Sensitivity Analysis and Worst-Case Analysis – Making use of netting effects when designing insurance contracts

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IAA LIFE Colloquium 2009 in Munich, Germany
Risks in life insurance

present value of future benefits and premiums depends on ...

(a) random pattern of states

\[ X_t - r \]

\[ X_{t_0} \]

\[ d \]

\[ b \]

biometrical risk:

unsystematic biometrical risk (diversifiable)

systematic biometrical risk (non-diversifiable)

(b) investment return / interest rate

\[ \phi(t) \]

financial risk (non-diversifiable)

(c) ...
Risks in life insurance

present value of future benefits and premiums depends on ...

(a) random pattern of states ($X_t$)

(b) investment return / interest rate

(c) ...
Risks in life insurance

present value of future benefits and premiums depends on ...

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→ biometrical risk:
  • unsystematic biometrical risk (diversifiable)
Risks in life insurance

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(a) random pattern of states ($X_t$)

→ biometrical risk:
  - unsystematic biometrical risk (diversifiable)
  - systematic biometrical risk (non-diversifiable)
Risks in life insurance

present value of future benefits and premiums depends on ...

(a) random pattern of states \((X_t)\)

\[ X_t \rightarrow X_0 \]

\[ \text{biometrical risk:} \]
- unsystematic biometrical risk (diversifiable)
- systematic biometrical risk (non-diversifiable)

(b) investment return / interest rate \(\varphi(t)\)

\[ \varphi(t) \rightarrow \text{financial risk} \ (\text{non-diversifiable}) \]

(c) ...
Getting rid of the non-diversifiable risks?

(i) benefits guaranteed

\[
\text{Policyholder} \quad \text{Insurer} \quad \text{Third Party}
\]
Getting rid of the non-diversifiable risks?

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\[ \text{Policyholder} \quad \text{Insurer} \quad \text{Third Party} \]

(ii) benefits according to surplus

e.g. with profits policies such as DC pension plans

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• contrary to customer demand (in Germany)!
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e.g. reinsurance, securitisation, ...

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e.g. reinsurance, securitisation, ...

\[\text{Policyholder} \quad \text{Insurer} \quad \text{Third Party}\]

- enough capacity for risk transfer?
Getting rid of the non-diversifiable risks?

(iv) reality (in Germany): compositions of (i) – (iii)

\[
\begin{array}{c}
\text{Policyholder} & \text{Insurer} & \text{Third Party} \\
\end{array}
\]
Getting rid of the non-diversifiable risks?

(iv) reality (in Germany): compositions of (i) – (iii)

Policyholder ──── Insurer ──── Third Party

• still a significant load of systematic risks for the insurer!
Getting rid of the non-diversifiable risks?

(iv) **reality** (in Germany): compositions of (i) – (iii)

<table>
<thead>
<tr>
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- still a significant load of systematic risks for the insurer!

**Question A:** effect of policy design on risk load of the insurer?

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\[
\text{Policyholder} \quad \leftrightarrow \quad \text{Insurer} \quad \text{Third Party}
\]

\[
\text{±}
\]

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\text{±}
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\[
\text{±}
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(v) **netting effects**

e.g. survival benefits vs. death benefits

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**Question B:** which combinations give strong netting effects?
TOOL 1:

Sensitivity Analysis
Effect of changes of the valuation basis on premiums and reserves

**qualitative results:**
Effect of changes of the valuation basis on premiums and reserves

qualitative results:

quantitative results:
Effect of changes of the valuation basis on premiums and reserves

**qualitative results:**

**quantitative results:**

\[
f(x + \Delta x) \approx f(x) + \langle \nabla_x f, \Delta x \rangle
\]

\[\sim\] interpret \( \nabla_x f = f'(x) \) as sensitivity of \( f \) at \( x \)
Effect of changes of the valuation basis on premiums and reserves

qualitative results:

quantitative results:

\[ f(x + \Delta x) \simeq f(x) + \langle \nabla_x f, \Delta x \rangle \]

\( \sim \) interpret \( \nabla_x f = f'(x) \) as sensitivity of \( f \) at \( x \)

- general approach for all valuation basis parameters and with continuous time?
General sensitivity analysis (C., 2008)

LET

\[ V_{t,a}(\varphi, \mu_{ad}, \mu_{ai}, \ldots) := \text{prospective reserve at time } t \text{ in state } a \]

with a valuation basis \((\varphi, \mu_{ad}, \mu_{ai}, \ldots)\) whose entries are either
(a) vectors of yearly interest rates, mortality probabilities, etc.
(b) or intensity functions for interest, mortality, etc.
(c) or cumulative intensity functions for interest, mortality, etc.
General sensitivity analysis (C., 2008)

