AFIR LIFE MUNICH 2009

Building blocks for a mortality index in an international context

Tiziana Torri

Max Planck Institute for Demographic Research

Munich, 7th September 2009

Section Finance de l'AAI
IAA Finance Section
AFIR

Section Vie de l'AAI
IAA Life Section
LIFE

DAV
DEUTSCHE AKTUARVEREINIGUNG e.V.

Outline

- Longevity risk:
  - Identification
  - Assessment
  - Management
- Securitization → Mortality Indexes
- Stochastic forecasts of life expectancy
  - Univariate ARIMA
  - Multivariate VAR → Vector Error Correction

Identification

- "Mortality risk" is divided into:
  - – Process risk: random fluctuations due to the stochastic nature of mortality
  - – Catastrophe risk: unexpected shocks of mortality
  - – Uncertainty risk: random deviations due to
    - choice of the projection model (model risk)
    - estimated parameters of the model (param.risk)

  → **Longevity Risk** refers to old age mortality

Assessment

- The impact of the LR is measured via:
  - Discounted cash flows
  - Random annual cash flows
  - Random number of annuitants alive in t
  - Loss function

- Combined with different indexes:
  - Expected value
  - Variance
  - Quantile

Management

- Portfolio strategies of loss control
  - Loss prevention
  - Loss severity

- Portfolio strategies of loss financing
  - Natural hedging
  - Risk transfer → Securitization
  - Risk retention

Securitization

- LR is transferred to capital markets via mortality-linked securities:
  - Longevity Bonds (EIB/BNP in 2004)
  - Survivor Swaps (Swiss Re in 2007)
  - Mortality Futures
  - Mortality Forwards (JP Morgan in 2007)
  - Mortality Options

Securitization

- LR is transferred to capital markets via mortality-linked securities:
  - Longevity Bonds (EIB/BNP in 2004)
  - Failed due to:
    - Presence of basis risk
    - Lack of transparency
    - Capital strain

Mortality indexes

- Recently proposed:
  - Longevity Index by Credit Suisse (2005)
  - LifeMetrics Indices by JP Morgan (2007)
  - Xpect-Indices by Deutsche Börse (2008)
- Our proposal:
  - Life expectancy at birth
  - Publicly available mortality data (HMD)
  - Multivariate analysis between countries

Stochastic models to forecast mortality

- Univariate ARIMA models
  - De Beer and Alders (1999)
  - Keilman, Pham and Hetland (2001)
  - Alders Keilman and Cruijsen (2007)

- Multivariate VAR models
- Multivariate VEC models

→ Economics

ARIMA models

- Based on the theory of stationary stochastic processes. ARIMA(p, d, q):

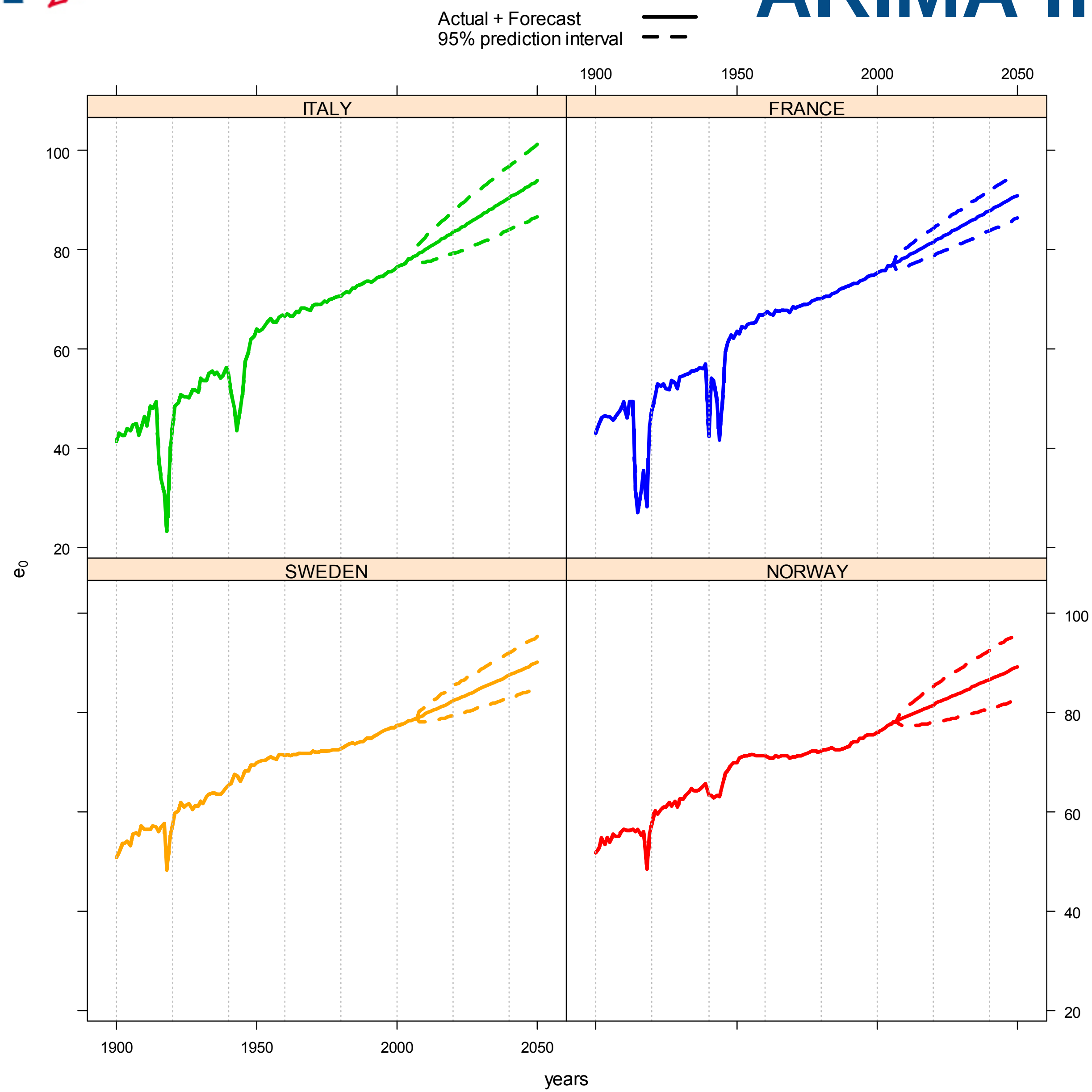
$$\nabla^d Y_t = \delta + \phi_1 \nabla^d Y_{t-1} + \ldots + \phi_p \nabla^d Y_{t-p} + \epsilon_t + \theta_1 \epsilon_{t-1} + \ldots + \theta_q \epsilon_{t-q}$$

- Box and Jenkins' iterative model selection strategies:
  - Model identification
  - Model estimation
  - Diagnostic checking of model adequacy

ARIMA models

	Male			
	Italy	France	Sweden	Norway
ARIMA(p,d,q)	(0,1,1)	(0,1,1)	(0,1,1)	(2,1,0)
δ	0.3479	0.3127	0.2601	0.3086
s.e.	(0.0542)	(0.0335)	(0.0385)	(0.0508)
ϕ_1	-	-	-	-0.2188
s.e.	-	-	-	(0.0964)
ϕ_2	-	-	-	0.2670
s.e.	-	-	-	(0.1028)
θ_1	-0.3092	-0.4979	-0.2871	-
s.e.	(0.0886)	(0.0829)	(0.0901)	-

ARIMA models



VAR models

- Variables are explained by their own past values and the past values of all the other variables in the system.

