BAYESIAN STOCHASTIC MORTALITY MODELLING FOR TWO POPULATIONS

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Plan

- Background
- 2-population APC model plus MCMC
- Case studies
  - England and Wales versus CMI assured lives
  - Males
  - Females

Motivation

A: Risk assessment for pension plans and insurers

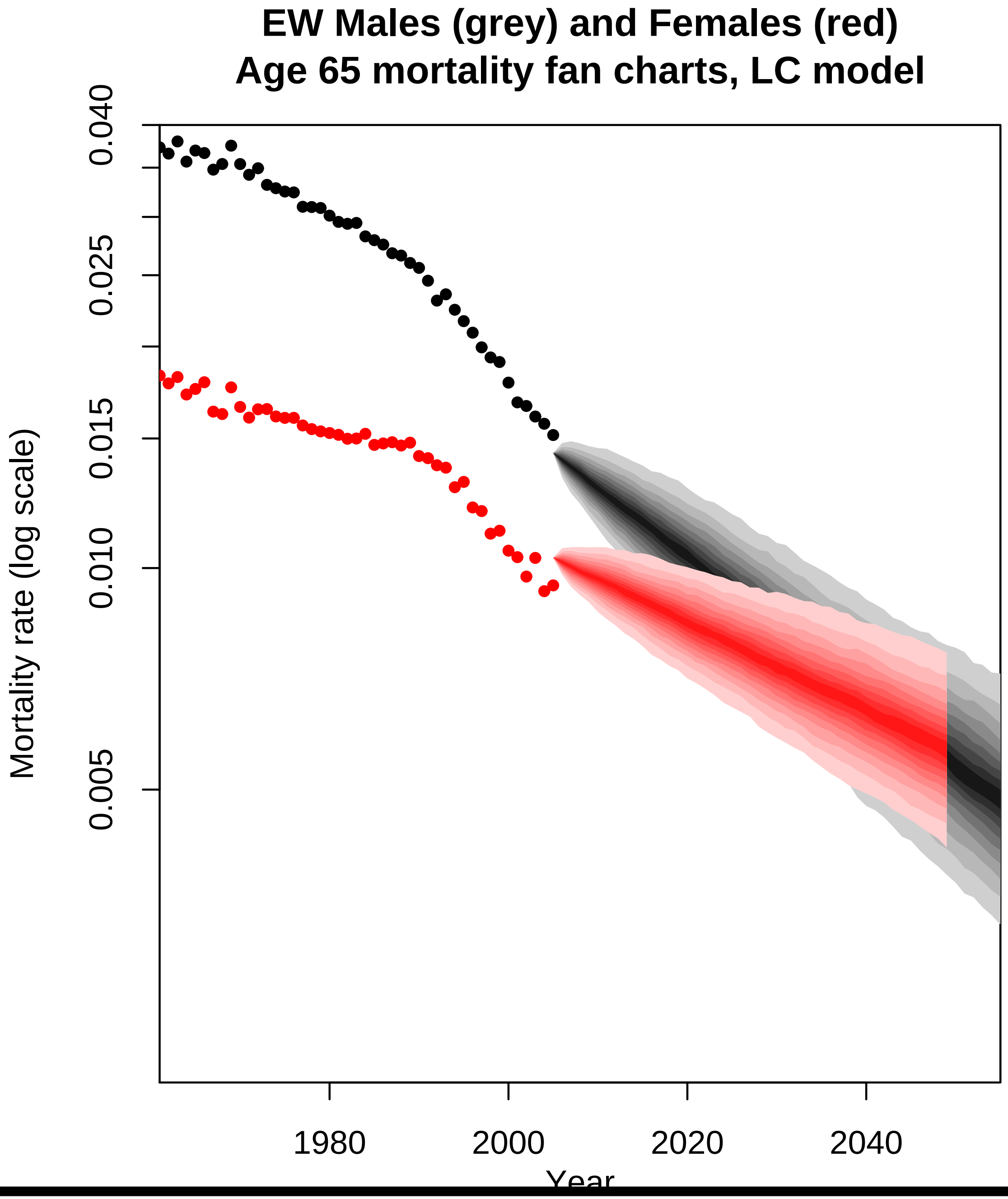
- Males/Females
- Blue/White collar
- Smokers/Non-smokers
- UK/Europe
- Annuities/Life insurance
- Limited data $\Rightarrow$ learn from other populations

Motivation

B: Risk management for pension plans and insurers

- Retain systematic mortality risk; versus:
- OTC deals (e.g. longevity swap)
  - own experience
  - potentially expensive and illiquid
- Standardised mortality-linked securities
  - linked to national mortality index
  - basis risk
  - less illiquid
- Catastrophe bonds

The problem with single population projections



Two populations

- Similar characteristics
- But not identical
- Desire for consistent forecasts
  - distributions
  - pathwise

Key hypothesis

- $q_1(t,x) =$ pop. 1 mortality rate in year t at age x
- $q_2(t,x) =$ pop. 2 mortality rate in year t at age x
- Hypothesis (e.g. Li and Lee, 2005):

  For each age x, $\dfrac{q_1(t,x)}{q_2(t,x)}$ does not diverge over time

- Spread $= \log q_1(t,x) - \log q_2(t,x)$ is stochastic with some form of mean reversion

Age-Period-Cohort model (APC)