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\[ V_{t,a}(\varphi, \mu_{ad}, \mu_{ai}, ...) \] := prospective reserve at time ’t’ in state ’a’

with a valuation basis \((\varphi, \mu_{ad}, \mu_{ai}, ...)\) whose entries are either

(a) vectors of yearly interest rates, mortality probabilities, etc.
(b) or intensity functions for interest, mortality, etc.
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THEN there exists a generalized first-order Taylor expansion

\[ V_{t,a}(\varphi + \Delta \varphi, \mu_{ad} + \Delta \mu_{ad}, ...) \simeq V_{t,a}(\varphi, \mu_{ad}, ...) \]

\[ + \langle \nabla_{\varphi} V_{t,a}, \Delta \varphi \rangle + \langle \nabla_{\mu_{ad}} V_{t,a}, \Delta \mu_{ad} \rangle + ... \]
General sensitivity analysis (C., 2008)

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THEN there exists a generalized first-order Taylor expansion

\[ V_{t,a}(\varphi + \Delta\varphi, \mu_{ad} + \Delta\mu_{ad}, ...) \approx V_{t,a}(\varphi, \mu_{ad}, ...) + \langle \nabla_{\varphi} V_{t,a}, \Delta\varphi \rangle + \langle \nabla_{\mu_{ad}} V_{t,a}, \Delta\mu_{ad} \rangle + ... \]

with generalized gradients / sensitivities

\[ \nabla_{\varphi} V_{t,a} (s) = -1_{s > t} \cdot v(t, s) \cdot \sum_{k} P(X_s = k | X_t = a) \cdot V_{s-,k} \]

\[ \nabla_{\mu_{ad}} V_{t,a} (s) = 1_{s > t} \cdot v(t, s) \cdot P(X_s = a | X_t = a) \cdot S@R_{ad}(s) \]
Example

endowment insurance with disability waiver

sensitivities of the prospective reserve at time $t = 0$ in state $a =$ 'active'
Example: mortality sensitivities

Policyholder at time $t = 0$ is a 30 year old male.

Mortality sensitivities of the prospective reserve at time $t = 0$ in state $a =$’active’.
Example

combinations of insurance contracts:
sensitivity of \( \left( \sum_i \alpha_i \cdot policy_i \right) \) = \( \sum_i \alpha_i \cdot \left( \text{sensitivity of } policy_i \right) \)
Example

combinations of insurance contracts:
sensitivity of \( \left( \sum_i \alpha_i \cdot \text{policy}_i \right) = \sum_i \alpha_i \cdot \left( \text{sensitivity of policy}_i \right) \)

pure endowment ins.  +  temporary life ins.  =  combined policy
Example

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pure endowment ins. \( + \) temporary life ins. \( = \) combined policy

\[ 2 \times \]
Conclusion for Tool 1: Sensitivity Analysis

Pro

- yields graphic images of risk structures

Con

- disregards the variability of the arguments
- no real-valued risk measure
  - how to compare policies (with respect to their systematic mortality risk)
Conclusion for Tool 1: Sensitivity Analysis

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- sensitivity is linear with respect to combinations of policies

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  ✔️ very helpful tool to find netting effects

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Conclusion for Tool 1: Sensitivity Analysis

Pros

- yields graphic images of risk structures
- sensitivity is linear with respect to combinations of policies
  - very helpful tool to find netting effects

Cons

- disregards the variability of the arguments
- no real-valued risk measure
  - how to compare policies (with respect to their systematic mortality risk) ?
Comparing policies

*temporary life ins.*

![Graph comparing temporary life insurance policies](image)

- **Yearly premium**
- **Lump sum premium**

**clear**
Comparing policies

**temporary life ins.**

- Yearly premium
- Lump sum premium

**pure endowment ins.**

- Yearly premium
- Lump sum premium

† clear
Comparing policies

*temporary life ins.*

*pure endowment ins.*

*disability vs. pure endowment ins.*

*yearly premium*

*yearly premium*

*yearly premium*

*pure endowment ins.*

*disability ins.*

*unclear*
Comparing policies

*temporary life ins.*

*pure endowment ins.*

*disability vs. pure endowment ins.*

*pure endowment & temporary life ins.*
TOOL 2: Worst-Case Analysis
Comparing / Measuring policies

IDEA (risk measure for systematic mortality risk):
use the SCR according to the standard formula in Solvency II

\[
\text{systematic mortality risk} = \text{Life}_{long} \& \text{Life}_{mort}
\]
Comparing / Measuring policies

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- reflects the practical consequences for the insurer
- approximation of the risk measure V@R
- cost-of-capital method: proportional to some (hypothetical) market price
Comparing / Measuring policies

IDEA (risk measure for systematic mortality risk):
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\[ \text{systematic mortality risk} = L_{\text{long}} \& L_{\text{mort}} \]

- reflects the practical consequences for the insurer
- approximation of the risk measure \( \text{V@R} \)
- cost-of-capital method: proportional to some (hypothetical) market price

BUT risk approximation by the standard formula is too rough here!
Calculation of $\text{Life}_{\text{mort}}$ and $\text{Life}_{\text{long}}$

$\Delta \text{NAV} =$ changes in the net value of assets and liabilities due to ...

