IAA AFIR Colloquium 2009

September 10th–11th,2009

Coffee Break

Breakout Session Topic 10:

Solvency, guarantees and risk capital

10 September 2009

An integrated Cost of Risk model and its application to company valuation

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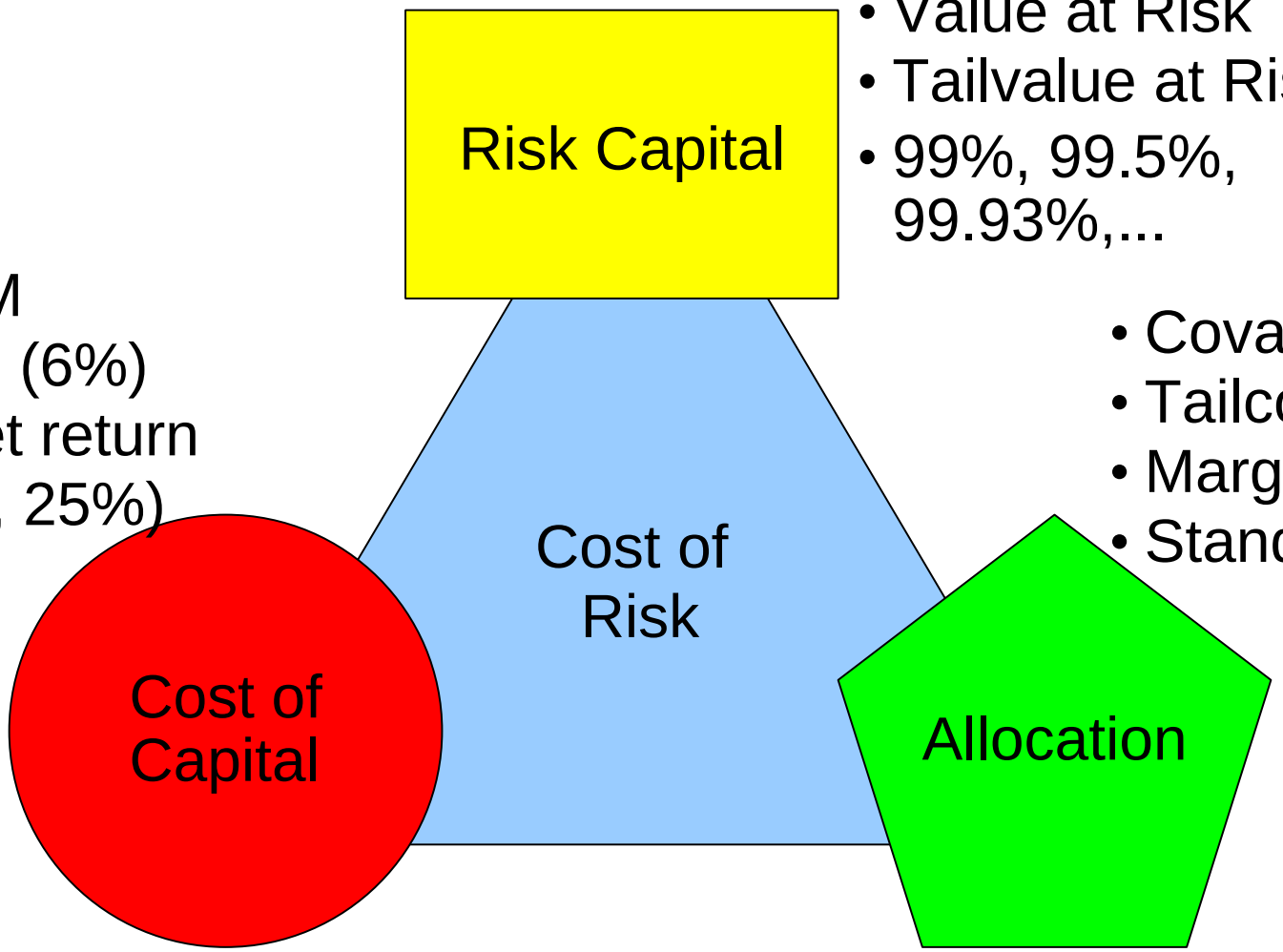
Agenda

- Motivation
- Development of model
- Properties
- Application

Conventional approach



Example: Cat bonds

- QIS 4 parameters: 99.5% VaR, 6%CoC
- Investment volume 100m€

- Cat bond A:
  - E(Claim) 0.5m€
  - VaR 99.5m€
  - Risk load 5.97m€
  - Multiple 13

- Cat bond B:
  - E(Claim) 2.0m€
  - VaR 98.0m€
  - Risk load 5.88m€
  - Multiple 4

▶ Almost constant risk loads!

