p) = \text{VaR}_{X|\theta}(p) + \frac{1}{1-p} \left[ E[ X - \text{VaR}_{X|\theta}(p)], ] \right] \)

IX. \( E[X] = E[X \wedge d] + E[|x-d|] \)

The formulas I and II have the nomenclature of the appendix like Software EasyFit 5.4 and similar to [4]. The formulas III – VI have the nomenclature like [4], using the name of parameters from EasyFit 5.4. The formulas VII and VIII use the definitions from [2] and IX can be consulted in [3].

7.1.1 DEFINITION OF THE PARAMETRIC SPACE

The estimator \( \hat{\theta} \) with the random sample of size 5,000 has an estimation of:

\( \hat{\theta} = (\hat{\alpha}, \hat{\beta}, \hat{k}) = (4.000585, 5.986607, 2.006663) \)

The space \( W_\Omega \) was built with \( W_1 = (3.7, 4.3) \), \( W_2 = (5.5, 6.5) \) and \( W_3 = (1.5, 2.5) \). Then, uniform random vectors \( \{ U_i \}_{i=1}^{200,000} \) were generated in \( W_\Omega \) from which 5,899 were in the set \( \Theta_{0.95} \), giving as result vectors in the parametric space at 95% confidence level.
Fig. 7.1.1. Shows the Parametric Space at 95% confidence level.

![Parametric Space](image1)

Fig. 7.1.2. Shows the Projections of $\Theta_{.95}$ in $\mathbb{R}^2$, the red axis are $\hat{\theta}$.

![Projections](image2)

7.1.2 **CALCULUS OF** $P[\theta|Z_n]$

Given the values $\{\theta_i, I(\theta_i)\}$ $\forall i \in M_2$ the function $P^*[\theta|Z_n]$ can be calculated as stated the section 4.
Fig. 7.1.3. Distribution of the parameters and Log-Likelihood

Fig. 7.1.4. Distribution of the probabilities $P^*[\theta|Z_n]$

7.1.3 SIMULATION OF $\theta|Z_n$

With uniform random variables $\{r_i\}_{i=1}^{5,000}$ and $P^*[\theta|Z_n]$ the sample $\{\theta_i\}_{i=1}^{5,000}$ was generated, and the values $\{VaR_{X|\theta_i}(p)\}_{i=1}^{5,000}$ and $\{TVaR_{X|\theta_i}(p)\}_{i=1}^{5,000}$ were obtained with $p = .95$. 

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7.1.4 Sensitivity of VaR & TVaR

With the random samples \( \{\text{VaR}_{X|\theta_i}(p)\}_{i=1}^{5,000} \) and \( \{\text{TVaR}_{X|\theta_i}(p)\}_{i=1}^{5,000} \), is possible to compute some statistics, the behavior is presenting in Tables 6.1 and 6.2.

Table 7.1.1. Empirical and Parametric Statistics from the random variable \( \text{VaR}_{X|\theta}(p) \)

<table>
<thead>
<tr>
<th>Estatistics</th>
<th>Empirical</th>
<th>Parametric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Value</td>
<td>8.157797</td>
<td>8.1578</td>
</tr>
<tr>
<td>Variance</td>
<td>0.003684346</td>
<td>0.00368</td>
</tr>
<tr>
<td>CI 95%</td>
<td>(8.043541, 8.276959)</td>
<td>(8.0417, 8.2796)</td>
</tr>
<tr>
<td>Mode</td>
<td>8.15604857907939</td>
<td>8.1548</td>
</tr>
<tr>
<td>( \text{VaR}_{905} )</td>
<td>8.312699</td>
<td>8.3198</td>
</tr>
</tbody>
</table>

Table 7.1.2. Empirical and Parametric Statistics from the random variable \( \text{TVaR}_{X|\theta}(p) \)

<table>
<thead>
<tr>
<th>Estatistics</th>
<th>Empirical</th>
<th>Parametric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Value</td>
<td>9.546565</td>
<td>9.5466</td>
</tr>
<tr>
<td>Variance</td>
<td>0.01212801</td>
<td>0.01213</td>
</tr>
<tr>
<td>CI 95%</td>
<td>(9.342187, 9.769140)</td>
<td>(9.3408, 9.7722)</td>
</tr>
<tr>
<td>Mode</td>
<td>9.5778466284898</td>
<td>9.5361</td>
</tr>
<tr>
<td>( \text{VaR}_{995} )</td>
<td>9.837143</td>
<td>9.8501</td>
</tr>
</tbody>
</table>

The parametric statistics were compute under a density function Log-Pearson 3, the goodness of fit were tested with the Kolmogorov Smirnov Test at 5% significance level, the p-value in both cases was bigger than 0.05, this calculations were made on EasyFit 5.4 with the sample generated by the code.
Table 7.1.3 Resume of the focus statistics

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$VaR_{95}$</td>
<td>8.190312</td>
<td>8.111643</td>
<td>8.158575</td>
<td>8.312699</td>
</tr>
<tr>
<td>$TVaR_{95}$</td>
<td>9.592636</td>
<td>9.615824</td>
<td>9.551571</td>
<td>9.837143</td>
</tr>
</tbody>
</table>

Table 7.1.4. Indicators about table 6.3.

