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Optimal Risk Classification and Underwriting Risk for Substandard Annuities

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Agenda

- Introduction
- The model framework
  - Basic model
  - Optimal risk classification
  - Optimal risk classification and costs of underwriting risk
- Market entry barriers and advantages
- Summary

Introduction
Motivation

- Substandard annuities offer increased pension payments for individuals with below-average life expectancy

- Surprisingly rare except for in the U.K. market

- Risk classification generally increases profitability (see Doherty, 1981), e.g., non-life

- Private pensions for person's with impaired health

- Reluctance of insurance companies to offer substandard annuity products

Introduction
Motivation

- Selling substandard annuities is a challenging task:

- Establish classification system based on insured's life expectancy

- Adequate underwriting guidelines are necessary

  Underwriting criteria: medical conditions or lifestyle factors

- Include classification costs when pricing the contract

- Demand for product is determined by annuity amount

Introduction
Motivation

- Aim of this paper:
- Comprehensive analysis with respect to substandard annuities
- Combine two strands of literature: substandard annuities and risk classification
- Develop a model to determine optimal profit-maximizing risk classification system for substandard annuities
- Crucial: account for classification costs and underwriting risk
- For (prospective) providers and standard insurers

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The Model Framework

Basic Model

- General population of potential risks with average population mortality
- Mortality heterogeneity in the general population considered by means of a frailty model

– Individual probabilities of death by application of a stochastic frailty factor to the population mortality table

$$q_x(d) = \begin{cases} d \cdot q'_x, & d \cdot q'_x < 1 \\ 1, \quad x = \min\left[\tilde{x} \in \{0,\ldots,\omega\} : d \cdot q'_{\tilde{x}} \geq 1\right] & \\ 0, & \text{otherwise} \end{cases} \qquad \text{for } x \in \{0,\ldots,\omega\}$$

The Model Framework
Basic Model

– Frailty factor specifies individual's state of health

– Frailty distribution represents distribution of different states of health (different life expectancies) in the general population