$m_k(t,x) =$ population k death rate

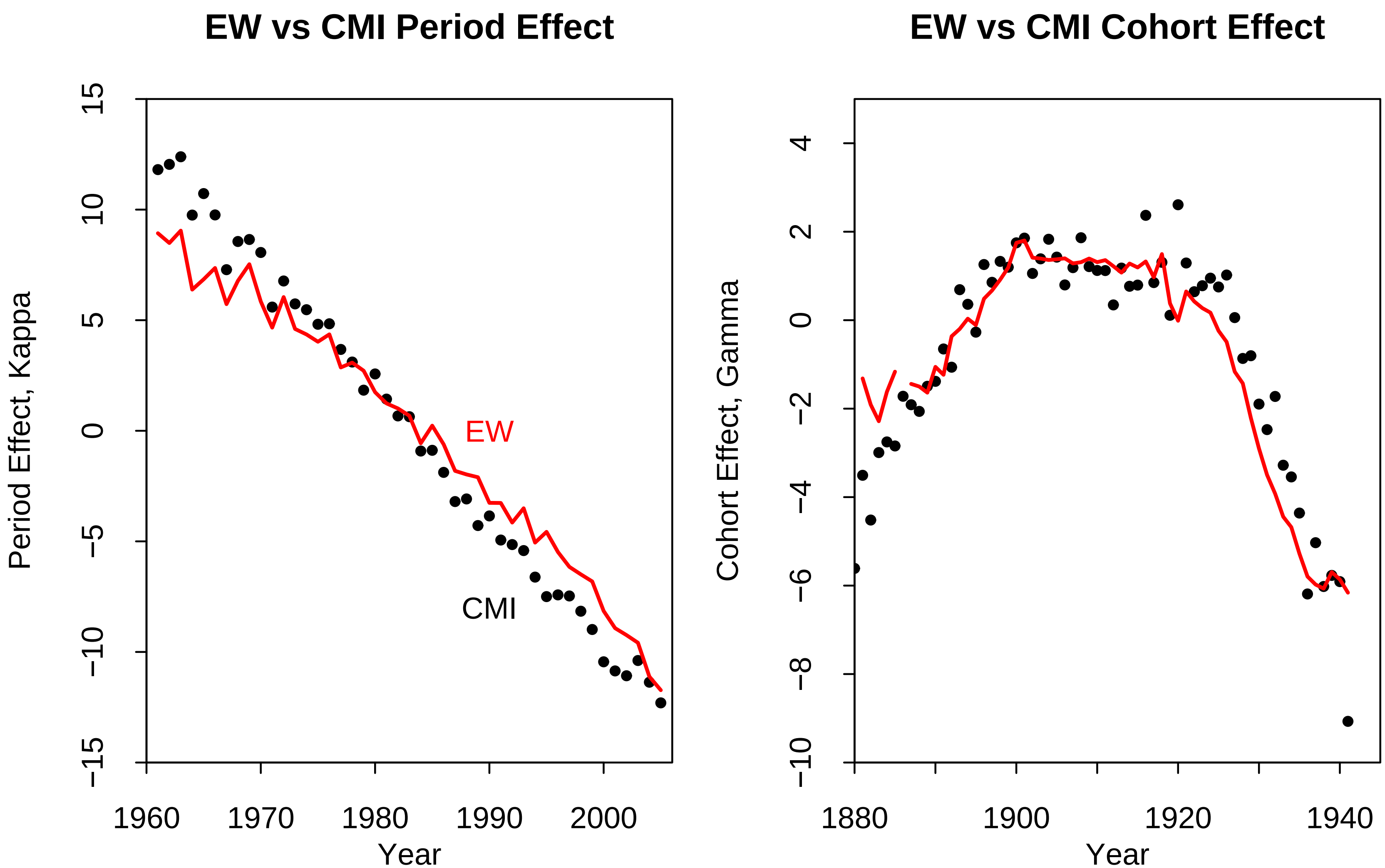
$n_a =$ number of age groups

$$\log m_k(t,x) = \beta^{(1k)}(x) + n_a^{-1}\kappa^{(2k)}(t) + n_a^{-1}\gamma^{(3k)}(t-x)$$

Hypothesis $\Rightarrow$

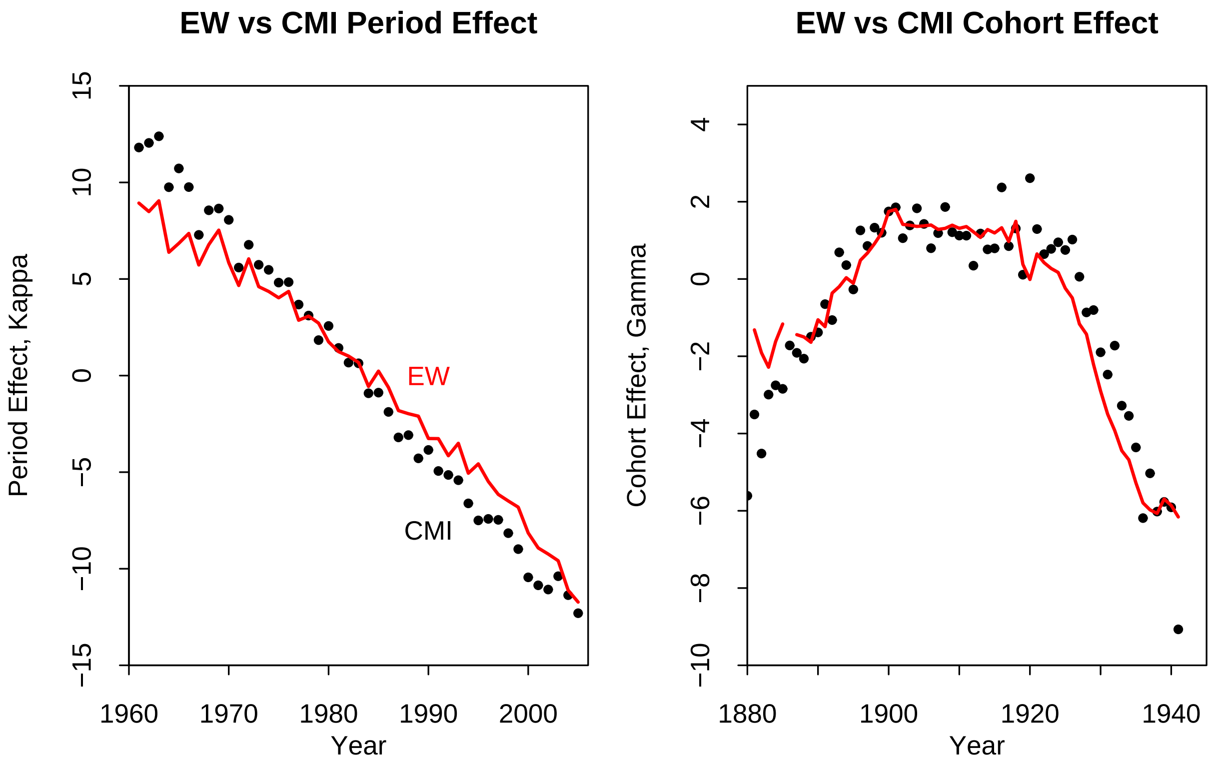
- $\kappa^{(21)}(t) - \kappa^{(22)}(t)$ mean reverting
- $\gamma^{(31)}(t-x) - \gamma^{(32)}(t-x)$ mean reverting