VAR models

- Variables are explained by their own past values and the past values of all the other variables in the system. VAR(p):

$$Y_t = \mu + A_1 Y_{t-1} + \ldots + A_p Y_{t-p} + \varepsilon_t$$

  – Model identification
  – Model estimation
  – Diagnostic checking of model adequacy

VAR(2) model

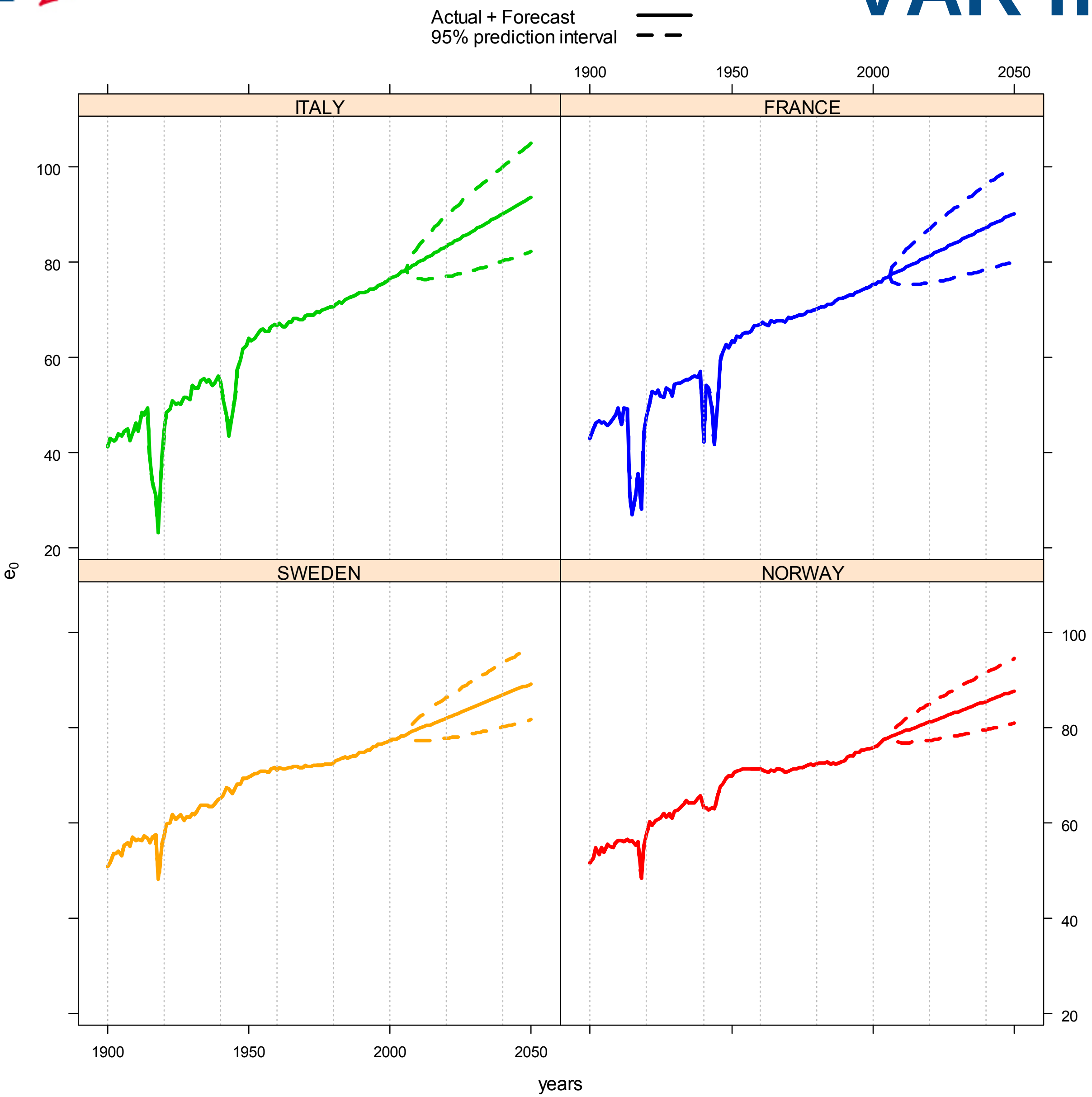
$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \varepsilon_t$$

$$\Delta e_0^{Ita}(t)$$
$$\Delta e_0^{Fra}(t)$$
$$\Delta e_0^{Swe}(t)$$
$$\Delta e_0^{Nor}(t)$$

$$a_1^{Ita}$$
$$a_1^{Fra}$$
$$a_1^{Swe}$$
$$a_1^{Nor}$$

$$\Delta e_0^{Ita}(t-1)$$
$$\Delta e_0^{Fra}(t-1)$$
$$\Delta e_0^{Swe}(t-1)$$
$$\Delta e_0^{Nor}(t-1)$$

VAR models



Actual + Forecast
95% prediction interval
ITALY
FRANCE
SWEDEN
NORWAY
1900
1950
2000
2050
100
80
60
40
20
e0
years

Cointegration

Cointegration

- "If each element of a vector of time series Y_t achieves stationarity after differencing, but a linear combination $Z_t = \beta' Y_t$ is already stationary, the time series are said to be cointegrated"

Cointegration

- “If each element of a vector of time series Y_t achieves stationarity after differencing, but a linear combination $Z_t = \beta' Y_t$ is already stationary, the time series are said to be cointegrated”
- Grangers' representation theorem: “For each cointegrated system exists a VEC representation; if exists a VEC representation and the series are integrated, then they are also cointegrated”

VEC models

- A VAR in a VEC representation:

$$\Delta Y_t = \Gamma_1 \Delta Y_{t-1} + \ldots + \Gamma_{p-1} \Delta Y_{t-p+1} - \Pi Y_{t-1} + \varepsilon_t$$

where $\Gamma_i = -(I - A_{i+1} - \ldots - A_p)$, $\Pi = -(I - A_1 - \ldots - A_p)$