  Non-negative

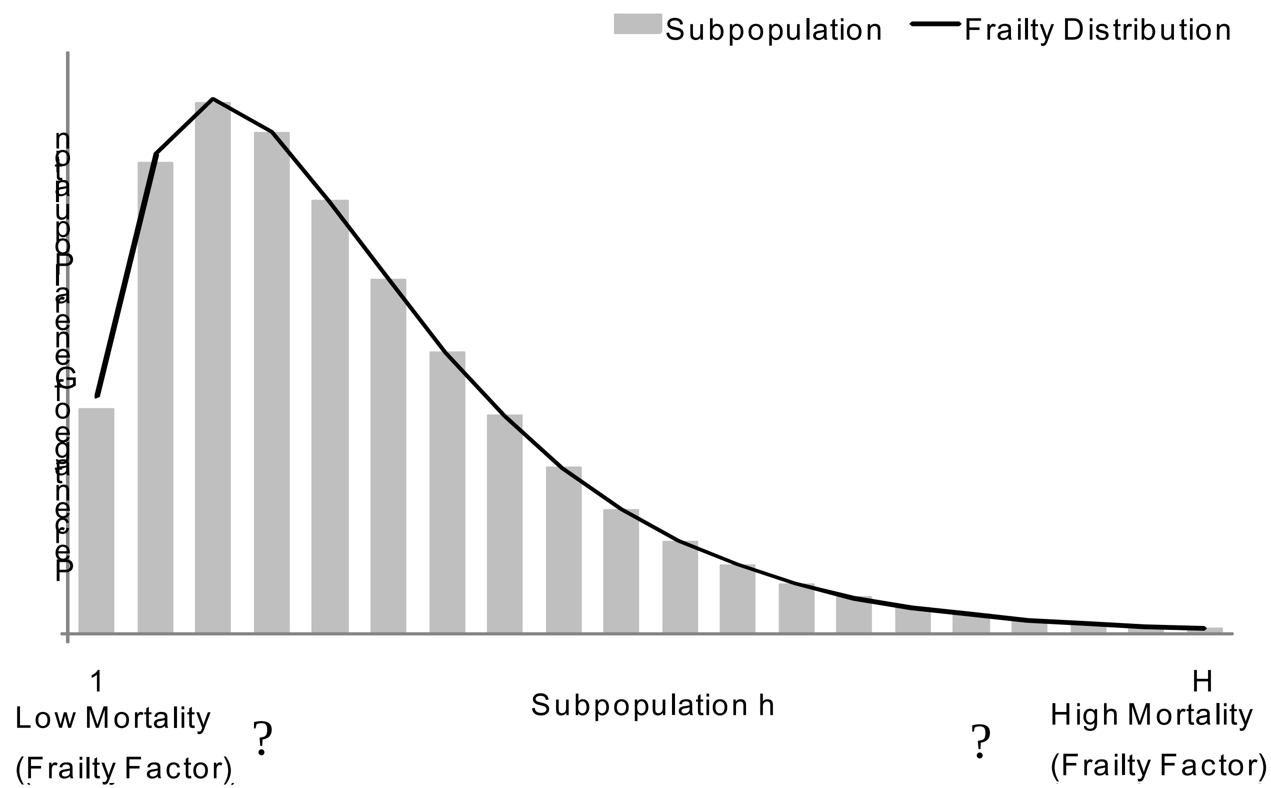
  Continuous

  Flat at 0, right-skewed

  Expected value of 1

The Model Framework

Basic Model

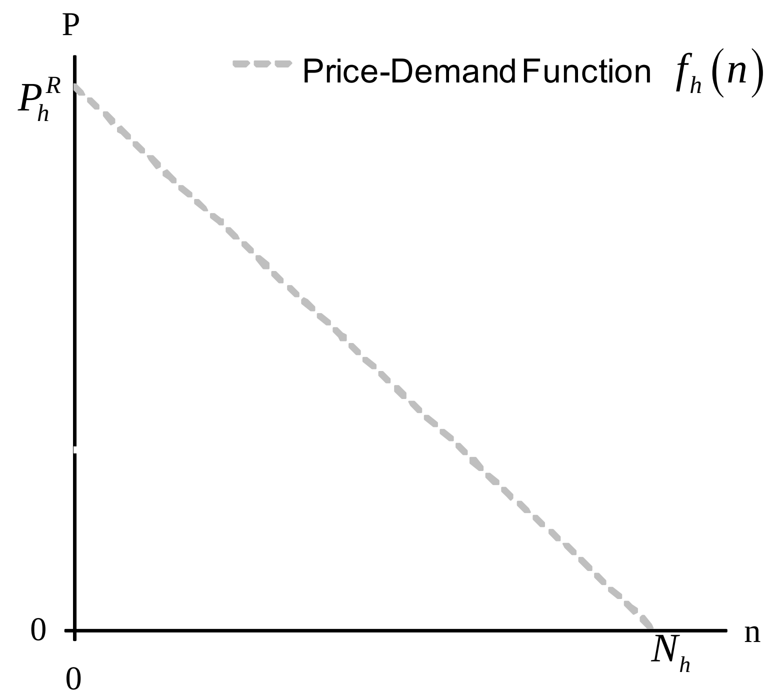


- *H* different subpopulations with differing mortality level

The Model Framework
Basic Model

- Characteristics of subpopulation *h, h=1,…,H*
  - Number of risks N_h
  - Price-demand function $f_h(n)$
    - Monotonously decreasing
    - Reservation price P_h^R decreases with increasing *h:*