+10% mortality shock

mortality rate

−25% longevity shock
Calculation of $Life_{mort}$ and $Life_{long}$

$\Delta NAV = \text{changes in the net value of assets and liabilities due to ...}$

- $+10\%$ mortality shock
- mortality rate
- $-25\%$ longevity shock
- $\pm$ mixed shock

- Do we really study the crucial scenarios?
For those contracts that provide benefits both in case of death and survival, one of the following two options should be chosen [...]:

1. Contracts [...] should not be unbundled. [...] the mortality scenario should be applied fully allowing for the netting effect provided by the 'natural' hedge between the death benefits component and the survival benefits component. [...]
For those contracts that provide benefits both in case of death and survival, one of the following two options should be chosen [...]:

1. **Contracts [...] should not be unbundled. [...] the mortality scenario should be applied fully allowing for the netting effect provided by the ’natural’ hedge between the death benefits component and the survival benefits component. [...]**

2. **All contracts are unbundled into 2 separate components: one contingent on the death and other contingent on the survival of the insured person(s). [...]**
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For those contracts that provide benefits both in case of death and survival, one of the following two options should be chosen [...]:

1. **Contracts [...] should not be unbundled. [...] the mortality scenario should be applied fully allowing for the netting effect provided by the 'natural' hedge between the death benefits component and the survival benefits component. [...] → not the crucial scenarios**

2. **All contracts are unbundled into 2 separate components: one contingent on the death and other contingent on the survival of the insured person(s). [...] → no netting effect**
Finding the crucial scenario(s)

• Which scenarios lead to the highest/lowest $\Delta$NAV?

+10% shock UPPER BOUND

−25% shock LOWER BOUND
Finding the crucial scenario(s)

• Which scenarios lead to the highest $\Delta NAV$?

+10% shock UPPER BOUND
worst/best scenario?
−25% shock LOWER BOUND
Optimization problem

If we focus on the liabilities only then $\Delta NAV = \Delta V_{0,a}$. 
Optimization problem

If we focus on the liabilities only then \( \Delta NAV = \Delta V_{0,a} \).

Problem Find scenario \( \overline{\mu}_{ad} \) with

\[
V_{0,a}(\overline{\mu}_{ad}) = \max \left\{ V_{0,a}(\mu_{ad}) \mid LowBound \leq \mu_{ad} \leq UppBound \right\}
\]
Optimization problem

If we focus on the liabilities only then $\Delta NAV = \Delta V_{0,a}$.

**Problem** Find scenario $\overline{\mu}_{ad}$ with

$$V_{0,a}(\overline{\mu}_{ad}) = \max \{ V_{0,a}(\mu_{ad}) \mid LowBound \leq \mu_{ad} \leq UppBound \}$$

**first-order Taylor approximation**

$$V_{0,a}(\overline{\mu}_{ad} + \Delta \mu_{ad}) = V_{0,a}(\overline{\mu}_{ad}) + \langle \nabla_{\mu_{ad}} V_{0,a}, \Delta \mu_{ad} \rangle + \text{Remainder}$$

where $\nabla_{\mu_{ad}} V_{0,a}(s) = \mathbf{1}_{s>0} \cdot v(0, s) \cdot P(X_{s-} = a \mid X_0 = a) \cdot S@R_{ad}(s)$
Optimization problem

If we focus on the liabilities only then $\Delta NAV = \Delta V_{0,a}$.

Problem Find scenario $\mu_{ad}$ with

$$V_{0,a}(\mu_{ad}) = \max \left\{ V_{0,a}(\mu_{ad}) \mid LowBound \leq \mu_{ad} \leq UppBound \right\}$$

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$$V_{0,a}(\mu_{ad} + \Delta \mu_{ad}) = V_{0,a}(\mu_{ad}) + \langle \nabla_{\mu_{ad}} V_{0,a}, \Delta \mu_{ad} \rangle + \text{Remainder}$$

where $\nabla_{\mu_{ad}} V_{0,a}(s) = \mathbf{1}_{s>0} \cdot v(0,s) \cdot P(X_{s-} = a | X_0 = a) \cdot S \otimes R_{ad}(s)$

Conclusion 1 For all $s$

$$\text{sign} \left( \nabla_{\mu_{ad}} V_{0,a}(s) \right) = \text{sign} \left( \overline{S \otimes R_{ad}(s)} \right)$$
Optimization problem

If we focus on the liabilities only then \( \Delta \text{NAV} = \Delta V_{0,a} \).