- Example of a VAR(1): $Y_t = AY_{t-1} + \varepsilon_t$

$\Delta Y_t = \Pi Y_{t-1} + \varepsilon_t$ where $\Pi = A - I$

VEC models

- Studying the rank of Π we obtain information on the number of cointegrating relationship:

  $$0 \le r \le n$$

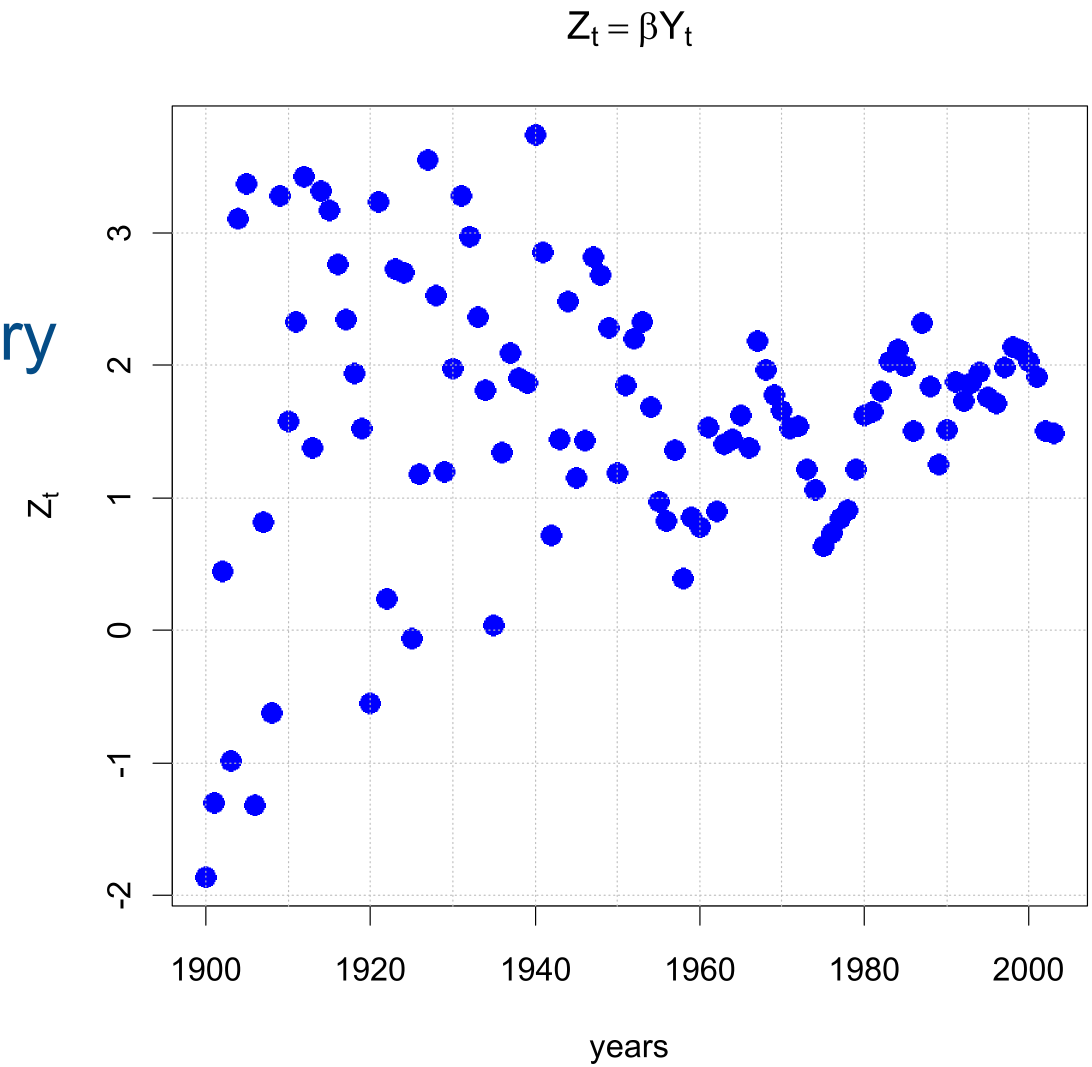
- $\Pi = \alpha' \beta$ is decomposed into the loading matrix α and the cointegrating matrix β

$$\Delta Y_t = \Gamma_1 \Delta Y_{t-1} + \ldots + \Gamma_{p-1} \Delta Y_{t-p+1} - \alpha Z_{t-1} + \varepsilon_t$$

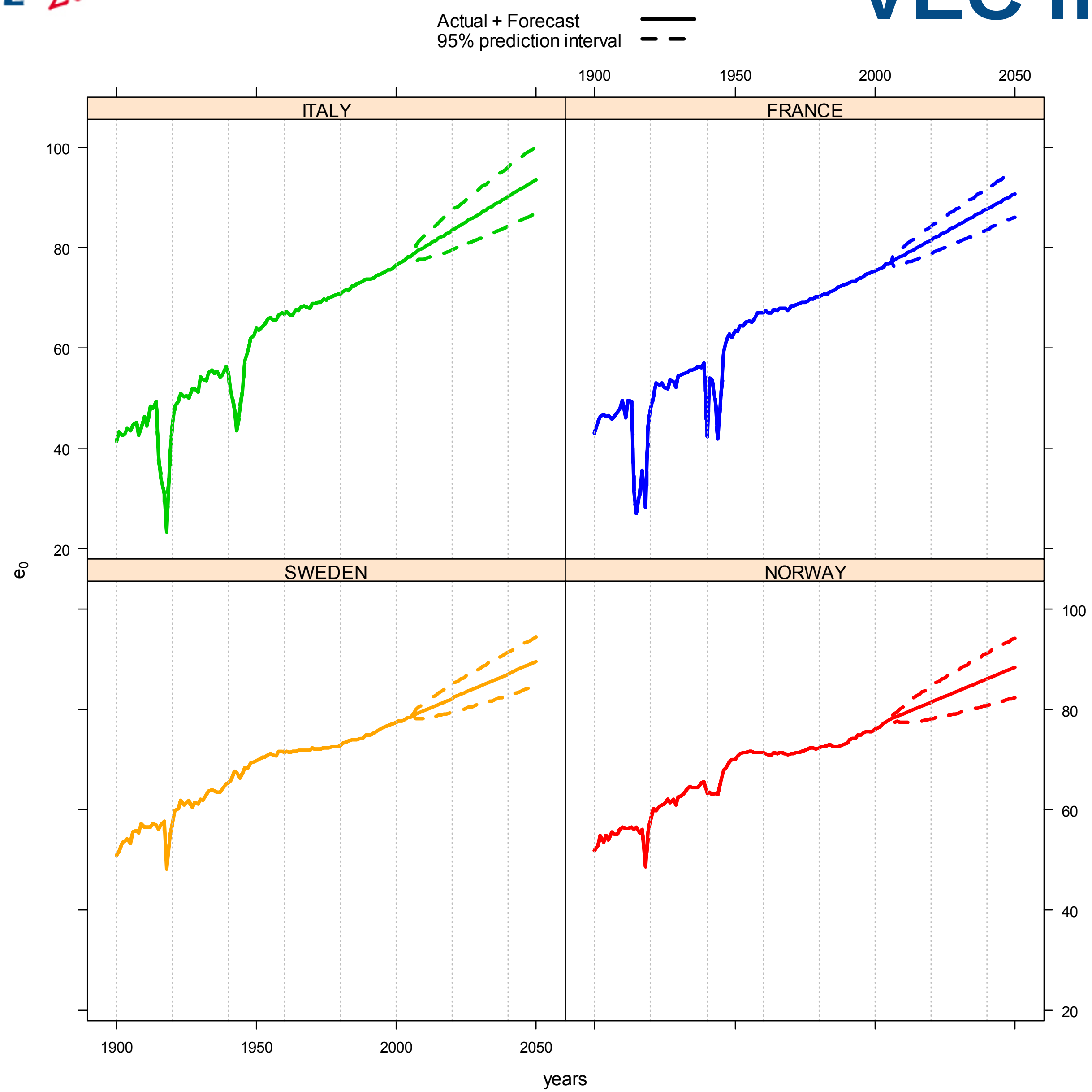
- VEC is a VAR model on the first differences plus a vector of cointegrating residuals