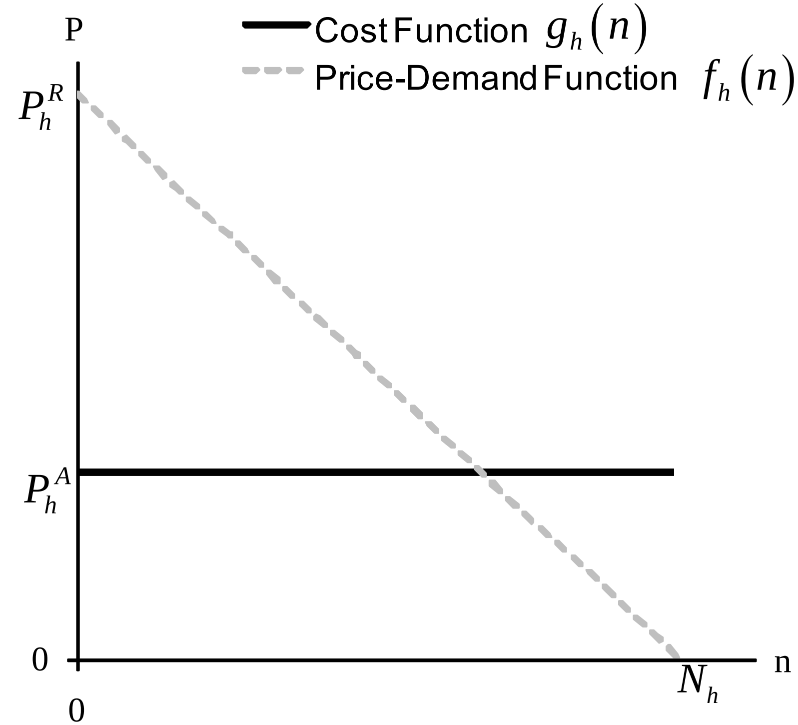
      $P_1^R > P_2^R > \cdots > P_H^R$
    - $f_h(N_h) = 0$



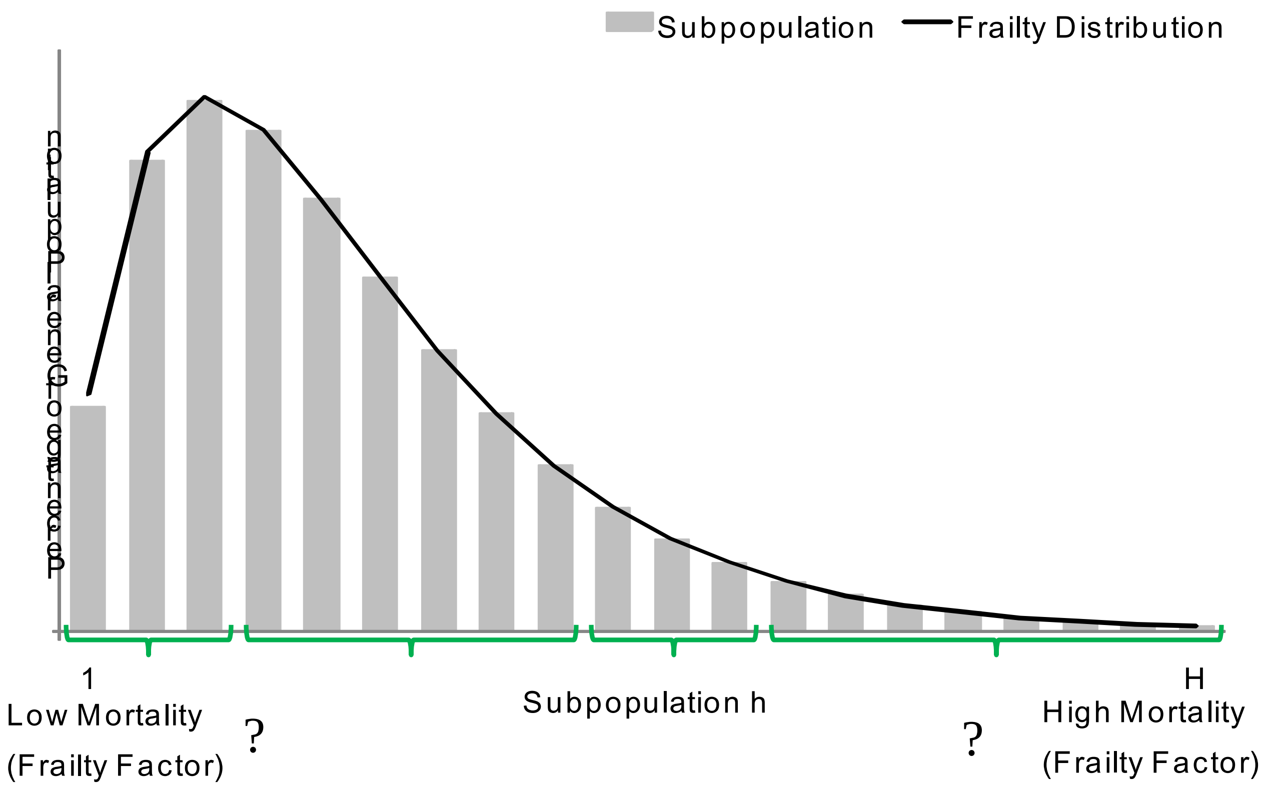
The Model Framework
Basic Model

– Cost function $g_h(n) = P_h^A$

- Actuarial premium for covering the cost of (one unit of) annuity insurance for the average potential insured in subpopulation *h*)
- Depending on the average frailty factor $\overline{d}_h$



