Optimal Risk Classification and Underwriting Risk for Substandard Annuities

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Agenda

• Introduction

• The model framework
  – Basic model
  – Optimal risk classification
  – Optimal risk classification and costs of underwriting risk

• Market entry barriers and advantages

• Summary
Introduction
Motivation

• Substandard annuities offer increased pension payments for individuals with below-average life expectancy
  - Surprisingly rare except for in the U.K. market
  - Risk classification generally increases profitability (see Doherty, 1981), e.g., non-life
  - Private pensions for person's with impaired health
  - Reluctance of insurance companies to offer substandard annuity products
Introduction
Motivation

- Selling substandard annuities is a challenging task:
  - Establish classification system based on insured's life expectancy
  - Adequate underwriting guidelines are necessary
    Underwriting criteria: medical conditions or lifestyle factors
  - Include classification costs when pricing the contract
  - Demand for product is determined by annuity amount
Introduction
Motivation

• Aim of this paper:
  - Comprehensive analysis with respect to substandard annuities
  - Combine two strands of literature: substandard annuities and risk classification
  - Develop a model to determine optimal profit-maximizing risk classification system for substandard annuities
  - Crucial: account for classification costs and underwriting risk
  - For (prospective) providers and standard insurers
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The Model Framework

Basic Model

- General population of potential risks with average population mortality
- Mortality heterogeneity in the general population considered by means of a frailty model
  - Individual probabilities of death by application of a stochastic frailty factor to the population mortality table

\[
q_x(d) = \begin{cases} 
  d \cdot q'_x, & d \cdot q'_x < 1 \\
  1, & x = \min \left\{ \tilde{x} \in \{0, \ldots, \omega\} : d \cdot q'_x \geq 1 \right\} \\
  0, & \text{otherwise}
\end{cases}
\] for \( x \in \{0, \ldots, \omega\} \).
The Model Framework
Basic Model

- Frailty factor specifies individual's state of health

- Frailty distribution represents distribution of different states of health (different life expectancies) in the general population
  Non-negative
  Continuous
  Flat at 0, right-skewed
  Expected value of 1
The Model Framework

Basic Model

- $H$ different subpopulations with differing mortality level
The Model Framework

Basic Model

- Characteristics of subpopulation $h$, $h=1,\ldots,H$
  - Number of risks $N_h$
  - Price-demand function $f_h(n)$
    - Monotonously decreasing
    - Reservation price $P^R_h$ decreases with increasing $h$:
      \[ P^R_1 > P^R_2 > \cdots > P^R_H \]
    - $f_h(N_h) = 0$
The Model Framework

Basic Model

- Cost function \( g_h(n) = P_h^A \)
  
  - Actuarial premium for covering the cost of (one unit of) annuity insurance for the average potential insured in subpopulation \( h \)
  
  - Depending on the average frailty factor \( d_h \)
The Model Framework
Optimal Risk Classification

- $I_m$ risk classes $i=1,\ldots, I_m$ in classification system $m$
- $M$ set of all possible classification systems $m$

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The Model Framework
Optimal Risk Classification

- Aggregate cost and price-demand function in risk class $i$ consisting of 2 subpopulations
The Model Framework
Optimal Risk Classification

- Aggregate cost and price-demand function in risk class $i$, general formulas

$$f_i(n) = f_i(f_i^{-1}(P_i)) = f_i\left(\sum_{s=1}^{\nu} f_s^{-1}(P_i)\right)$$

$$g_i(n) = \frac{1}{n} \sum_{s=1}^{\nu} f_s^{-1}(f_i(n)) \cdot g_s(f_i(n)) = \frac{1}{n} \sum_{s=1}^{\nu} f_s^{-1}(P_i) \cdot g_s(P_i) = \frac{1}{n} \sum_{s=1}^{\nu} n_s \cdot P_s^A$$

if $n \in I_\nu = \left[\left[\sum_{s=1}^{\nu} f_s^{-1}(P_{\nu^R}),\sum_{s=1}^{\nu+1} f_s^{-1}(P_{\nu^R+1})\right] \quad \text{for } \nu = 1, \ldots, S_i - 1\right.$

$$\left[\sum_{s=1}^{\nu} f_s^{-1}(P_{\nu^R}), N_i\right] \quad \text{for } \nu = S_i.$$
The Model Framework
Optimal Risk Classification

• Profit in risk class $i$
  $$\Pi_i(n) = E_i(n) - C_i(n) = n \cdot f_i(n) - n \cdot g_i(n), n = 1, \ldots, N_i$$

• Classification costs $k(I_m - 1)$

• Total profit from classification system $m$
  $$\Pi(n_1, \ldots, n_{I_m}) = \sum_{i=1}^{I_m} \Pi_i(n_i) - k(I_m - 1)$$

• Optimization problem
  $$\max_{m \in M} \max \left\{ \Pi(n_1, \ldots, n_{I_m}) \right\}$$
The Model Framework
Optimal Risk Classification

- Maximization in recurrent steps:
  
  Find optimal price-demand combinations for each risk class $i$ within each classification system $m \in M$

\[
\begin{align*}
    n_i^* &= \arg\max_{n_i^* \in \nu, \nu=1,...,S_i} \Pi_i (n_i^*) \\
    m^* &= \arg\max_{m \in M} \Pi (n_1, ..., n_{M})
\end{align*}
\]
The Model Framework
Optimal Risk Classification & Costs of Underwriting Risk

- Underwriting risk is one of the main reasons why insurers are reluctant to engage in risk classification
- Ex.: Effect of underwriting errors for two risk classes
The Model Framework
Optimal Risk Classification & Costs of Underwriting Risk

- Expected profit in risk class \( i \), given probability \( p_{ij} \) of wrongly classifying (risk class \( j \) instead of \( i \))

\[
\tilde{\Pi}_i = \sum_{j \geq i} p_{ij} \Pi_i(n_{ij})
\]

- Error probability depends on classification system:
  - Larger number of risk classes
    => smaller differences between risk classes
    => higher error probabilities for adjacent classes, but smaller effects of wrong classification

- Extended optimization problem:
  \[
m^{**} = \arg\max_{m \in M} \tilde{\Pi}(n_1, ..., n_{l_m})
\]
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Market entry barriers and advantages

• Market entry barriers
  – Classification costs
  – Underwriting risk
  – Market very competitive
  – Competition by other financial products
  – Low market awareness requires strong distribution system
  – Attractive and innovative product design needed
  – Impact on existing portfolios
Market entry barriers and advantages

- Advantages
  - Huge market potential
  - Attractive for new market players
  - Possibility to reach broader population or niche markets
  - Early involvement prevents defensive reactions
  - Benefit of competitive advantage
  - Avoidance of adverse selection problems
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Summary

- Propose a model for a risk classification system in a mortality heterogeneous general population
  - Solve for optimal number and size of risk classes
  - Profit-maximizing price-demand combination in each risk class
  - Extension: account for costs of underwriting risk in optimization

- Discuss market entry barriers and advantages

- In summary: Risk classification in annuity markets
  ...not only increases profitability of insurance companies
  ...but also benefits society at large, since formerly uninsurable persons gain access to private pensions
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Thank you very much for your attention!

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