Economic Values of Contribution Cashflows and Measures to Bring the EVs under Control

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3. Partial ring-fencing of plan assets by payout year
4. Idea of the payout-year-specific funding standard and probability of full funding
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1-1. Covariance pricing formula

\[ q = \frac{\text{E}(v)}{R_F} + \text{cov}(\xi, v) \]

- A cashflow increasing in market downturn and decreasing in market upturn correlates positively to the state price density.
- The second term thus becomes positive.
1-2. Evaluation of employer contributions

• The EV of employer contributions would be evaluated higher than their best-estimated PV.

• The greater the volatility of contributions is, the higher the EV of the contributions is evaluated.
1-3. In the case of benefit cashflows

\[ q = \frac{E(v)}{R_F + \delta} \]

- The risk premium delta with regard to the benefit volatility originated from the beta risk of investments should be **positive**.
- The risk premium delta with regard to the benefit volatility originated from the *macro* longevity risk should be **negative**.
2. A benefit design sustainable under the ‘Japan Scenario’

\[
\text{if } L^{(1)} \leq A \quad \text{, then } \quad B = B^{(1)} \\
\text{if } A = L^{(0)} + \alpha (L^{(1)} - L^{(0)}) \quad (0 \leq \alpha < 1) \quad ,
\]

\[
\text{then } B = B^{(0)} + \alpha (B^{(1)} - B^{(0)})
\]

\[
\text{if } A < L^{(0)} \quad , \quad \text{then } \quad B = B^{(0)}
\]

- \(B(0)\) and \(B(1)\) are both indexed to inflation.
- The portion exceeding \(L(0)\) functions as a virtual risk buffer.
3-1. A serious deficiency in DB plans

• No specific correspondence between individual liabilities and plan assets

• As a result,
  – the interests of beneficiaries are given the most privileged status.
  – no effective and timely mechanism of keeping under control the risks of active participants is incorporated.
3-2. Framework of the payout-year-specific (PYS) funding standard

• Divide contributions by payout year
• Load the decomposed contributions on the ‘sequentially chained containers’
• And assign each container
  – a payout year, and
  – a minimum permissible funding ratio
3-3. Partial ring-fencing of assets

• The assets can only be used for paying benefits in the year assigned to the container.

• A surplus may be used for filling up the shortfalls of other containers
  – a kind of *intertemporal* risk-sharing

• But cannot fill up a shortfall with aggravating deficiencies of other containers.
3-4. How to decompose contributions

• Naturally given by discounting back the accrued benefits using appropriate discount rates

• Then, how to determine the appropriate discount rates for each container?

• The gist of payout-year-specific (PYS) funding standard lies in this point!
3-5. Using the expected return of the actual portfolio in not prudent

\[
dA_u = rA_u \, du + \sigma A_u \, dW_u \quad u \in [t, T]
\]

\[
A_t = L_T \exp\{-r(T-t)\}
\]

Then,

\[
P(A_T > L_T) = N\left(-\frac{1}{2} \sigma \sqrt{T-t}\right)
\]
4-1. Main ideas of the PYS standard (1/2)

P1 = the probability that the portfolio value will attain the liability value \textit{at some time}.

P0 = probability that the portfolio value will surpass the liability value \textit{at the year of maturity}.

Then,

P1 > P0
4-1. Main ideas of the PYS standard (2/2)

• Assume switching to a liability-hedging portfolio at the time of hitting the upper barrier, not waiting the year of maturity

• Determine the maximum discount rate so as to satisfy the condition:
  – \( P_1 > p_1 \) (given from outside)

• Need to evaluate \( P_1 \)
4-2. The probability of hitting the upper barrier (1/4)

\[ dA_u = r A_u \, du + \sigma \, A_u \, dW_u \quad u \in [t, T] \]

\[ A_t = L^{(1)} \exp \{ - (\mu \theta_t + r_F) (T - t) \} \]

\[ \mu = r - r_F \]

\[ dB_u = r_F B_u \, du \quad u \in [t, T] \]

\[ B_t = L^{(1)} \exp \{ - r_F (T - t) \} \]
4-2. The probability of hitting the upper barrier (2/4)

\[ X_u = \log \frac{A_u}{B_u} \]

Then,

\[ d X_u = \left( \mu - \frac{1}{2} \sigma^2 \right) dt + \sigma dW_u \]

\[ X_t = \alpha_t = -\mu \theta_t (T - t) < 0 \]
4-2. The probability of hitting the upper barrier (3/4)

Consider a *running maximum process*:

\[ M_X(u) = \sup_{t \leq s \leq u} X_s \]

\[ F_{M(u)}(x): \text{ the distribution function} \]