VEC models

- Lag-order: 2
- Rank(Π)=1
- $Z_t = \beta' Y_t$ stationary



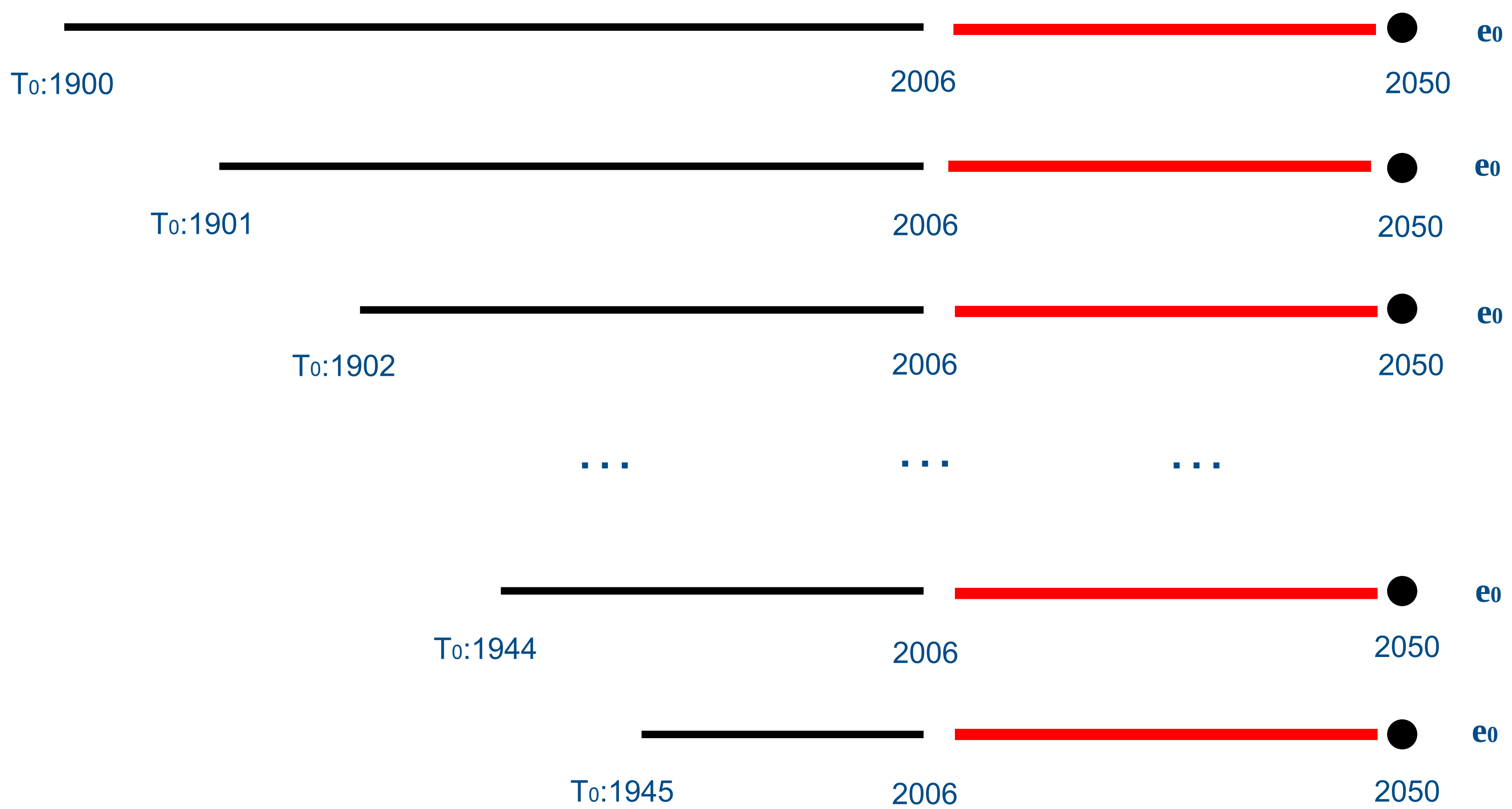
VEC models


Actual + Forecast
95% prediction interval
ITALY
FRANCE
SWEDEN
NORWAY
100
80
60
40
20
1900
1950
2000
2050
e0
years