EW versus CMI (assured lives) males



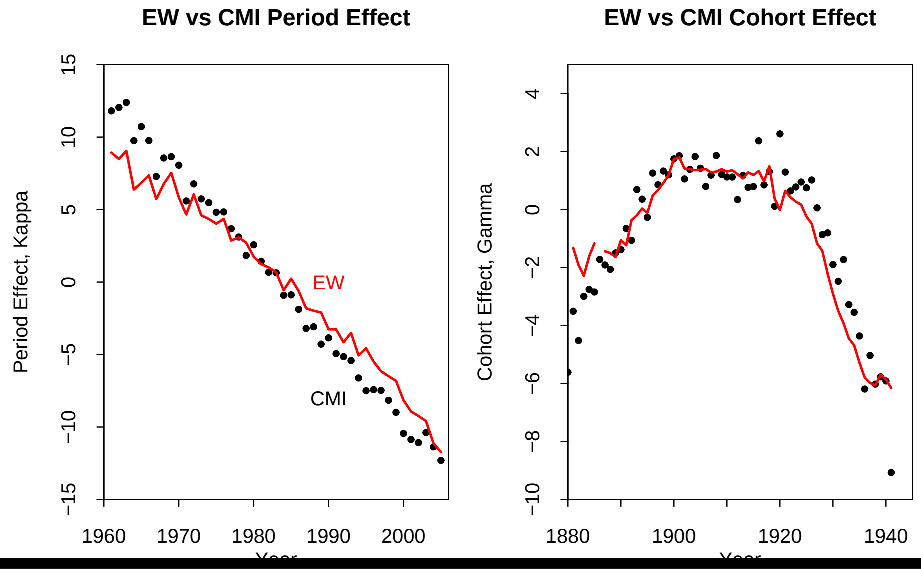
Stylised facts: period effects

- Annual innovations in $\kappa^{(21)}(t)$ and $\kappa^{(22)}(t)$: moderate correlation
- Limited data $\Rightarrow$ proving mean reversion difficult



Stylised facts: cohort effects

- Annual innovations in $\gamma^{(31)}(c)$ and $\gamma^{(32)}(c)$ NOT highly correlated
- Longer-term shapes of $\gamma^{(31)}(c)$ and $\gamma^{(32)}(c)$ very similar
- Small population 2 $\Rightarrow \gamma^{(32)}(c)$ noisy



Stylised facts: inferences

- “True” $\kappa^{(21)}(t)$ and $\kappa^{(22)}(t)$ incorporate significant correlated randomness
- “True” $\gamma^{(31)}(c)$ and $\gamma^{(32)}(c)$:
  - Relatively smooth in the short term
  - Stochastic trends
  - (+ hypothesis) Mean reverting spread
- Estimated $\gamma_k(c)$ affected by Poisson noise

A 2-population model (one large, one small)

Underlying processes	APC processes
$\beta^{(11)}(x), \quad \beta^{(12)}(x)$	$\beta^{(11)}(x), \quad \beta^{(12)}(x)$
$R_2(t)$ $S_2(t)$	$\kappa^{(21)}(t) = R_2(t)$ $\kappa^{(22)}(t) = R_2(t) - S_2(t)$
$R_3(c)$ $S_3(c)$	$\gamma^{(31)}(c) = R_3(c)$ $\gamma^{(32)}(c) = R_3(c) - S_3(c)$

A 2-population model

Underlying processes	Model
$\beta^{(11)}(x), \quad \beta^{(12)}(x)$	no model
$R_2(t)$ $S_2(t)$	Random walk AR(1) around μ_{S2}
$R_3(c)$ $S_3(c)$	AR(2)+trend or ARIMA(1,1,0) AR(2) around μ_{S3}

Bayesian Priors

- Mostly weak uninformative priors
- priors for ARIMA parameters
  - less weak
  - aim: to push parameters away from unit roots

Enhanced priors for the cohort effect

Biological reasonableness $\Rightarrow \gamma^{(31)}(c)$ and $\gamma^{(32)}(c)$ have

- similar short term volatility
- similar long term variability
- moderate/strong correlation in the long term