Cost of Risk – Idea (single bond)

- Nominal value: N
- Probability of default: α
- Spread: $s(\alpha)$
- Result at redemption date: X

$$F_X = \alpha \cdot 1_{[-N;0[} + 1_{[0;\infty[}$$

- Cost of Risk: $N \cdot s(\alpha) = -Q_X(0) \cdot s(\alpha)$

$Q_X(\omega)$:=upper ω quantile of X

Cost of Risk – Idea (multiple bonds)

- Nominal value: $N = n_1 + \ldots + n_k$
- Partial defaults: $l_i = n_i + \ldots + n_k$
- Probabilities of default: $\alpha_1, \ldots, \alpha_k$
- Spreads: $s(\alpha_1), \ldots, s(\alpha_k)$
- Distribution at redemption

$$F_X = \sum_{i=1}^{k} \alpha_i \cdot 1_{[-l_i;-l_{i+1}[} + 1_{[0;\infty[}$$

- Cost of Risk:

$$\sum_{i=1}^{k} n_i \cdot s(\alpha_i) = \sum_{i=1}^{k} (Q_X(\alpha_i) - Q_X(\alpha_{i-1})) \cdot s(\alpha_i)$$

Cost of Risk Model

Transition from differential sum to integral

$$\mathrm{CoR}_s(X) = \int_0^1 s(\alpha)\,dQ_X(\alpha) - s(1)Q(1)$$

If s is smooth and $s(0) = 0$

$$\mathrm{CoR}_s(X) = -\int_0^1 Q_X(\omega)\,ds$$

Properties of CoR_s

- Under regularity conditions for s, CoR_s is a spectral risk measure.
- x%VaR and x%TVaR can be represented by appropriate selection of s.
- If s is concave CoR_s is coherent on all centered random variables.
- For discrete $X=X_1+X_2$ CoR_s has a natural decomposition to X_1 and X_2.