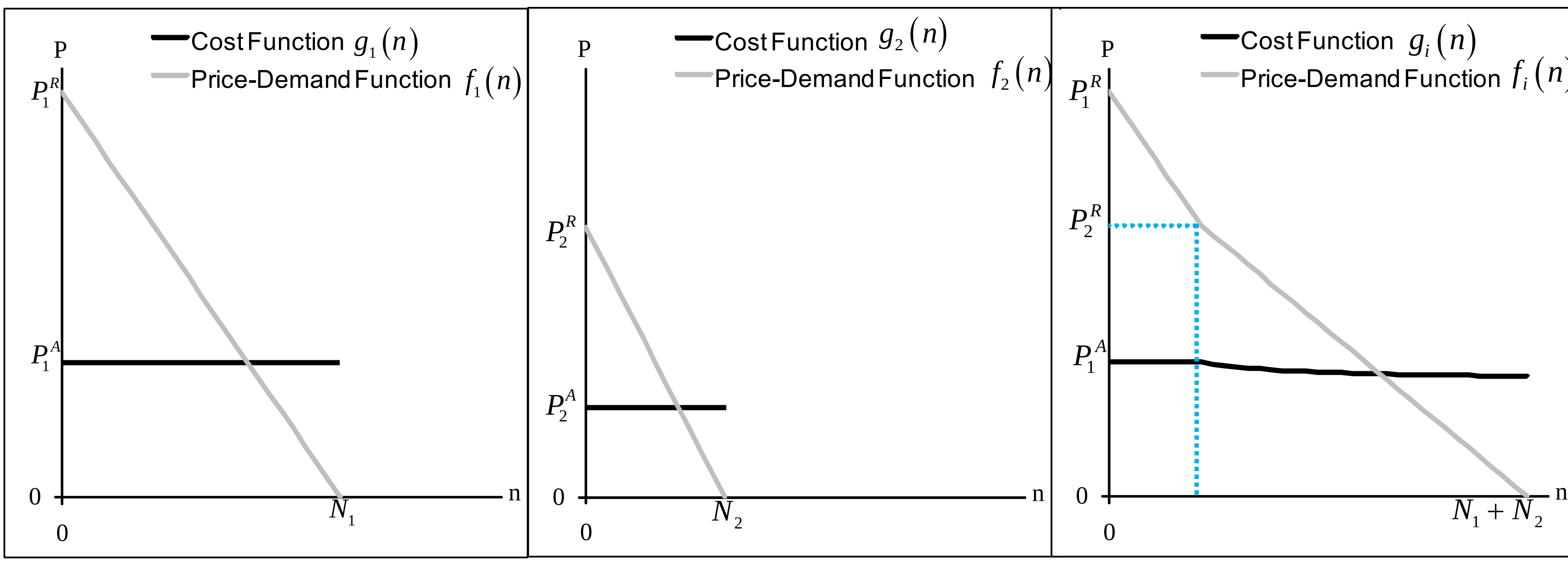
The Model Framework
Optimal Risk Classification



- I_m risk classes $i=1,\ldots, I_m$ in classification system m
- M set of all possible classification systems m

The Model Framework
Optimal Risk Classification

- Aggregate cost and price-demand function in risk class *i* consisting of 2 subpopulations



The Model Framework
Optimal Risk Classification

- Aggregate cost and price-demand function in risk class i, general formulas

$$f_i(n) = f_i\left(f_i^{-1}(P_i)\right) = f_i\left(\sum_{s=1}^{\nu} f_s^{-1}(P_i)\right)$$

$$g_i(n) = \frac{1}{n}\sum_{s=1}^{\nu} f_s^{-1}\left(f_i(n)\right)\cdot g_s\left(f_i(n)\right) = \frac{1}{n}\sum_{s=1}^{\nu} f_s^{-1}(P_i)\cdot g_s(P_i) = \frac{1}{n}\sum_{s=1}^{\nu} n_s \cdot P_s^A$$

$$\text{if } \; n \in I_\nu = \begin{cases} \left[\sum_{s=1}^{\nu} f_s^{-1}\left(P_\nu^R\right), \sum_{s=1}^{\nu+1} f_s^{-1}\left(P_{\nu+1}^R\right)\right) & \text{for } \nu = 1,\ldots,S_i - 1 \\ \left[\sum_{s=1}^{\nu} f_s^{-1}\left(P_\nu^R\right), N_i\right] & \text{for } \nu = S_i. \end{cases}$$