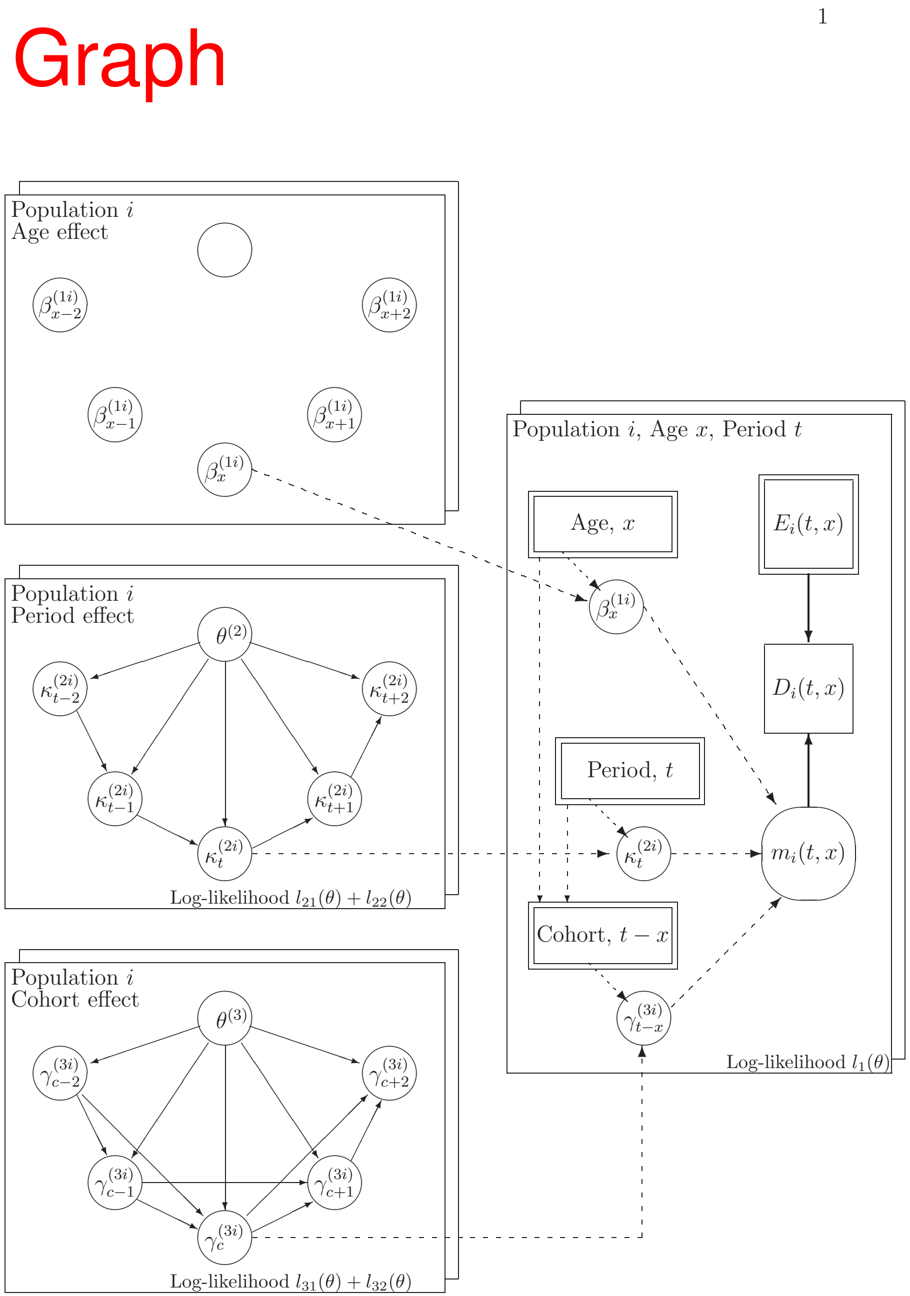
Markov chain Monte Carlo

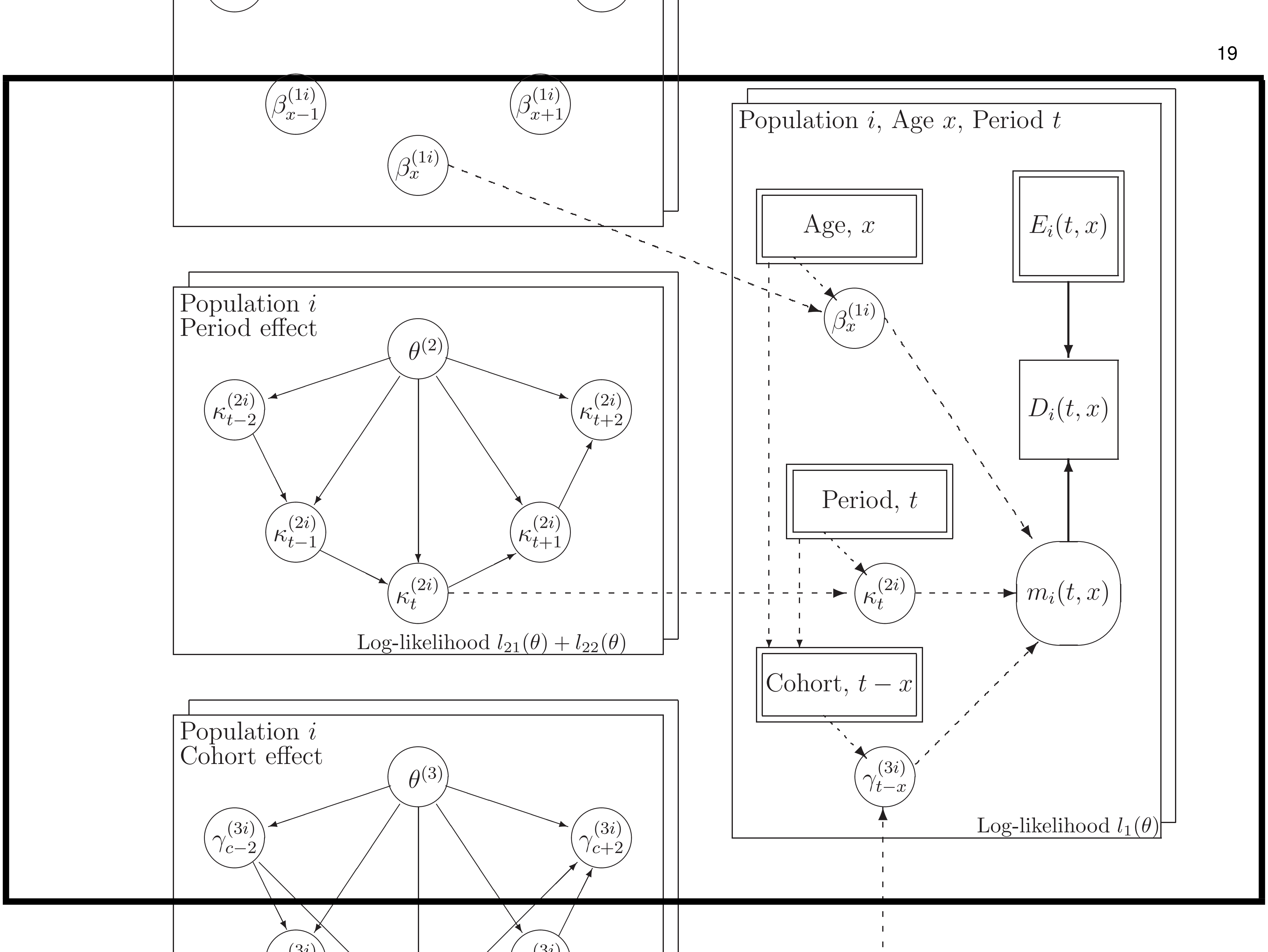
- $\theta =$ parameter vector
  - process parameters
  - latent processes

    $\beta^{(11)}(x), \beta^{(12)}(x), R_2(t), S_2(t), R_3(c), S_3(c)$
- Metropolis-Hastings algorithm
- update elements or blocks of θ