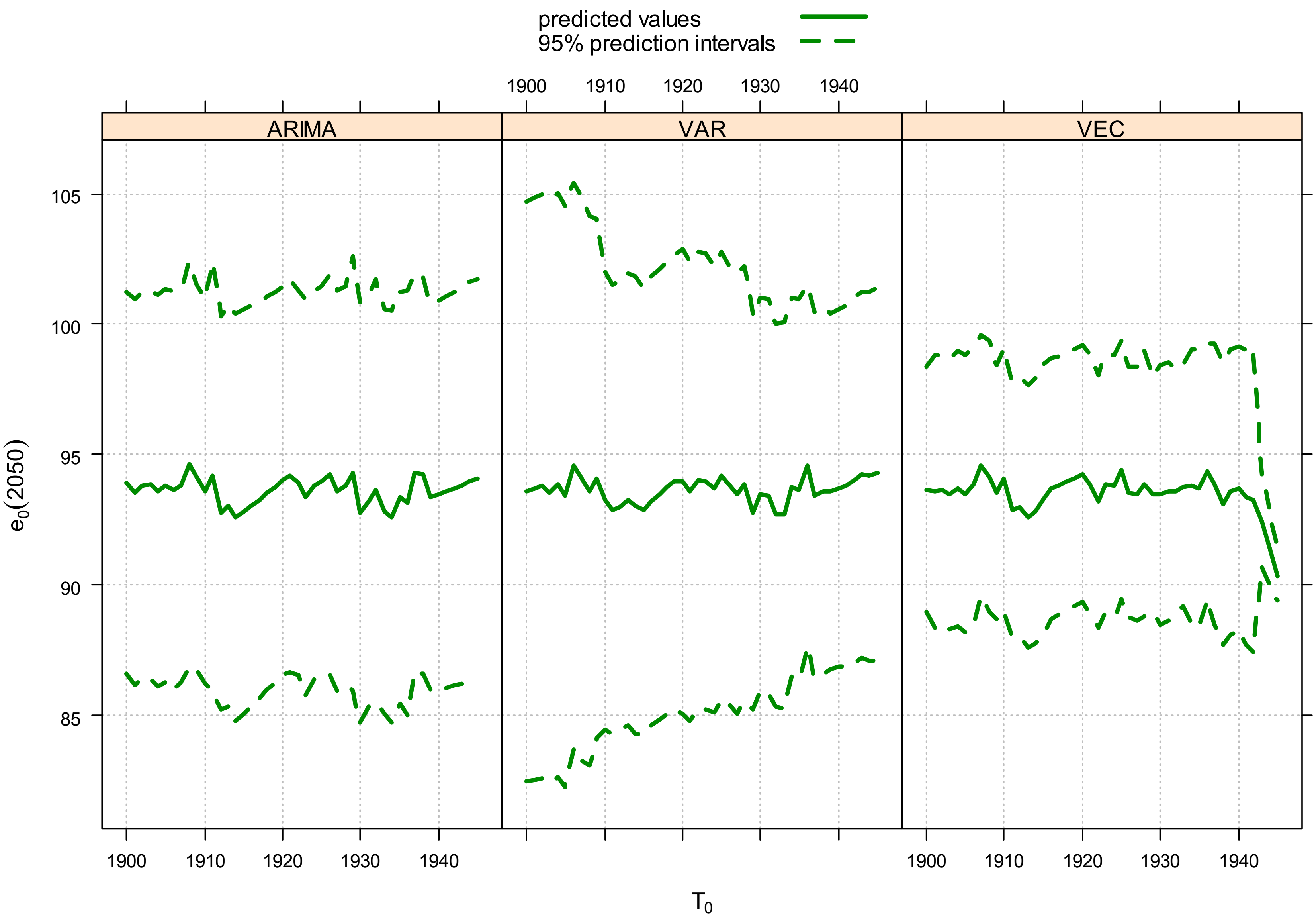
Comparison of results: 2050

	Italy	France	Sweden	Norway
ARIMA				
e_0	93.91	90.93	87.47	86.06
95% CI	(86.59–101.22)	(86.35–95.52)	(82.21–92.75)	(80.33–91.79)
VAR				
e_0	93.66	90.29	89.34	87.79
95% CI	(82.32–105.01)	(80.21–100.37)	(81.71–96.96)	(81.03–94.57)
VEC				
e_0	93.18	90.11	86.80	88.22
95% CI	(88.03–98.33)	(85.46–94.76)	(80.59–93.01)	(82.31–94.13)