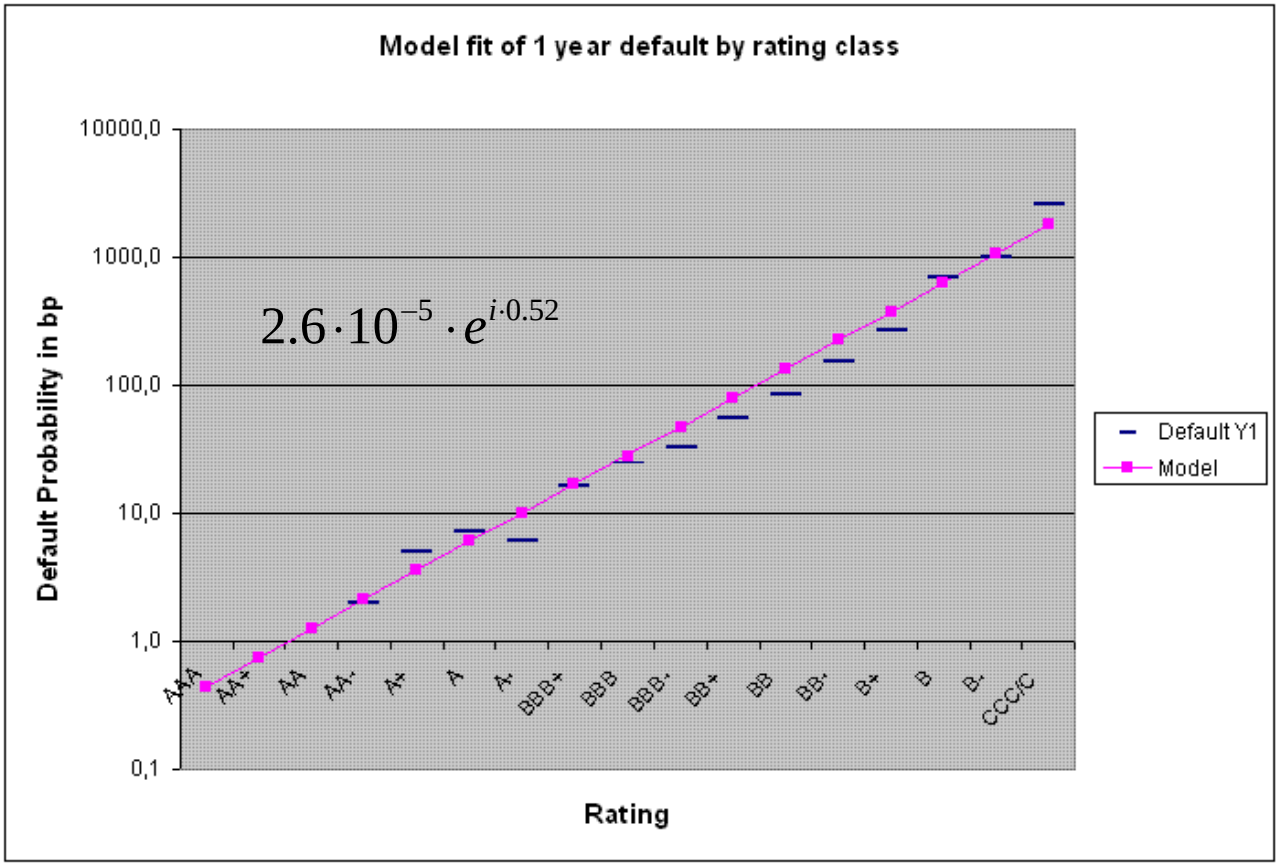
Application

1. Estimate result distribution for company.
2. Value with best estimate assumptions.
3. Calibrate spread by linking to default probabilities via rating classes.
4. Compute integral numerically, e.g. Monte Carlo simulation.
5. Risk adjusted value is 2. less 4.

Example company

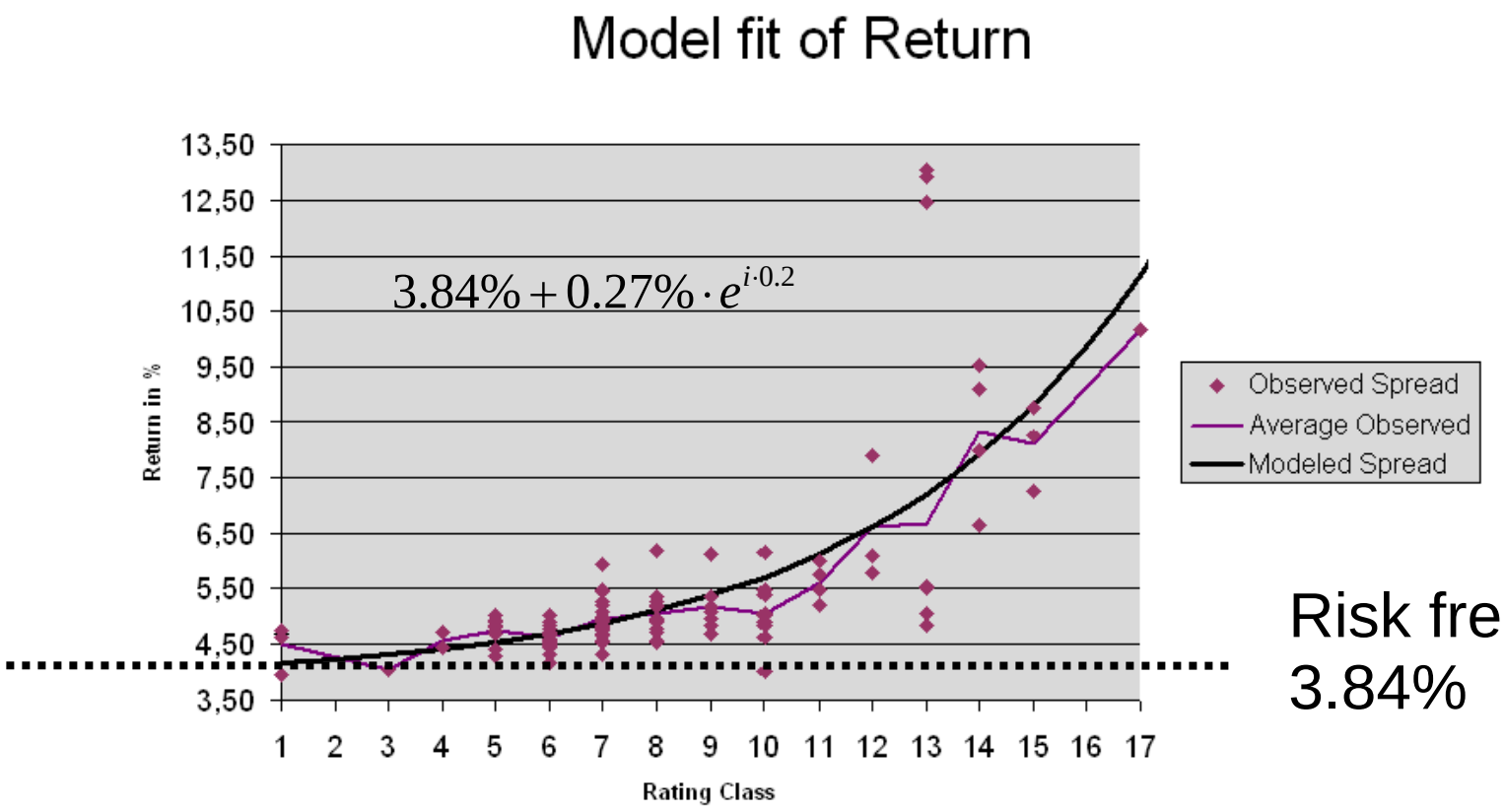
- Motor monoliner
- Premium and reserve risk as in QIS 3
- Assets as at 31.12. (risk free) 85m€
- Annual premium (written 1.1.) 20m€
- Expected C/R: 100%
- Reserves (best estimate) 59m€
- Accident year pays out over 8 years:
  20%; 15%; 15%; 10% each other

Calibration – Rating to default



Data: S&P annual 2006 global corporate default study.