  $\Rightarrow$ Markov chain $\theta(u)$ with stationary dist'n $=$ posterior

Directed Acyclic Graph

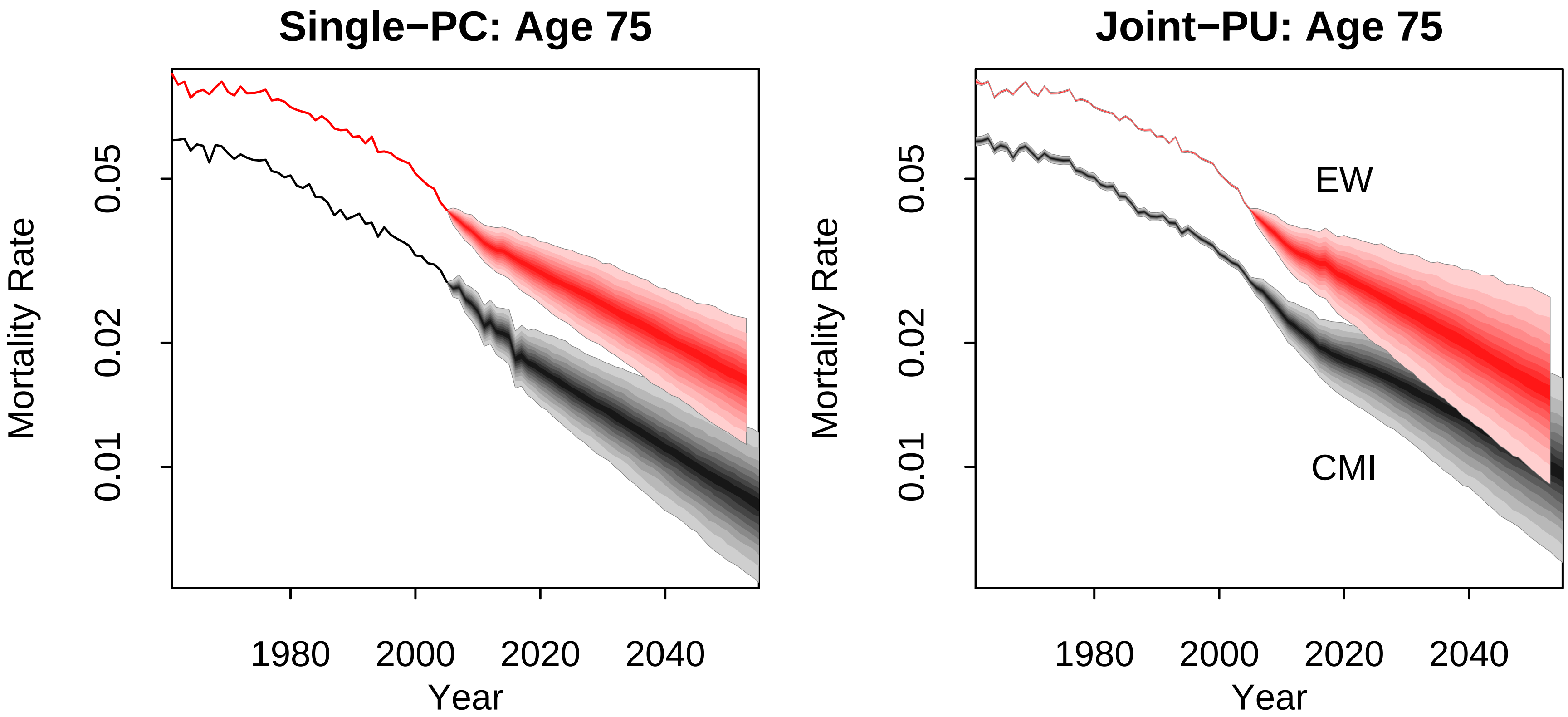



Population i, Age x, Period t
Age, x
$E_i(t,x)$
$D_i(t,x)$
Period, t
$m_i(t,x)$
Cohort, $t-x$
$\beta_x^{(1i)}$
$\kappa_t^{(2i)}$
$\gamma_{t-x}^{(3i)}$
Log-likelihood $l_1(\theta)$
$\beta_{x-1}^{(1i)}$
$\beta_{x+1}^{(1i)}$
Population i
Period effect
$\theta^{(2)}$
$\kappa_{t-2}^{(2i)}$
$\kappa_{t+2}^{(2i)}$
$\kappa_{t-1}^{(2i)}$
$\kappa_{t+1}^{(2i)}$
Log-likelihood $l_{21}(\theta)+l_{22}(\theta)$
Population i
Cohort effect
$\theta^{(3)}$
$\gamma_{c-2}^{(3i)}$
$\gamma_{c+2}^{(3i)}$

Case Study: EW versus CMI males

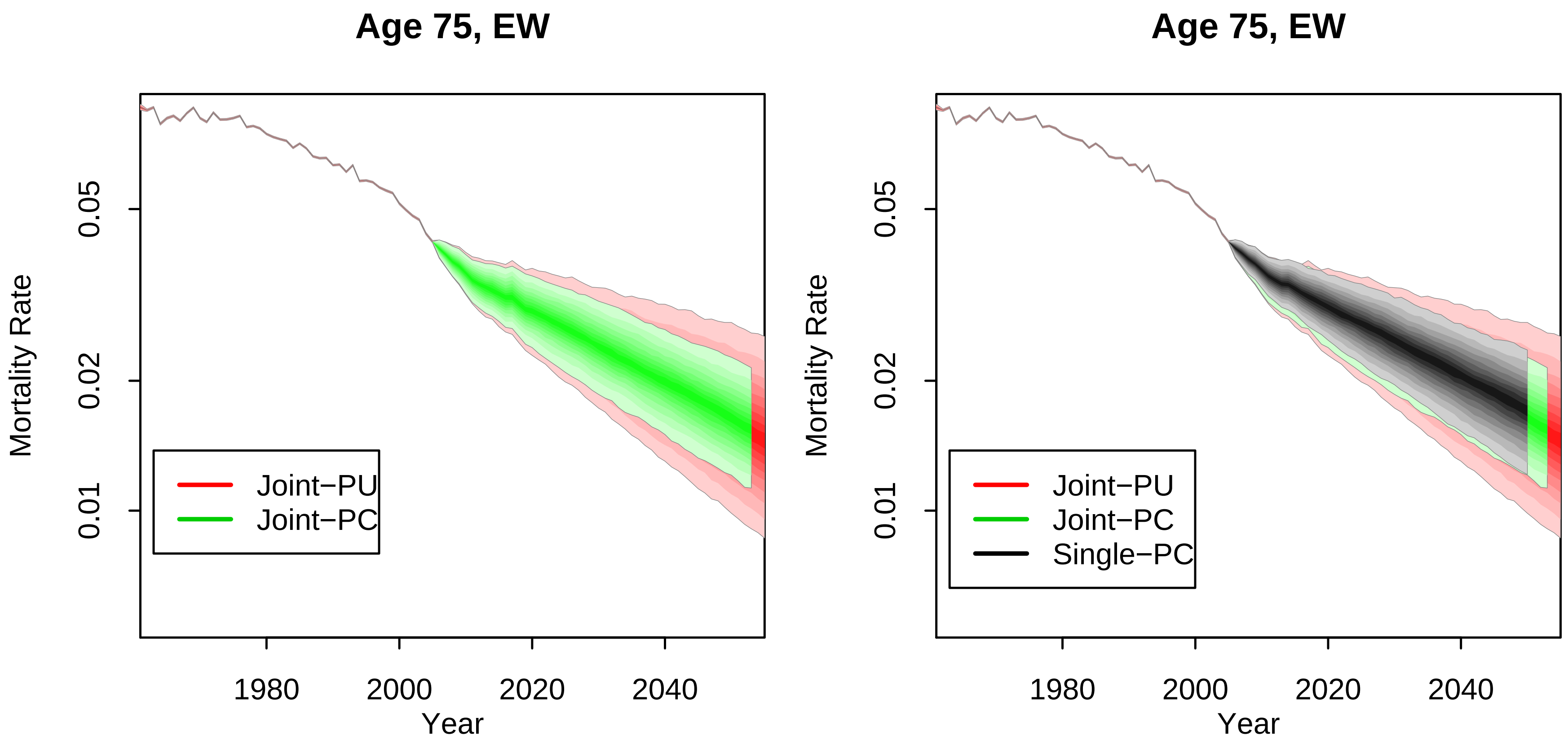
- Data: 1961-2005
- Ages: 61-89
- CMI exposures $\sim 10\%$ of EW
- EW Exclusions:
  - 1886 cohort
  - 1961-1970, ages 85-89