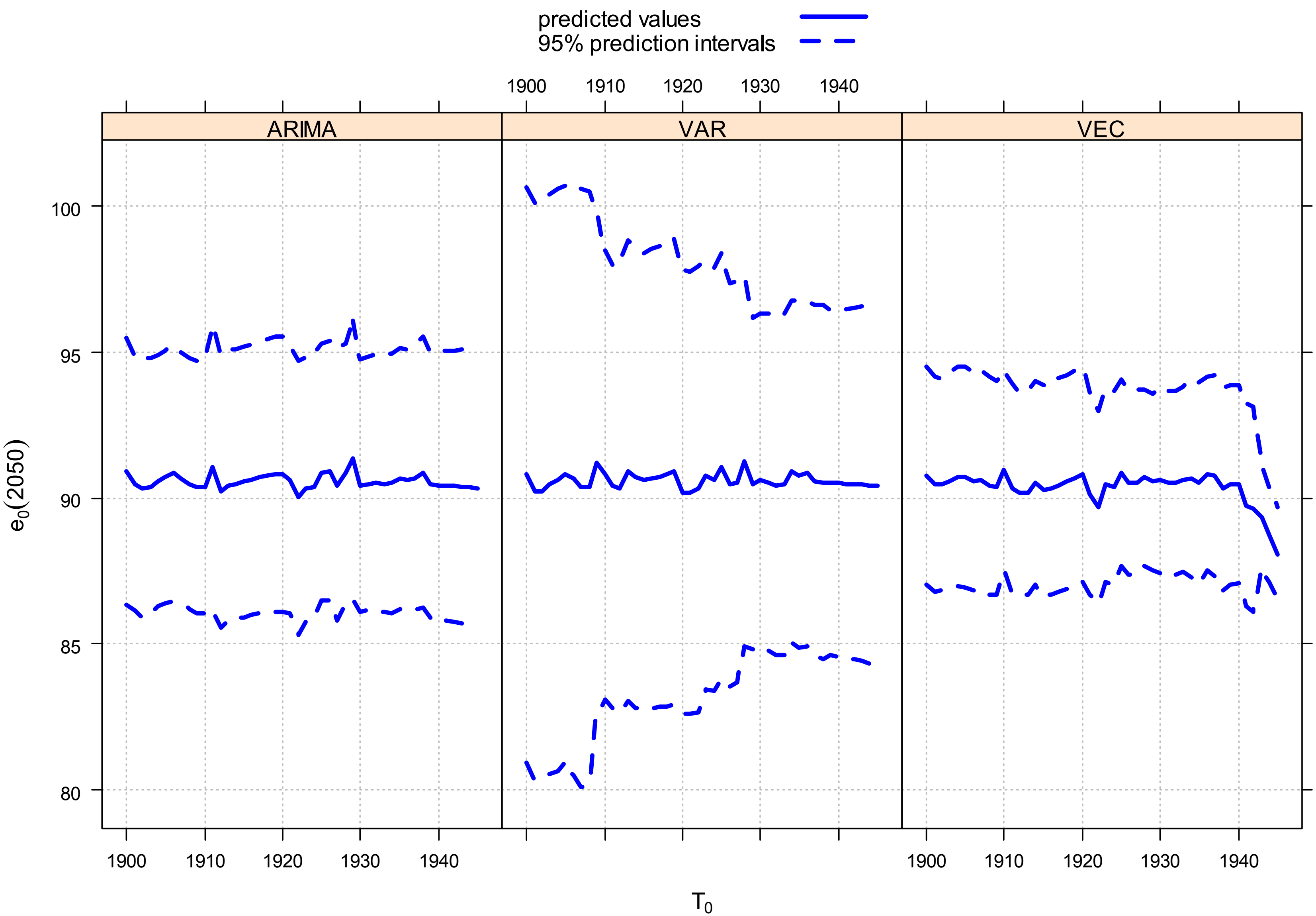
Changing time-window



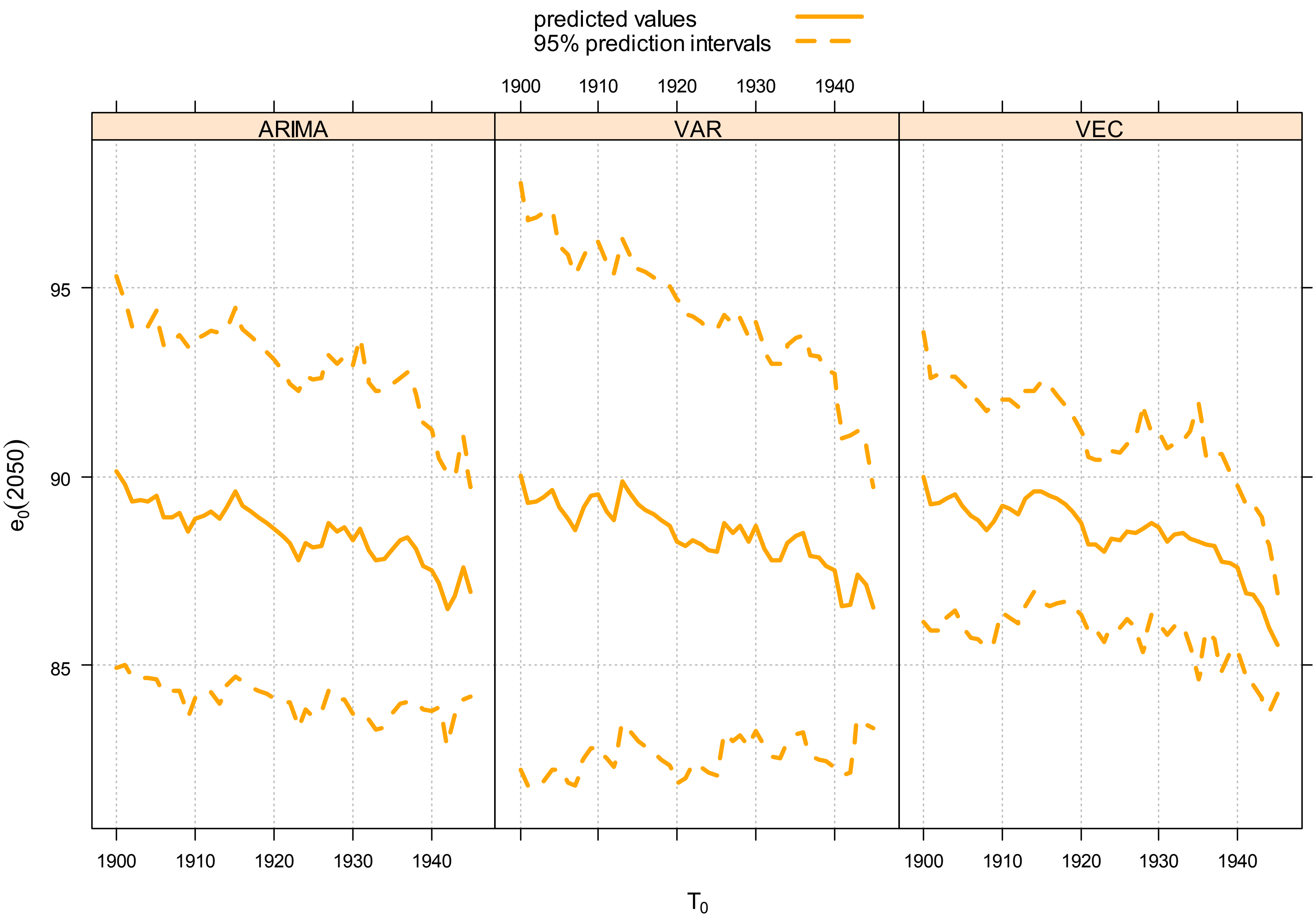
Changing time-window: ITALY



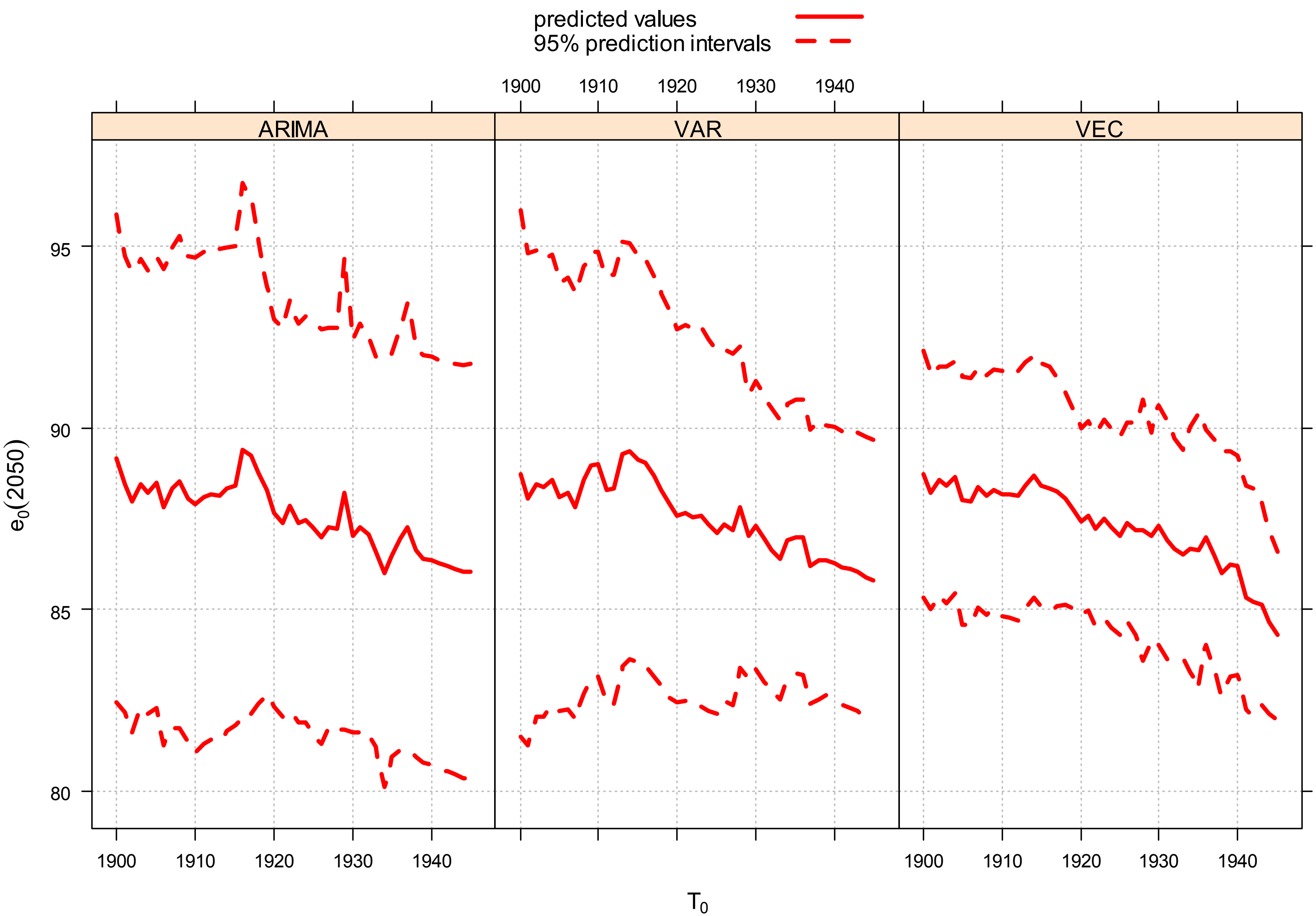
Changing time-window: FRANCE



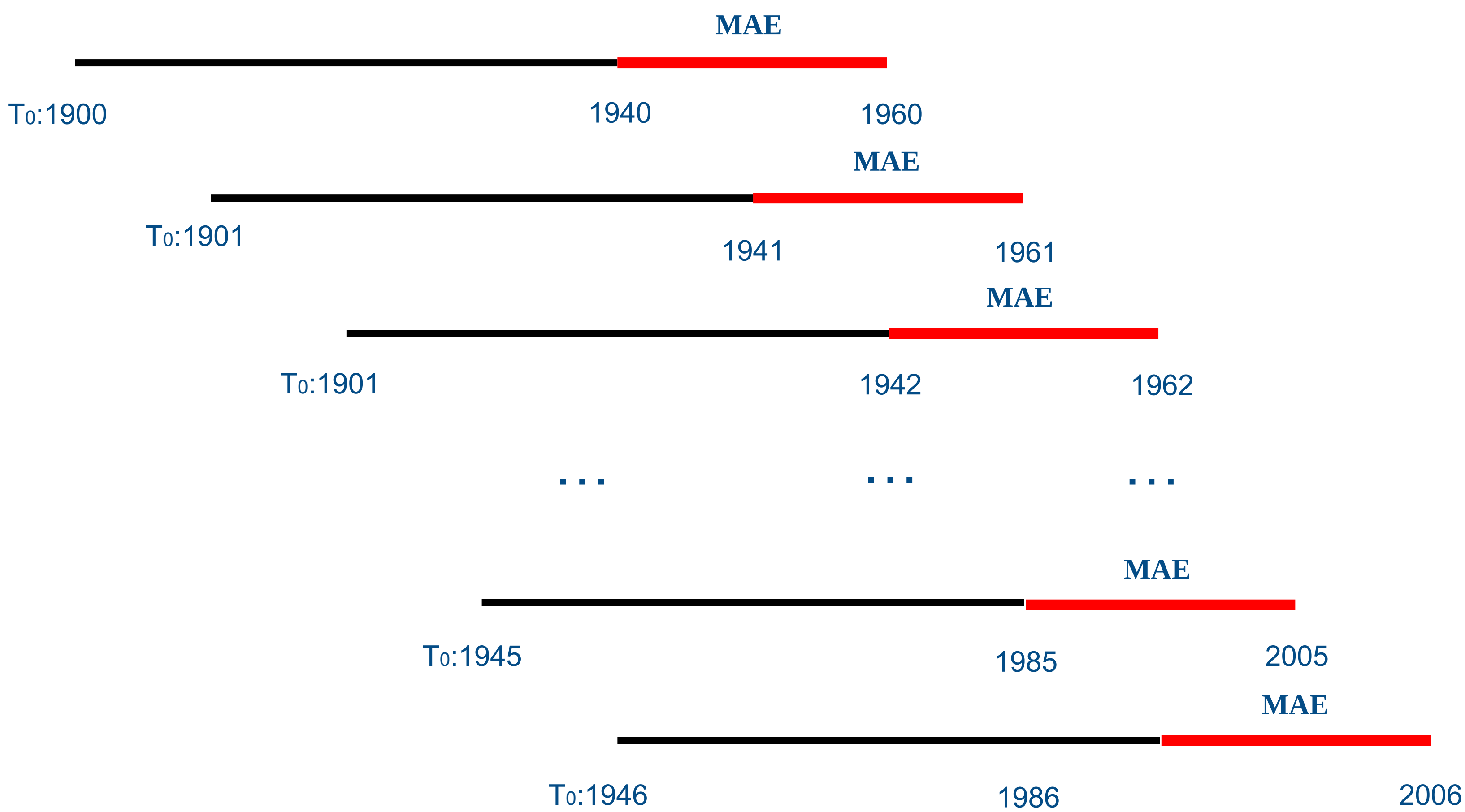
Changing time-window: SWEDEN



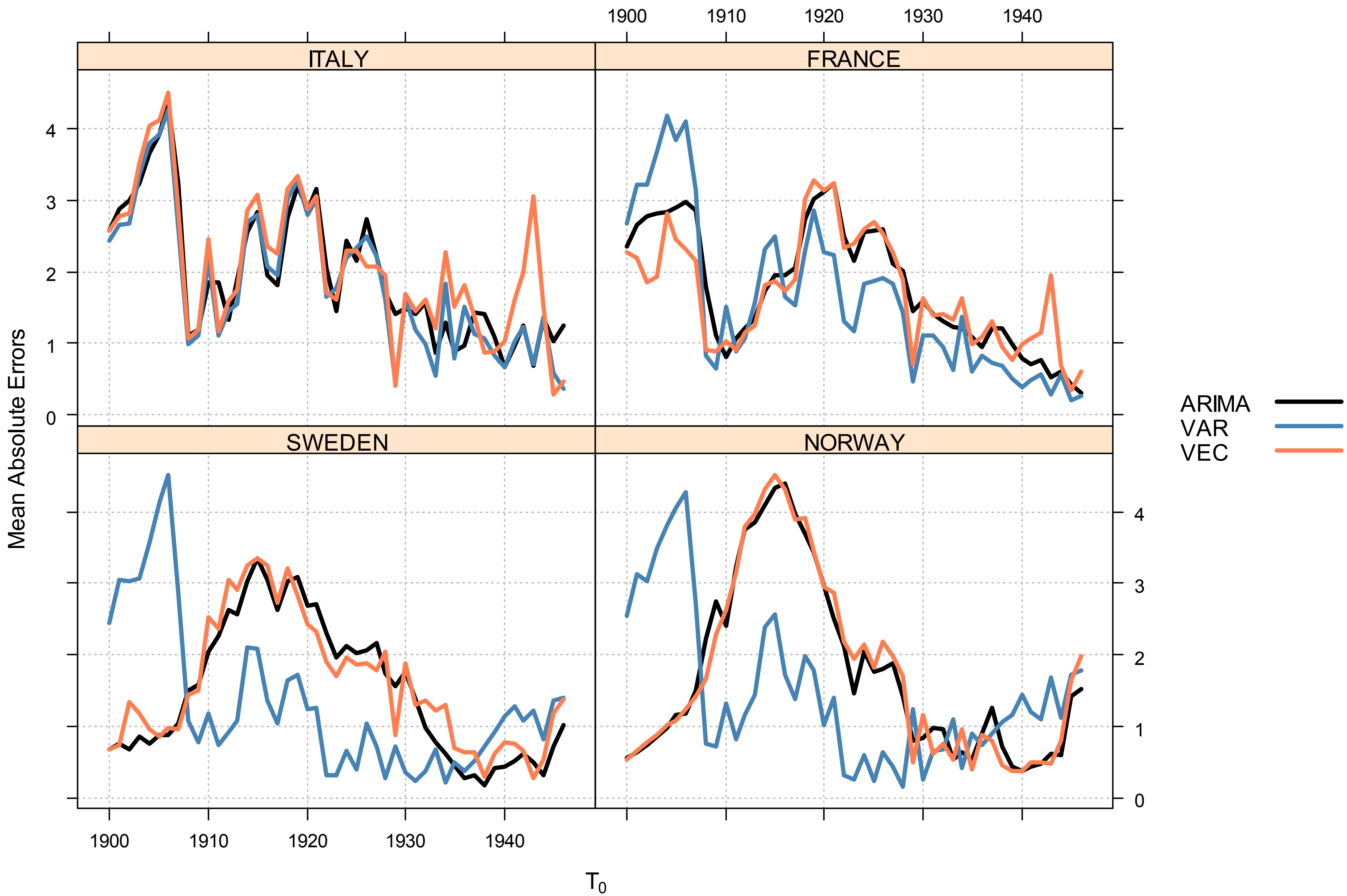
Changing time-window: NORWAY