**Problem** Find scenario \( \bar{\mu}_{ad} \) with

\[
V_{0,a}(\bar{\mu}_{ad}) = \max \{ V_{0,a}(\mu_{ad}) | \text{LowBound} \leq \mu_{ad} \leq \text{UppBound} \}
\]

**first-order Taylor approximation**

\[
V_{0,a}(\bar{\mu}_{ad} + \Delta \mu_{ad}) = V_{0,a}(\bar{\mu}_{ad}) + \langle \nabla_{\bar{\mu}_{ad}} V_{0,a} , \Delta \mu_{ad} \rangle + \text{Remainder}
\]

where \( \nabla_{\mu_{ad}} V_{0,a}(s) = \mathbf{1}_{s > 0} \cdot v(0, s) \cdot P(X_{s-} = a \mid X_0 = a) \cdot S@R_{ad}(s) \)

**Conclusion 1** For all \( s \)

\[
\text{sign} \left( \nabla_{\bar{\mu}_{ad}} V_{0,a}(s) \right) = \text{sign} \left( S@R_{ad}(s) \right)
\]

**Conclusion 2** Because of the maximality property of \( \bar{\mu}_{ad} \):

\[
\langle \nabla_{\bar{\mu}_{ad}} V_{0,a} , \Delta \mu_{ad} \rangle \leq 0 \text{ for all } \text{LowBound} \leq \bar{\mu}_{ad} + \Delta \mu_{ad} \leq \text{UppBound}
\]
Solution of the optimization problem

Hence

\[
\bar{\mu}_{ad} = \begin{cases} 
    \text{UppBound}_{ad} : \quad \text{sign} \quad \overline{S@R_{ad}} > 0 \\
    \text{LowBound}_{ad} : \quad \text{sign} \quad \overline{S@R_{ad}} < 0
\end{cases}
\]
Solution of the optimization problem

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\[
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\& \quad \text{Thiele’s rekursion/differential/integral equation}

\Rightarrow \quad \text{Worst-Case rekursion/differential/integral equation}
Solution of the optimization problem

Hence

\[ \overline{\mu}_{ad} = \begin{cases} 
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& Thiele's rekursion/differential/integral equation

\[ \equiv \text{Worst-Case rekursion/differential/integral equation} \]

THEOREM (C., 2008)

The Worst-Case rekursion/differential/integral equation has a unique solution \( \overline{V}_{t,a} \) that is maximal.
Solution of the optimization problem

Hence

\[ \overline{\mu}_{ad} = \begin{cases} UppBound_{ad} : \text{sign} \overline{S@R}_{ad} > 0 \\ LowBound_{ad} : \text{sign} \overline{S@R}_{ad} < 0 \end{cases} \]

& Thiele's rekursion/differential/integral equation

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The Worst-Case rekursion/differential/integral equation has a unique solution \( \overline{V}_{t,a} \) that is maximal.

Conclusion
solution \( \overline{V}_{t,a} \) \( \Rightarrow \) \( \overline{S@R}_{ad} \) \( \Rightarrow \) \( \overline{\mu}_{ad} \)
Example 1

Task Design a combination of
\[ 1 \times \text{pure endowment ins.} + \beta \times \text{temporary life ins.} \]
Example 1

Task Design a combination of
\[ 1 \times pure\ endowment\ ins. + \beta \times temporary\ life\ ins. \]

Question Which ratio
\[ \text{death benefit : survival benefit} = \beta : 1 \]
leads to the lowest systematic mortality risk?
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Question Which ratio

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Example 2

Task Design a combination of
\[ 1 \times \text{disability ins.} + \beta \times \text{temporary life ins.} \]

Question Which ratio
\[ \text{death benefit : yearly disability benefit} = \beta : 1 \]
leads to the lowest systematic mortality risk?
Example 2

**Task** Design a combination of
\[ 1 \times \text{disability ins.} \quad + \quad \beta \times \text{temporary life ins.} \]

**Question** Which ratio
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synchronised changes of \((a, d)\) and \((i, d)\)
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Example 3

Task Design a combination of
1× *annuity ins.* + β× *temporary/whole life ins.*

Question Which ratio

death benefit : yearly annuity benefit = β : 1
leads to the lowest systematic mortality risk?
Example 3

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Example 3

Task Design a combination of
\[1 \times \text{annuity ins.} + \beta \times \text{temporary/whole life ins.}\]

Question Which ratio
\[\text{death benefit : yearly annuity benefit} = \beta : 1\]
leads to the lowest systematic mortality risk?
literature


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