1-population forecasts versus 2-pop MCMC



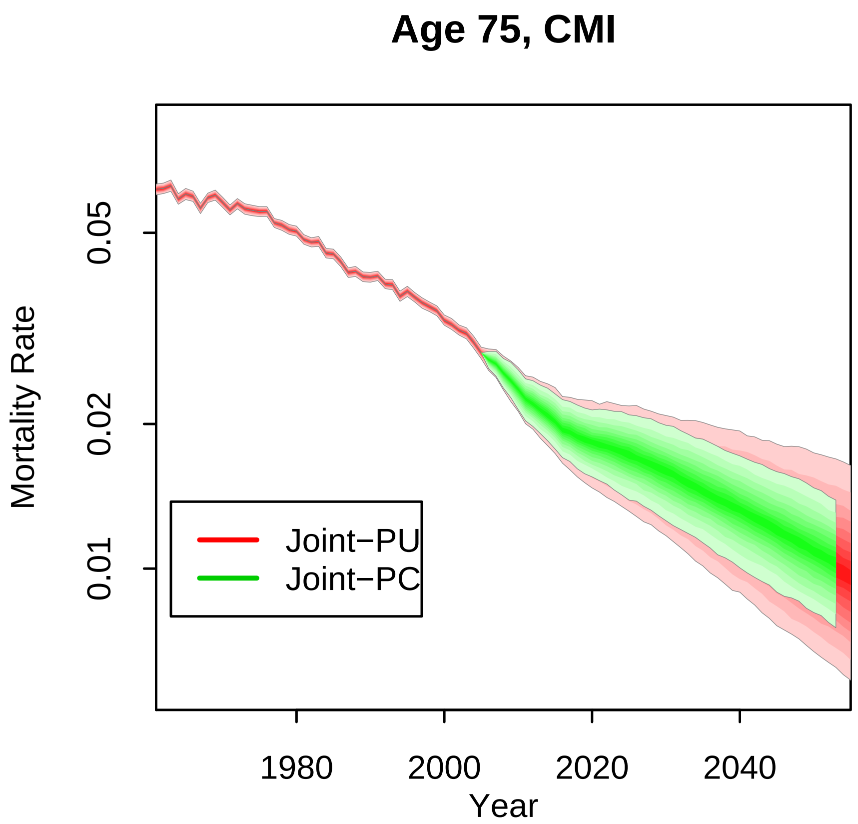
- Joint-PU: uses the 2-population model with parameter uncertainty
- Single-PC: uses the 1-population model with no parameter uncertainty

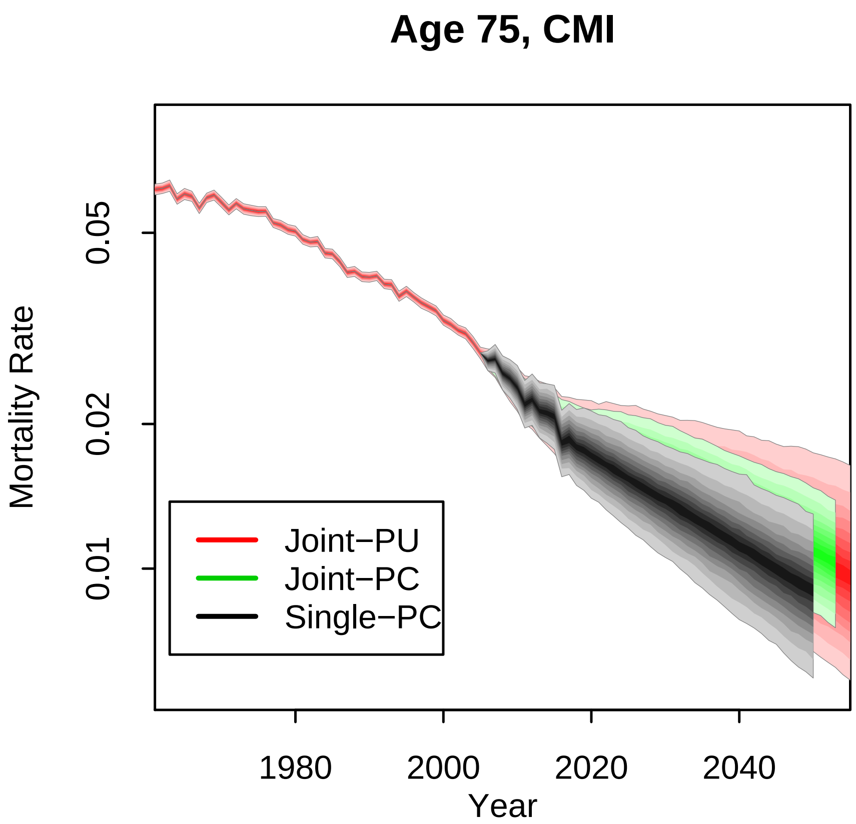
1-population forecasts versus 2-pop MCMC



- Joint: uses the 2-population model   Single: uses the 1-population model
- PC: parameters certain   PU: parameters uncertain