The Model Framework
Optimal Risk Classification

- Profit in risk class *i*

$$\Pi_i(n) = E_i(n) - C_i(n) = n \cdot f_i(n) - n \cdot g_i(n), n = 1,\ldots,N_i$$

- Classification costs $k(l_m - 1)$

- Total profit from classification system *m*

$$\Pi\left(n_1,\ldots,n_{l_m}\right) = \sum_{i=1}^{l_m} \Pi_i(n_i) - k(l_m - 1)$$

- Optimization problem

$$\max_{m \in M} \max_{\left\{\left(n_1,\ldots,n_{l_m}\right)\right\}} \Pi\left(n_1,\ldots,n_{l_m}\right)$$

The Model Framework

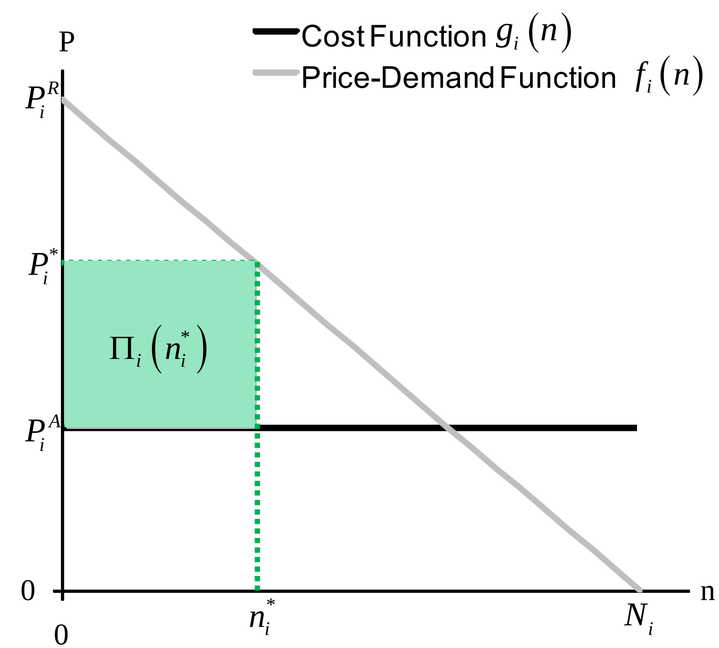
Optimal Risk Classification

- Maximization in recurrent steps:

  Find optimal price-demand combinations for each risk class *i* within each classification system $m \in M$

$$n_i^{\nu *} = \underset{n_i^{\nu} \in I_{\nu}, \nu=1,\ldots,S_i}{\operatorname{argmax}} \Pi_i\left(n_i^{\nu}\right)$$

$$m^* = \underset{m \in M}{\operatorname{argmax}} \Pi\left(n_1,\ldots,n_{I_m}\right)$$

