Mortality when Converting from Conventional Life Insurance to Universal Life Insurance Policies

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Conventional and universal life models

Conventional model

Prospective calculation: Savings = present value of future benefits – present value of future payments

$$V_{x+t} = A_{x+t:w}(*) \cdot S_{x+t} - B_{x+t:k-t]} \cdot \ddot{a}_{x+t:k-t]}$$

Universal life model

Retrospective calculation: Savings = payments added by given compensations and reduced by given charged premiums and loadings

(cf. bank accounts)

$$V_{x+t} = \sum_{k=0}^{t} \left\{ (1-\beta) \cdot B_{x+k} + \frac{i}{1-q_{x+t}} \cdot (V_{x+k-1} + B_{x+k}) - \frac{q_{x+t}}{1-q_{x+t}} \cdot \left[(1-\gamma) \cdot S_{x+t} - (V_{x+k-1} + B_{x+k}) \right] \right\}$$

Often repeated arguments

- Administration of products using conventional models is cheap
- We know our products
- Conventional models cannot be transfered into universal life models

Three scenarios

1: Neutron bomb

employees disappear – computers remain

2: Fahrenheit 2800

iron melts – employees remain

3: Achilles, the immortal

dignified aging

Scenario 1: Neutron bomb

Resignation known beforehand

- education
- orientation
- fall of efficiency

Resignation as a surprise

- as above, added by
- stronger commitment of the management and actuaries
- possible outsourcing services
- contemporary decrease of service level

Scenario 2: Fahrenheit 2800

Hardware repairable

- urgent reparation of the hardware (seller’s market)
- contemporary decrease of service level

New software

- Implementing/purchasing new software for new platform
- requires resources of professionals
- conversion

Scenario 3: Achilles, the immortal

Option to continue runing small portfolios in old computers

Option to transfer the portfolio e.g. to excel
- cooperation between actuaries and back-office personnel
- exceptional back-office processes

Argumenting the investment decision difficult because most of the costs have already been realized:
  - personnel turnover, hardware
  - software environment costs (interfaces, testing)

Substitute arrangements also in case of small portfolios

Summary of products using conventional methods

- Neither of the scenarios will become materialized as described, but there will be costs – some of them extraordinary

- Costs are not only software costs or only personnel costs

- Knowhow becomes out of date and personnel availability might be a problem

- Solving problems involves the management and actuaries

- Early conversion to universal life saves money

  - target: same processes as in existing systems
  - conversion to existing system fast

Understanding your business

Universal life model more informative

- shows clearly the components
- basis for all cashflow-models
- also non-actuaries understand the models

"It may be … taken for granted that the days of the glory for the commutation numbers now belong to the past".
Argument for this is the
-"advent of powerful computers" and
-"growing acceptance of models based on probability theory, which allows a more complete understanding of the essentials of the insurance"

(Hans U. Gerber)

Transfer to universal life model possible

...if the model is well-defined and

...if common calculation principles are used

The premium is the present value of future cash flows.

Part of a well-defined cash flow is well-defined itself.

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An actuary does trust on others...

... without testing by himself

... or without proving by himself

... or without seeing a proof by himself

Working paper (Actuarial Association of Finland): "Conversion from conventional life insurance policies into universal life policies"

An actuary is either right or can prove he is.

Modeling on annual basis

Payment

$$B_{x:k]} = \frac{A_{x:w}(*)}{\ddot{a}_{t:k]}} \cdot S_x$$

Prospective reserve

$$V_{x+t} = A_{x+t:w}(*) \cdot S_{x+t} - B_{x+t:k-t]} \cdot \ddot{a}_{x+t:k-t]}$$

Commutations numbers

Discrete model

$$D_x = l_x \cdot v^x$$

$$N_x = \sum_{i=x}^{w'} D_i$$

$$M_x = \sum_{i=x}^{w'} \left[\frac{q_x}{1+i} \cdot D_x \right]$$

Continuous model

$$D_x = l_x \cdot v^x$$

$$N_x = \sum_{i=x}^{w'} D_i$$

$$\overline{N}_x = N_x - D_x \cdot \left[\frac{1}{2} + \frac{1}{12} \cdot (\ln(1+i) + \mu_x) \right]$$

$$\overline{M}_x = D_x - \ln(1+i) \cdot \overline{N}_x$$

Continuity correction

Example: Pure endownment

$$V_{x+t+1} = \frac{D_w}{D_{x+t+1}} \cdot S_{x+t}$$

$$= \frac{D_{x+t}}{D_{x+t+1}} \cdot \frac{D_w}{D_{x+t}} \cdot S_{x+t}$$

$$= \frac{D_{x+t}}{D_{x+t+1}} \cdot V_{x+t}$$

$$= V_{x+t} + \frac{i}{1-q_{x+t}} \cdot V_{x+t} + \frac{q_{x+t}}{1-q_{x+t}} \cdot V_{x+t}$$