Changing the in-sample period



Changing the in-sample period



Conclusions

- Our models:
  - Multivariate framework
    - VEC more coherent, VAR performs better forecasts
- Questionable the use of e_0:
  - Not strictly representing the annuitants' mortality
    - Basis risk present for the insurance company

however:

  - A well known and understood measure
  - Easily available and constantly updated
    - Transparency guaranteed to investors

Thanks for your attention!

Comments
or
questions?

torri@demogr.mpg.de

VAR models

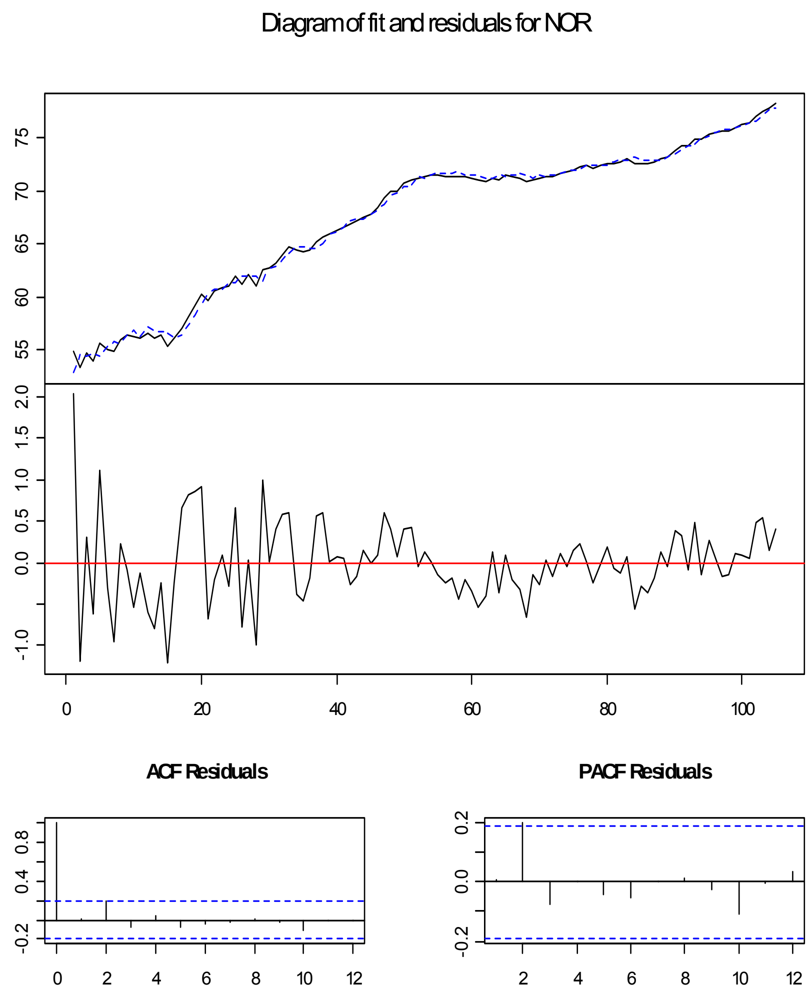


Diagram of fit and residuals for NOR
ACF Residuals
PACF Residuals

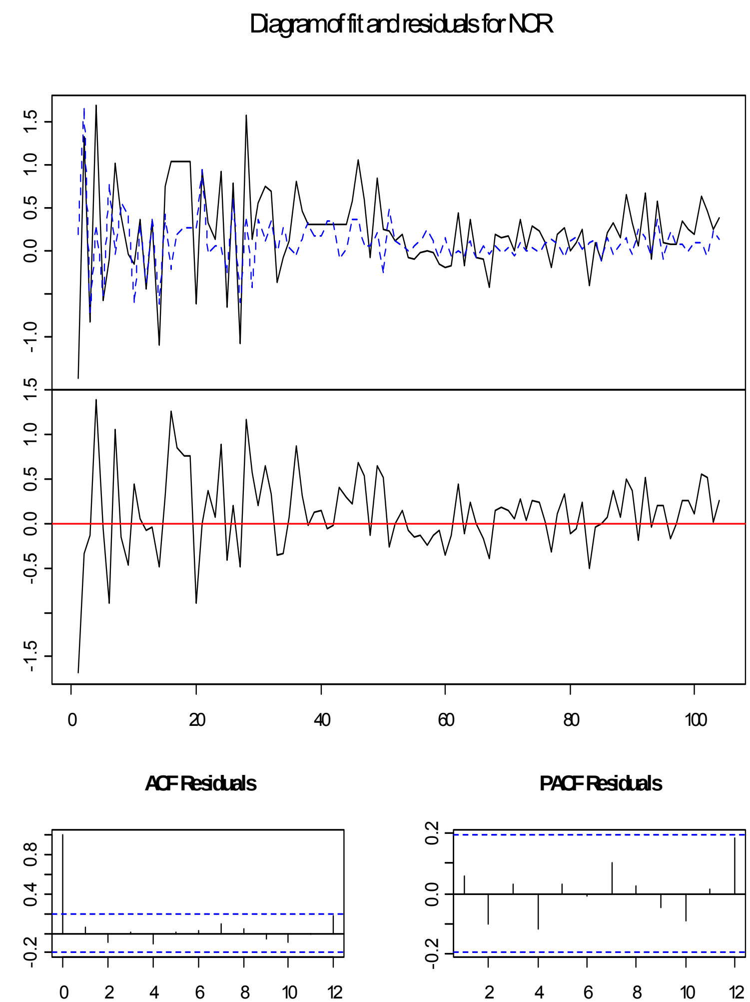


Diagram of fit and residuals for NOR
ACF Residuals
PACF Residuals