Then, probability \( P_1 \) is given by:

\[ P_1 = 1 - F_{M(T)}(0) \]
4-2. The probability of hitting the upper barrier (4/4)

\[
F_{M(u)}(x) = \left\{ \begin{array}{l}
\exp\left\{ -\frac{1}{2} \left( \frac{\mu - \frac{1}{2} \sigma^2}{\sigma^2} (x - \alpha_t) \right) \right\} N\left\{ -\frac{(x - \alpha_t) + \left( \mu - \frac{1}{2} \sigma^2 \right) (u - t)}{\sigma \sqrt{u - t}} \right\}
\end{array} \right.
\]

\[
\frac{1}{2} \left( (x - \alpha_t) - \left( \mu - \frac{1}{2} \sigma^2 \right) (u - t) \right)
\]

\[
\frac{\sigma \sqrt{u - t}}{2}
\]
5. Additional conditions (1/6)

- Condition $P1 > p1$ only determines the maximum proportion $\theta_t$ of the excess return $\mu = r - r_F$ for each combination of excess return and its volatility.

- Another condition has to be introduced, especially from the aspect of restricting the risk of underfunding to determine unique discount rates.
5. Additional conditions (2/6)

(1) Restricting the severity of loss when the portfolio value could not attain the liability value \textit{at any time}

(2) The conditional expectation of the portfolio value \textit{at the year of maturity} is within an affordable range:

$$E\left[ \frac{A_T | A_T < L^{(0)}}{L^{(0)}} \right] \ge q, \quad 0 < q < 1$$
5. Additional conditions (3/6)

(3) Probability $P_2$ that the log funded ratio is absorbed into the lower barrier at some time is less than constant $p_2$.

\[ dC_u = r_F C_u \, du \quad u \in [t, T] \]

\[ C_t = L^{(0)} \exp\{-r_F (T - t)\} \]
5. Additional conditions (4/6)

\[ Y_u = \log \frac{A_u}{C_u} \]

\[ dY_u = (\mu - \frac{1}{2} \sigma^2) \, du + \sigma \, dW_u \]

\[ Y_t = \beta_t = -\mu \theta_t \, (T - t) + \log \left( \frac{L^{(1)}}{L^{(0)}} \right) > 0 \]
5. Additional conditions (5/6)

Consider a *running minimum process*:

\[ m_Y(u) = \inf_{t \leq s \leq u} Y_s \]

\[ F_{m(u)}(y): \text{ the distribution function} \]

Then, probability P2 is given by:

\[ P_2 = F_{m(u)}(0) \]
5. Additional conditions (6/6)

\[ F_{m(u)}(y) = N \left( \frac{(y - \beta_t) - (\mu - \frac{1}{2} \sigma^2)(u - t)}{\sigma \sqrt{u - t}} \right) \]

\[ + \exp \left\{ \frac{1}{2} \frac{(\mu - \frac{1}{2} \sigma^2)(y - \beta_t)}{\sigma^2} \right\} N \left( \frac{(y - \beta_t) + (\mu - \frac{1}{2} \sigma^2)(u - t)}{\sigma \sqrt{u - t}} \right) \]
6-1. Maximum permissible proportions of excess returns --- condition (2)

<table>
<thead>
<tr>
<th>$L^{(1)}/L^{(0)}$</th>
<th>$T-t$</th>
<th>$p_1$</th>
<th>$q$</th>
<th>$w_t$</th>
<th>$\theta_t$</th>
<th>$r_P$</th>
<th>$\exp{(r_P-r_F)(T-t)}$</th>
<th>$\exp{\alpha_t}$</th>
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6-2. Maximum permissible proportions of excess returns --- condition_(3)

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<td>1.29 1.27 1.25</td>
<td>1.36 1.34 1.31</td>
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</tbody>
</table>
6-3. Implications for discount rates

• The discount rates thus determined include a proportion of the expected excess return.

• The proportion increases progressively as the period until maturity extends.
  – but only if acceptable risk also increases

• The graph of the proportion might be hump-shaped
  – if there is a due limit on acceptable risks
6-4. Implications for funding standards

• Pension funds may take larger investment risks as investment horizon extends
  – but only when funding deficiencies can be corrected gradually spending longer periods

• Any funding standard should strike a right balance between
  A) assuring stable employer contributions with reasonable prices, and
  B) ensuring that targeted benefits are paid with reasonably high probabilities
6-5. Implications for investment strategies

• Any portfolio is considered as a composite of target date funds (TDFs).
  – irrespective of the funding standards

• Decomposition of a portfolio by payout year makes the discussion on investment horizons extremely transparent.

• Any portfolio should be rebalanced along with the changes in benefit cashflow estimation and the degree of risk aversion
  – even if the prospect on the market is invariant
7. Concluding remarks

• Any funding standard cannot continue to exist without paying proper consideration to investments and vice versa.

• The PYS funding standard is a bridge connecting the financing issues and investment issues systematically.