Universal life model on annual basis

$$
\begin{aligned}
V_{x+t+1} =& \\
& V_{x+t} \\
& + B_{x+t:k-t]} \\
& - E_{x+t} \\
& + \frac{i}{1-q_{x+t}} \cdot \left[V_{x+t} + B_{x+t:k-t]} - E_{x+t} \right] \\
& - \frac{q_{x+t}}{1-q_{x+t}} \cdot \left[S_{x+t} - (V_{x+t} + B_{x+t:k-t]} - E_{x+t} \right]
\end{aligned}
$$

Savings =

- initial savings

+ payments

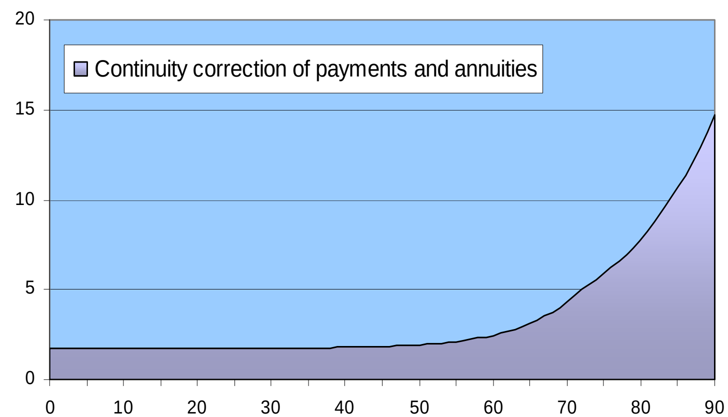
- pension outpayments

+ interest

- mortality premium/+mortality compensation

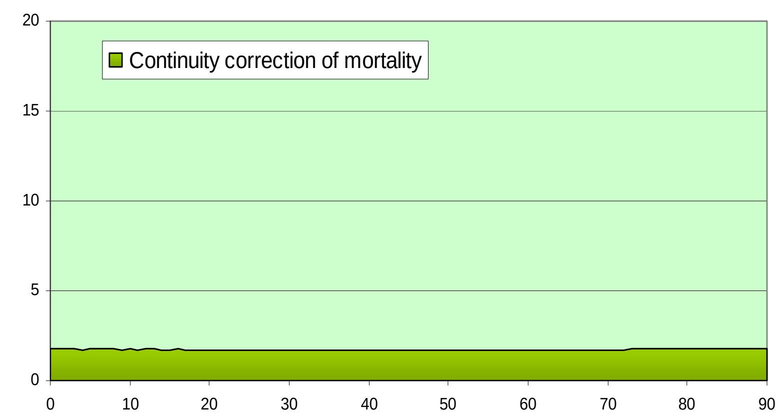
Effects of continuity corrections (%)

Premium



$$-\left\{\frac{1}{12}\cdot(\mu_{x+t}-\mu_{x+t+1})+\frac{i+q_{x+t}}{1+i}\cdot\left[\frac{1}{2}+\frac{1}{12}\cdot\left(\ln(1+i)+\mu_{x+t+1}\right)\right]\right\}\cdot B_{x+t}$$

Mortality



$$\left(-\frac{i}{1-q_{x+t}}+\ln(1+i)\cdot\frac{1+i}{1-q_{x+t}}\cdot\left\{1-\frac{1}{12}\cdot(\mu_{x+t}-\mu_{x+t+1})-\frac{i+q_{x+t}}{1+i}\cdot\left[\frac{1}{2}+\frac{1}{12}\cdot\left(\ln(1+i)+\mu_{x+t+1}\right)\right]\right\}\right)\cdot S_{x+t}$$

Modeling complex functions is possible

Example: sickness insurance formulas in Finland

$$A_{x:w}(*) = (1+\omega)\cdot\left[a\cdot \ddot{a}_{x:w} + b\cdot 10^{-3}\cdot \int_x^w (0{,}1\cdot u)^5 \cdot \frac{D_u}{D_x}\,du\right]$$

Annual risk premium

$$(1+\omega)\cdot\frac{1+i}{1-q_{x+t}}\cdot\left(a-b\cdot\frac{0{,}1^7}{4}\cdot(x+t)^4\cdot\left\{\frac{1}{5}\cdot(x+t)+\frac{1}{3}\cdot\left[5-0{,}1\cdot(x+t)\cdot(\delta+\mu_{x+t})\right]\right\}\right)$$

$$+(1+\omega)\cdot b\cdot\frac{0{,}1^7}{4}\cdot(x+t+1)^4\cdot\left\{-\frac{1}{5}\cdot(x+t+1)+\frac{1}{3}\cdot\left[5-0{,}1\cdot(x+t+1)\cdot(\delta+\mu_{x+t+1})\right]\right\}$$