<table>
<thead>
<tr>
<th>Function</th>
<th>$[2]/[1]$</th>
<th>$[3]/[1]$</th>
<th>$[4]/[1]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$VaR_{95}$</td>
<td>0.990394871</td>
<td>1.005785758</td>
<td>1.018891044</td>
</tr>
<tr>
<td>$TVaR_{95}$</td>
<td>1.002417271</td>
<td>0.993317993</td>
<td>1.029897909</td>
</tr>
</tbody>
</table>

The Table 7.1.4. show that the empirical and MVL estimates have estimations under the real values while with our methodology the estimates are over the real values, but just for 1.8% and 2.9%.

Remember that this statistics were obtained with the sample size $n=5,000$, then these estimates are strong estimates inside each method.

7.1.5 CONCLUSION EXAMPLE ONE

This numeric example works as we expected, in the sense that the estimators $VaR_{VaR}$ and $VaR_{TVaR}$ provide robust estimations that shows the worst possible values for the traditional $VaR$ and $TVaR$. Bringing the security that our estimations are for above the real just the enough.
7.2 EXAMPLE TWO ( $\text{Var}_{VaR}$ Vs $VaR$ )

Our model of loss is $f_{X|\lambda}(x) = \lambda e^{-\lambda x}$ and given a random sample $\{x_i\}_{i=1}^n$ is wanted to estimate $\hat{VaR}_X(p)$, $\text{Var}(\hat{VaR}_X(p))$ under a Bayesian perspective and delta method and finally compare under different sample sizes $\text{Var}_{VaR}$ Vs $VaR$

7.2.1 DELTA METHOD

This method establish that, for a maximum likelihood estimator the approximate variance of $\hat{g}(\theta)$ is $\text{Var}(\hat{g}(\theta)) \approx \left[ \frac{\partial}{\partial \theta} g(\theta) \right]_{\theta = \hat{\theta}}^2 \text{Var}(\hat{\theta})$ as state in [3]. For the exponential model and given that $\text{Var}(\hat{\theta}) = \frac{1}{I(\hat{\theta})}$ (where $I(\hat{\theta})$ is the Fisher information) is not hard to demonstrate that $\hat{\theta} = \frac{1}{\bar{X}}$ and $\text{Var}(\hat{\theta}) = \hat{\theta}^2/n$. Then, given that $g(\theta) = \text{VaR}_{X|\theta}(p) = F_{X|\theta}^{-1}(p)$ we have:

- $\text{VaR}_{X|\theta}(p) = \frac{1}{\hat{\theta}} \log \left( \frac{1}{1-p} \right) = \frac{1}{\hat{\theta}} \log \left( \frac{1}{1-p} \right)$

- $\text{Var}(\text{VaR}_{X|\theta}(p)) = \left[ \log \left( \frac{1}{1-p} \right) \right]^2 \frac{1}{\hat{\theta}^2/n} = \left[ \log \left( \frac{1}{1-p} \right) \right]^2 \frac{\bar{X}}{n}$

7.2.2 BAYESIAN METHOD

The space to work is $\Theta$, and $\pi_\theta(\theta) = 1$, hence $P[\theta|Z_n] \propto L(\theta)$ and $\theta \in \mathbb{R}^+$. Since $X|\theta \sim \text{exp}(\theta)$, then $L(\theta) = \theta^n \exp(-\theta \sum x_i)$ and since $P[\theta|Z_n] \propto \theta^n \exp(-\theta \sum x_i)$ we have that $\theta|Z_n \sim \text{Gamma}(\alpha = n+1, \theta = \frac{1}{\sum_{x_i}})$, in this case is not necessary to build a discrete parametric space because the model have an accessible and know distribution.
For the expression \( VaR_{X|\theta}(p) = \frac{1}{\theta} \log \left( \frac{1}{1-p} \right) \) the Bayesian variance is given by the following expression, \( \text{Var}(VaR_{X|\theta}(p)) = \left[ \log \left( \frac{1}{1-p} \right) \right]^2 \text{Var} \left[ \frac{1}{\lambda} \right] \). For Gamma \( (\alpha, \beta) \propto x^{\alpha-1} \exp(x/\beta) \) is known that \( E[X^k] = \frac{\theta^k \Gamma(\alpha+k)}{\Gamma(k)} \) for \( k > \alpha \) as stated in [4], then \( E[X^{-1}] = \frac{\Gamma(\alpha-1)}{\theta \Gamma(\alpha)} \) and 

\[
E[X^{-2}] = \frac{\Gamma(\alpha-2)}{\theta^2 \Gamma(\alpha)},
\]
then operating this we have \( \text{Var}(VaR_{X|\theta}(p)) = \left[ \log \left( \frac{1}{1-p} \right) \right]^2 \left[ \frac{\bar{X}}{n-1} \right] \).