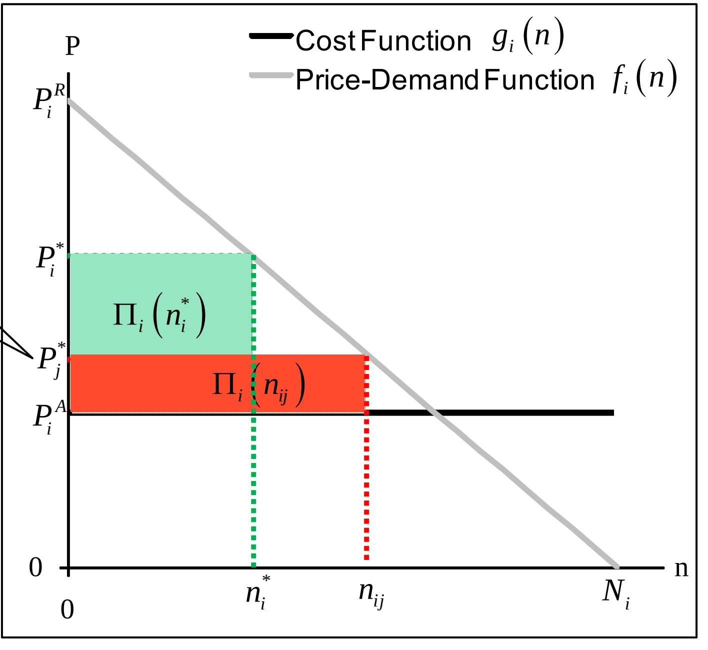


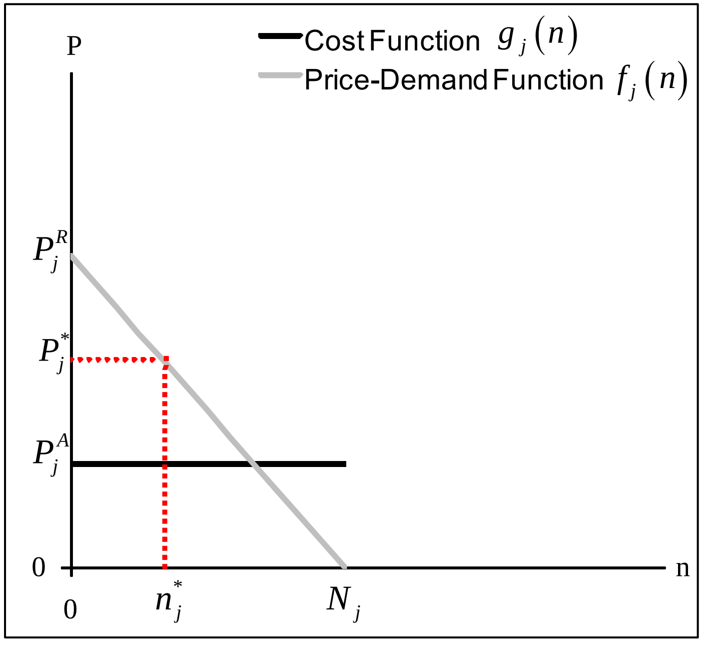
The Model Framework

Optimal Risk Classification & Costs of Underwriting Risk

- Underwriting risk is one of the main reasons why insurers are reluctant to engage in risk classification
- Ex.: Effect of underwriting errors for two risk classes

Underestimate true cost of insurance





The Model Framework

Optimal Risk Classification & Costs of Underwriting Risk

- Expected profit in risk class i, given probability p_{ij} of wrongly classifying (risk class j instead of i)

$$\tilde{\Pi}_i = \sum_{j \geq i} p_{ij} \Pi_i \left(n_{ij} \right)$$

- Error probability depends on classification system:

– Larger number of risk classes

=> smaller differences between risk classes

=> higher error probabilities for adjacent classes, but smaller effects of wrong classification

- Extended optimization problem: $m^{**} = \arg\max_{m \in M} \tilde{\Pi} \left(n_1, \ldots, n_{l_m} \right)$

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Market entry barriers and advantages

- Market entry barriers
  - Classification costs
  - Underwriting risk
  - Market very competitive
  - Competition by other financial products
  - Low market awareness requires strong distribution system
  - Attractive and innovative product design needed
  - Impact on existing portfolios

Market entry barriers and advantages

- Advantages
  - Huge market potential
  - Attractive for new market players
  - Possibility to reach broader population or niche markets
  - Early involvement prevents defensive reactions
  - Benefit of competitive advantage
  - Avoidance of adverse selection problems

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Summary

- Propose a model for a risk classification system in a mortality heterogeneous general population
  - Solve for optimal number and size of risk classes
  - Profit-maximizing price-demand combination in each risk class
  - Extension: account for costs of underwriting risk in optimization
- Discuss market entry barriers and advantages
- In summary: Risk classification in annuity markets

  …not only increases profitability of insurance companies

  …but also benefits society at large, since formerly uninsurable persons gain access to private pensions

Optimal Risk Classification and Underwriting Risk for Substandard Annuities

Thank you very much for your attention!

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