Monthly calculation

Mortality

- discont factor preserving method
- mortality risk preserving method

Discount factor (including mortality)

$$\frac{D_{x+t}}{D_{x+t+1}} = \frac{1+i}{1-q_{x+t}} = 1 + \frac{i}{1-q_{x+t}} + \frac{q_{x+t}}{1-q_{x+t}}$$

Additional loading related to several times a year paid premiums

Discount factor preserving method

When converting policies the basis should be preserving discount factor.

Mortality is defined so that the annual discount factor does not change:

$$\frac{D_x}{D_{x+1}} = \prod_{m=0}^{11} \frac{D_{x+m/12}}{D_{x+(m+1)/12}} = \frac{1+i}{1-q_x}$$

So, monthly discount factor is as follows:

$$\frac{D_{x+m/12}}{D_{x+(m+1)/12}} = \frac{\sqrt[12]{1+i}}{1-q_{x+m/12}}$$

Actuary spends three months deriving graduated rates of mortality to five decimal places and then decide in thirty seconds that the proper rate of interest to be used in premium calculations is 2,5 %.

Mortality models for non-integer ages

exact mortality from e.g. continuous mortality function	$l_{x+m/12} = l_0 \cdot e^{-\int_0^{x+m/12} \mu_s ds}$	$q_{x+m} = 1 - e^{-\int_{x+m/12}^{x+(m+1)/12} \mu_s ds}$
unified distribution of deaths (UDD) - modified	$l_{x+m/12} = \left(1-\frac{m}{12}\right)\cdot l_x + \frac{m}{12}\cdot l_{x+1}$	$q_{x+m/12} = \frac{q_x^u}{12 - m\cdot q_x^u}$
Balducci-model - modified	$\frac{1}{l_{x+m/12}} = \frac{1-\frac{m}{12}}{l_x} + \frac{\frac{m}{12}}{l_{x+1}}$	$q_{x+m/12} = \frac{q_x^b}{12-(12-m)\cdot q_x^b}$
Linear Dx-model	$D_{x+(m+1)/12} = D_{x+m/12} - \frac{1}{12}\cdot(D_x - D_{x+1})$	$q_{x+m/12} = 1-\frac{(12-m)\cdot(1+i)+(1-q_x)\cdot m}{(11-m)\cdot(1+i)+(1-q_x)\cdot(m+1)}\cdot\sqrt[12]{1+i}$
Linear discount factor model	$\frac{1}{12}\cdot\left(\frac{D_x}{D_{x+1}}-1\right) = \frac{1}{12}\cdot\left(\frac{1+i}{1-q_x}-1\right)$	$q_{x+m/12} = 1-\frac{12+m\cdot i-(12-m)\cdot q_x}{12+(m+1)\cdot i-(11-m)\cdot q_x}\cdot\sqrt[12]{1+i}$

Example: Linear discount factor model

Reserve changes linearly across non-integer years.

Monthly change is:

$$\frac{1}{12}\cdot\left(\frac{D_x}{D_{x+1}}-1\right)=\frac{1}{12}\cdot\left(\frac{1+i}{1-q_x}-1\right)=\frac{1}{12}\cdot\frac{i+q_x}{1-q_x}$$

After m months this is:

$$D_{x+m/12}=\frac{D_x}{1+\frac{m}{12}\cdot\frac{i+q_x}{1-q_x}}=12\cdot\frac{1-q_x}{12-12\cdot q_x+m\cdot(i+q_x)}\cdot D_x=12\cdot\frac{1-q_x}{12+m\cdot i-(12-m)\cdot q_x}\cdot D_x$$

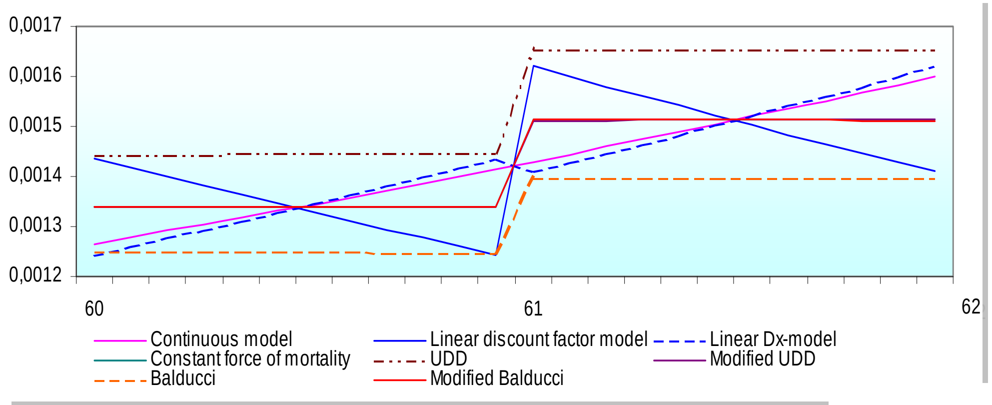
Discount factor preserving method yields:

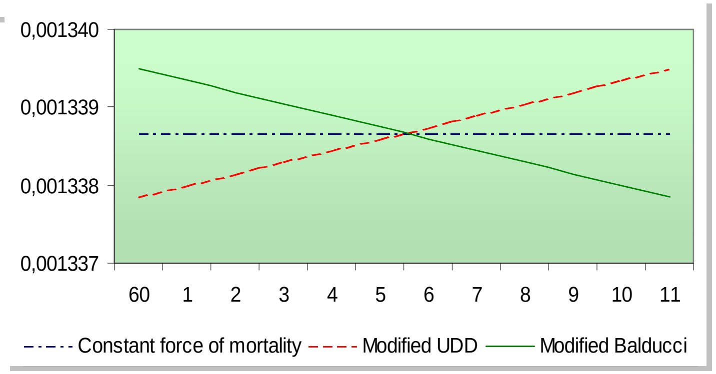
$$\frac{D_{x+m/12}}{D_{x+(m+1)/12}}=\frac{\sqrt[12]{1+i}}{1-q_{x+m/12}}=\frac{12\cdot\frac{1-q_x}{12+m\cdot i-(12-m)\cdot q_x}\cdot D_x}{12\cdot\frac{1-q_x}{12+(m+1)\cdot i-(12-(m+1))\cdot q_x}\cdot D_x}=\frac{12+(m+1)\cdot i-(11-m)\cdot q_x}{12+m\cdot i-(12-m)\cdot q_x}$$

From this we may solve the monthly mortality:

$$q_{x+m/12}=1-\frac{12+m\cdot i-(12-m)\cdot q_x}{12+(m+1)\cdot i-(11-m)\cdot q_x}\cdot\sqrt[12]{1+i}$$

Mortality models



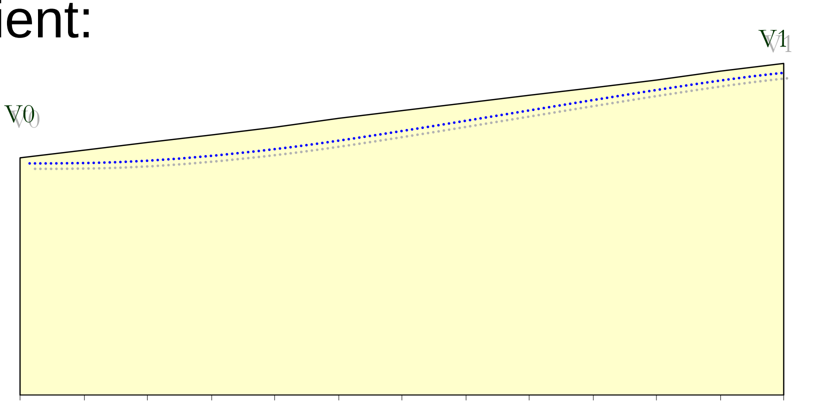


Mortality risk preserving method

Though mortality was based, the risk premiums should not. Some options:

1) Charge at the end of the insurance year
2) Charge in the beginning of the insurance year
3) Level premium
4) Monthly charge resulting in linear reserve change by modifying the cover with the following coefficient:

$$\frac{1}{12}\cdot\frac{1}{q_{x+t+m/12}}\cdot\left[\left(1-q_{x+t+m/12}\right)\cdot m-\sqrt[12]{1+i}\cdot(m-1)\right]$$



Further considerations

Fixed / flexible payment plans

Customer promises / approximations

A good actuary designs simple models...

... after he understands the behavior of the accurate model.

An actuary is the type of person that measures the length of a room by stepping one foot in front of the other, and then uses a micrometer to measure the final remaining position.

Seldom repeated arguments

- Administration of products using conventional models is expensive in the long run and when the portfolios become smaller
- It is useful to test the traditional products with a universal life -model
- Conventional models can be transfered into universal life models

... but this is the case

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More information:

Actuarial Association of Finland (Working paper):

”Jari Niittuinperä: Conversion from conventional life insurance policies into universal life policies"

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