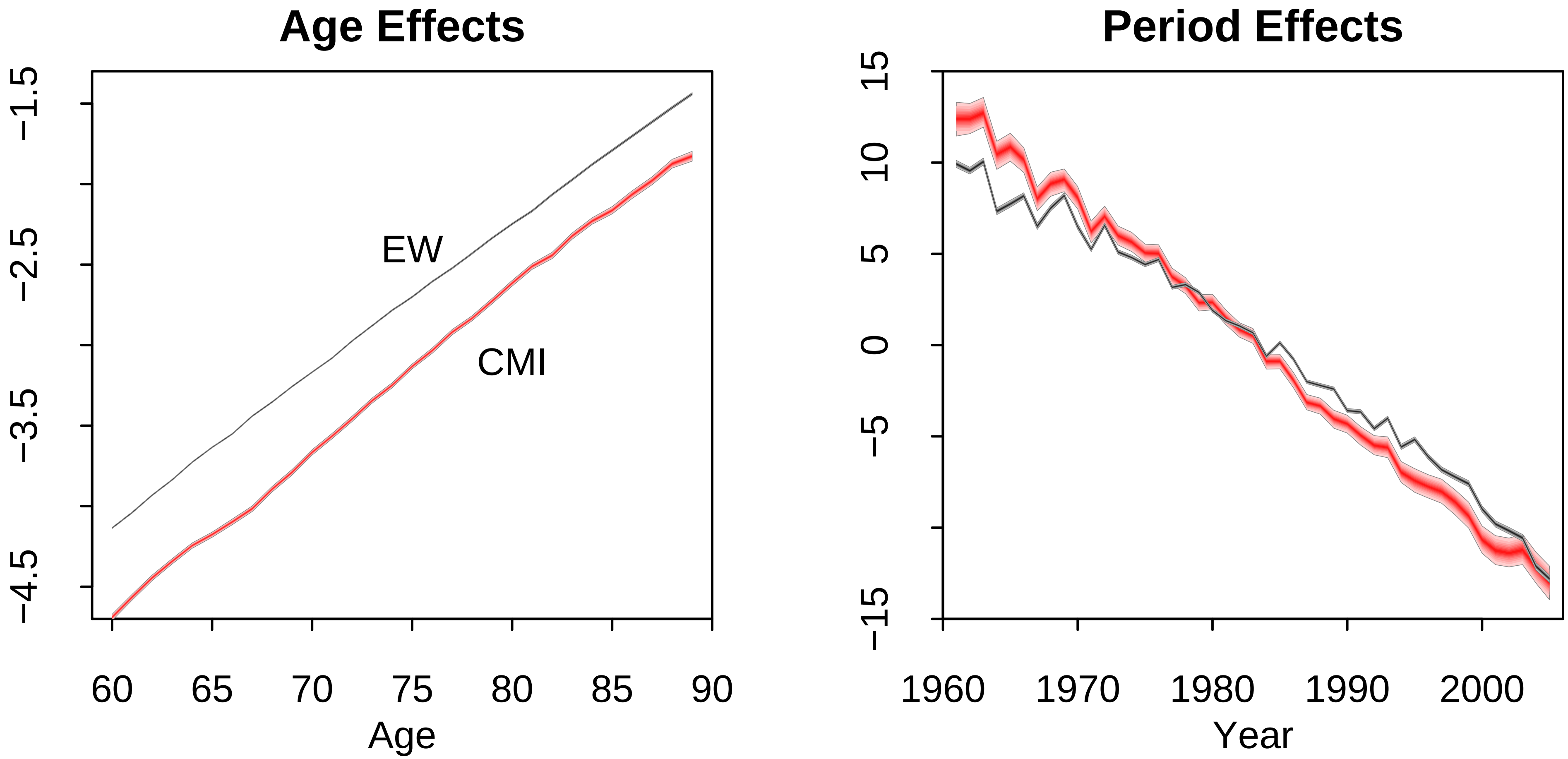
1-population forecasts versus 2-pop MCMC