Then, this shows that the variance for this statistic is similar under the delta method and the Bayesian method, will \( n \) gets bigger the estimate of the variance in both cases turns equal.

### 7.2.3 VaR Vs VaR

What about the estimation of the \( VaR \) and his stressed value \( VaR_{VaR} \) ? Remember that under a Bayesian context this is a random variable \( VaR_{X|\theta}(p) = \frac{1}{\theta} \log \left( \frac{1}{1-p} \right) \), then is possible to calculate the maximum possible value \( VaR_{VaR} = L_{\text{MAX}} \), this is such that \( P[VaR_{X|\theta}(p) \leq L_{\text{MAX}}] = p_2 \). With the last expression and knowing that \( \theta | Z_n \sim \text{Gamma}(\alpha = n+1, \theta = \frac{1}{\sum x}) \) is possible to demonstrate the following equity, where \( Q_{0|Z_n}(1-p_2) \) is the quantile \( 1-p_2 \) of \( \theta | Z_n \).

\[
VaR_{VaR_{VaR}(p)}(p_2) = \frac{\log \left( \frac{1}{1-p} \right)}{Q_{0|Z_n}(1-p_2)}
\]
The comparison between \( VaR \) and \( VaR_{VaR} \) will be shown with the simulations of five hundred different samples at same simple size, for different simple sizes. The results will be shown as the average of the simulated values \( VaR \) and \( VaR_{VaR} \) and the percentage of times that the estimation was bigger than the real statistics, this is the average of success.

Suppose that the real model is exponential and the true parameter is \( \theta = 1 \), hence the real value of \( VaR_{0.95} \) is 2.995732.

**Table 7.2.1 VaR Vs \( VaR_{VaR} \)**

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>Avarage</th>
<th>( VaR_{0.95} )</th>
<th>( VaR_{VaR(0.95)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>2.994336</td>
<td>3.889150</td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>2.987504</td>
<td>3.521833</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>2.997487</td>
<td>3.366267</td>
<td></td>
</tr>
<tr>
<td>1,000</td>
<td>3.002187</td>
<td>3.258347</td>
<td></td>
</tr>
<tr>
<td>2,500</td>
<td>2.993979</td>
<td>3.152803</td>
<td></td>
</tr>
</tbody>
</table>

**Table 7.2.2. Success of VaR Vs \( VaR_{VaR} \)**

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>Avarage Succes %</th>
<th>( VaR_{0.95} )</th>
<th>( VaR_{VaR(0.95)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>49</td>
<td>99.8</td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>49.2</td>
<td>99.4</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>50.4</td>
<td>99.4</td>
<td></td>
</tr>
<tr>
<td>1,000</td>
<td>51</td>
<td>99.6</td>
<td></td>
</tr>
<tr>
<td>2,500</td>
<td>48.8</td>
<td>99.6</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 7.2.1. Average Behavior of different estimators under different sample sizes.

By the Table 7.2.1 and Fig. 7.2.1 is possible to note that the MVL estimators have good estimations without high simple sizes, but have the inconvenient that is no possible to know certainly when this estimations are below the real value. In the case of the empirical estimators is needed to increase the sample size in order to have better estimations, but the problem of knowing if the estimations are below the real values persist. In the other hand, by Table 7.2.1 and Fig. 7.2.1 is possible to note that, with our methodology we can give an estimation that is almost certainly the maximum probable value or a bound of the real value and while the simple size increase this estimation turns close to the real one but almost always for above, in the Table 7.2.2 shows the percentage of success of the Bayesian estimation was in average almost the 99.5% effective will with the MVL estimation only an average of 50%

6.2.4 CONCLUSION EXAMPLE ONE

We conclude that the estimator $\text{VaR}_{\text{VaR}}$ provides a robust measure of $\text{VaR}$, almost always giving a bound for the real value and this bound decrease in function of the sample size. By the other hand about the estimation of the variation of $\text{VaR}$ was demonstrate that under two perspectives the estimators are almost the same. But the measure of $\text{VaR}$ and $\text{VaR}_{\text{VaR}}$ are totally different.
7. **GENERAL CONCLUSION**

Was shown that this methodology is helpful to understand the possible rational variations of the estimate of $\theta$ and his consequences over the risk measures $VaR$ and $TVaR$, measures of extreme scenarios. This methodology provide the tools to generate a risk measure that almost ensure that the stressed estimates of $VaR$ and $TVaR$ are just enough above the real one.

Therefore, this technique take into account not only the variation of model, but also taking into account the possible variation of the estimation of that model, so that, this technique is relevant in order to guarantee that funds reserved for the losses of a extreme scenario are sufficient.
BOOK REFERENCES


