Pricing European Options on Deferred Insurance Contracts

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This paper considers the pricing of European call options written on pure endowment and deferred life annuity contracts, also known as guaranteed annuity options. These contracts provide a guarantee value at maturity of the option. The contract valuation is dependent on stochastic interest rate and mortality processes. We assume single-factor stochastic square-root processes for both interest rate and mortality intensity, with mortality being a time inhomogeneous process. We then derive the pricing partial differential equation (PDE) and the corresponding transition density PDE for options written on deferred contracts. The general solution of the pricing PDE is derived as a function of the transition density function. We solve the transition density PDE by first transforming it to a system of characteristic PDEs using Laplace transform techniques and then applying the method of characteristics. Once an explicit expression for the density function is found, we then use sparse grid quadrature techniques to generate European call option prices on deferred insurance products. This approach can easily be generalised to other contracts which are driven by similar stochastic processes presented in this paper. We test the sensitivity of the option prices by varying independent parameters in our model. As option maturity increases, the corresponding option prices significantly increase. The effect of mis-pricing the guaranteed annuity value is analysed, as is the benefit of replacing the whole-life annuity with a term annuity to remove volatility of the old age population.