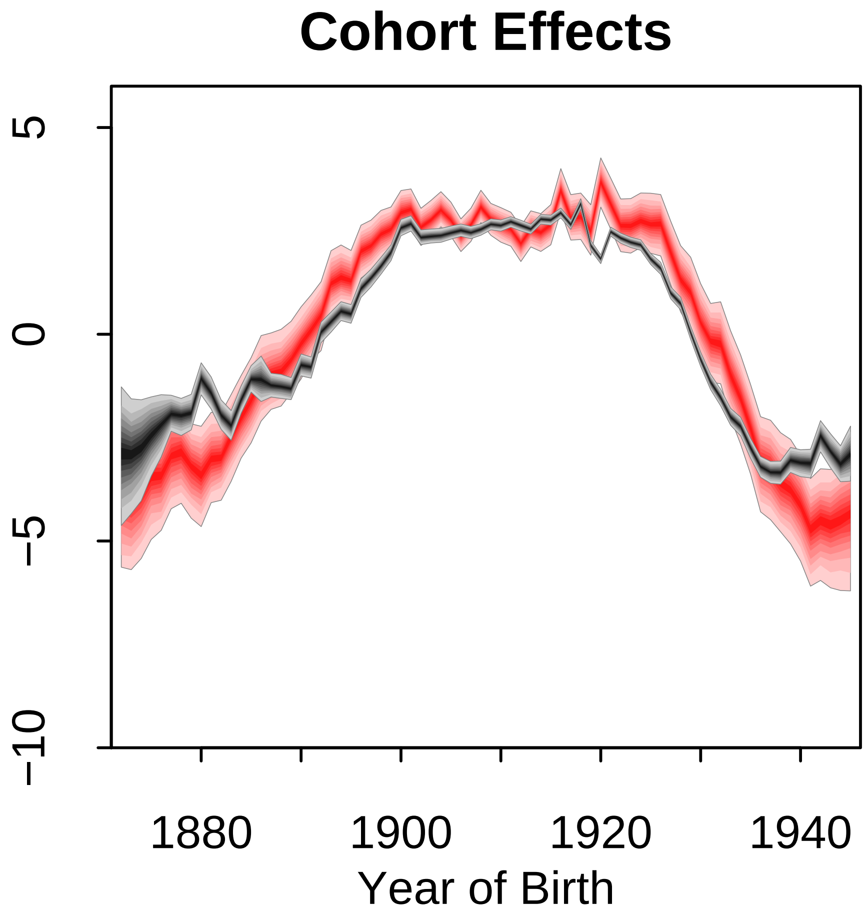


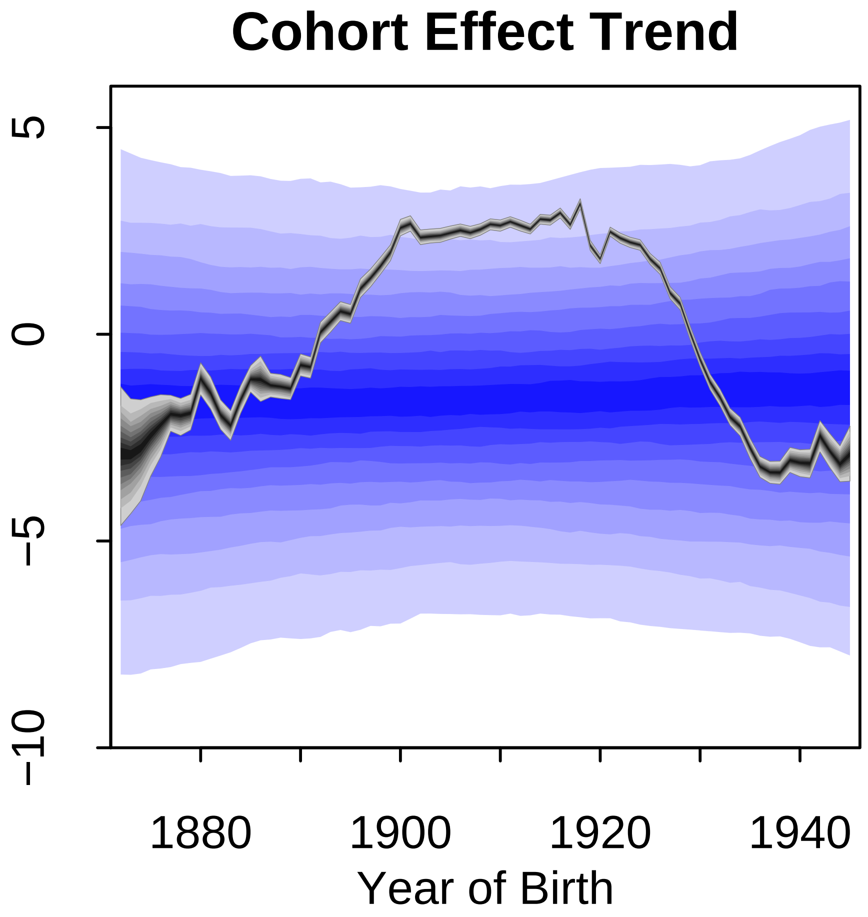
- Joint: uses the 2-population model   Single: uses the 1-population model
- PC: parameters certain   PU: parameters uncertain

EW (grey) and CMI (red) Age and Period Effects



EW (grey) and CMI (red) Cohort Effects



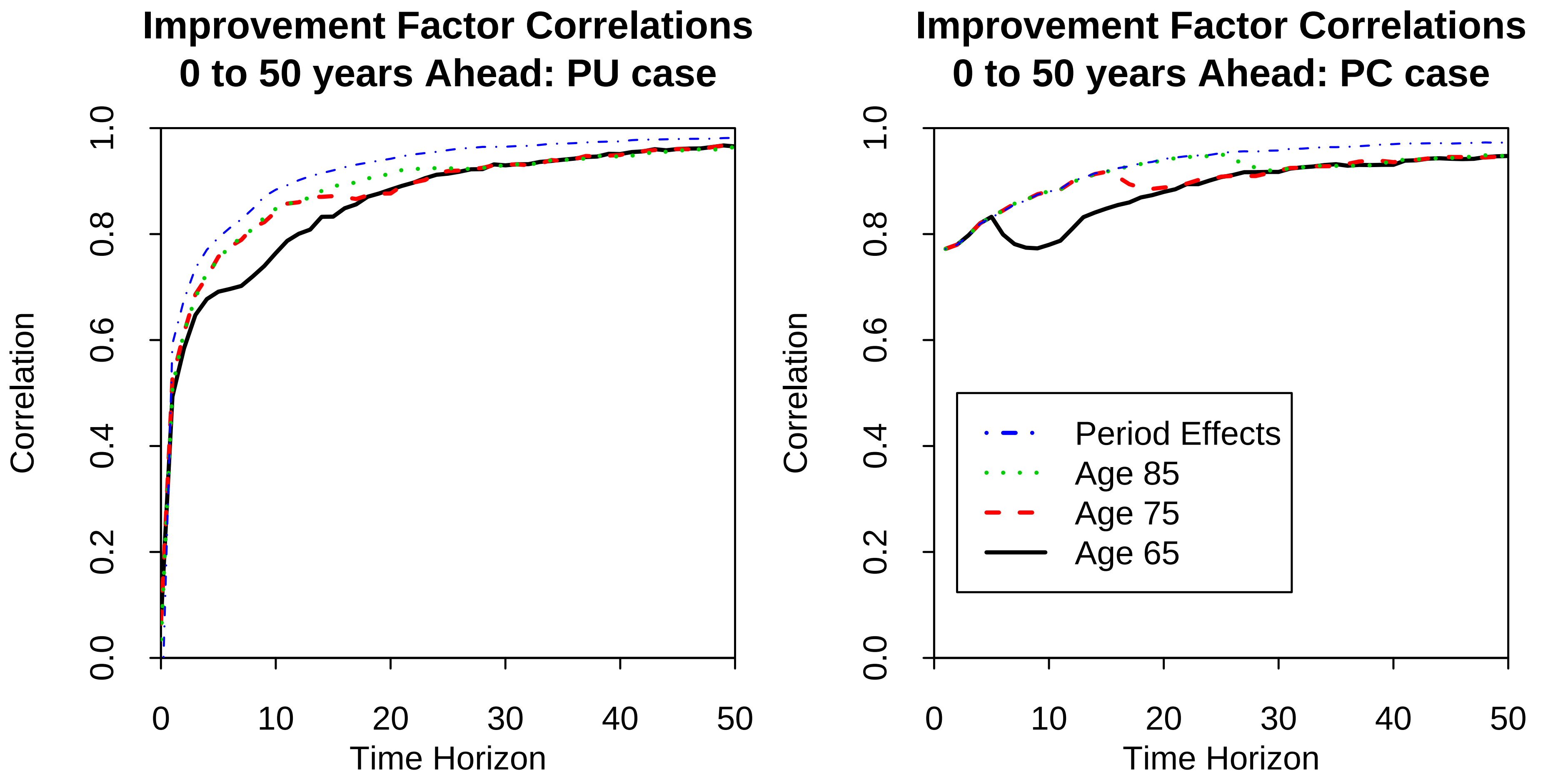


Reduction Factors

Population k, t years ahead from 2005

$$IF_k(t,x) = \frac{q_k(2005+t,x)}{q_k(2005,x)}$$

Correlation between $RF_1(t,x)$ and $RF_2(t,x)$



Missing data: extra calendar years

- CMI females: data 1983-2003 (much lower exposures than males)
- EW females:
  - 1983-2003
  - 1961-2003
  - 1961-2007

Missing data: extra calendar years

- Adding 1961-1982 EW data:
  - Small shift in both EW and CMI trend
  - Small changes in forecast uncertainty

- Adding 2004-2007 EW data:
  - EW now fans out from 2007 instead of 2003
  - EW generally narrower after 2007
  - CMI still fans out from 2003
  - CMI generally a bit narrower
  - EW and CMI small parallel shift in trajectory

Conclusions

- Sythesis of
  - Consistent 2-population projections
  - Bayesian approach
  - Ability to deal with small populations
  - Ability to deal with missing data
  - Full parameter uncertainty
- Full APC model to assess basis risk

Reference:
Bayesian Stochastic Mortality Modelling for Two Populations
LifeMetrics Working Paper, available shortly!
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