Calibration – Rating to spread



Risk free rate 3.84%

Data: Börse online 3rd calendar week 2008.

Economic valuation (excl. risk)

• Assets	85.0m€
• Reserves	-59.0m€
• Balance sheet capital	26.0m€
• Discounting of reserves	+5.0m€
• PV of new business (1 year)	+2.4m€
• Economic value (excl. risk)	33.4m€
• Return (interest on assets)	3.65m€
• Return on capital	14.0%

Risk adjusted valuation

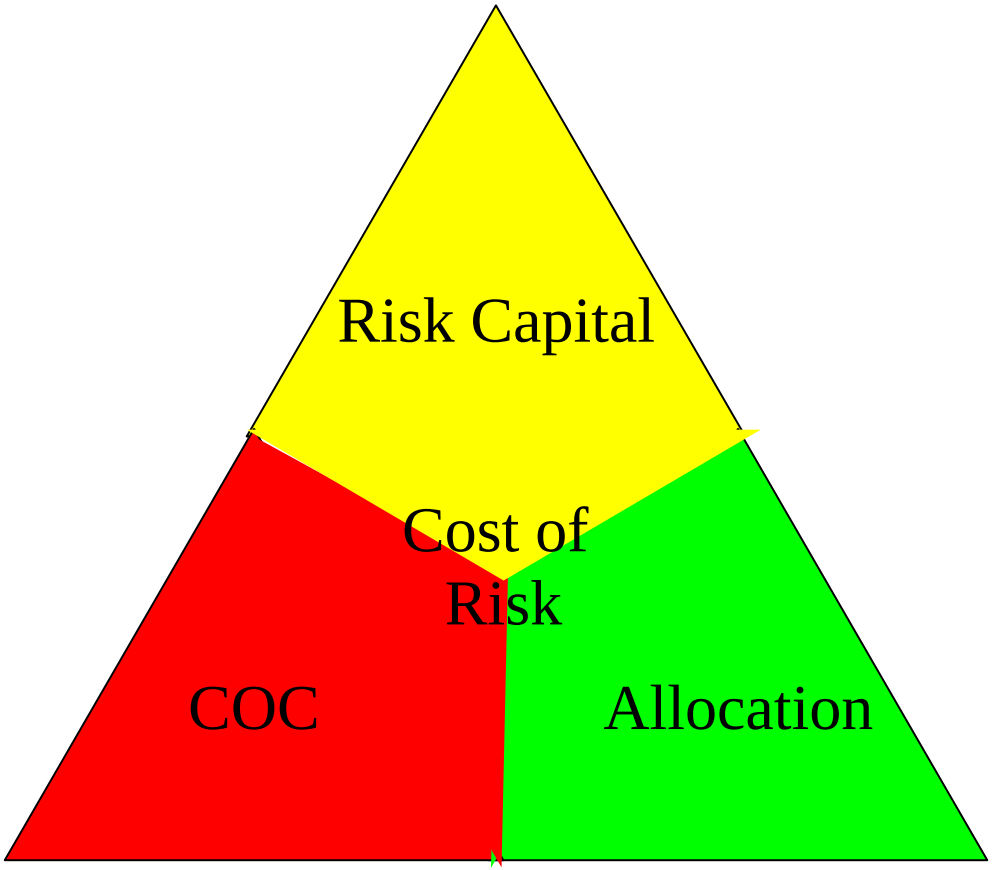
- Balance sheet capital 26.0m€
- Economic value (excl. risk) (A) 33.4m€
- 99.5% VaR (1 year; simulated) 24.9m€
- Cost of Risk (8 years; nominal) 4.9m€
- Cost of Risk (present value) (B) 4.6m€
- Risk adjusted value (A)-(B) 28.8m€

Shares of this company should trade at 111% of the book value per share.

Suggestions for further research

- How consistent does this model work in the market tourmoil of 2008?
- How to take information coded in a CAPM (if available) into account?
- How can the spread function be estimated without using ratings, which include also other aspects?

Thank you for your attention.



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