AFIR LIFE MUNICH 2009

Modelling and Hedging Synthetic CDO Tranche Spread Risks

by

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Presentation to the AFIR Colloquium 2009 Munich Germany

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September 6 to 11, 2009

Overview

- Outline of Synthetic CDOs
- Market methods (correlation mapping) for CDO tranche hedging/pricing bespoke credit portfolios
- Credit spread risk
- Results and key conclusions

Synthetic CDOs

Synthetic CDO tranches - derivatives on the default process of a portfolio of companies. Traded indices and tranches - iTraxx, CDX

Tranches	0-3%	3-6%	6-9%	9-12%	12-22%	Index
Prices	31.48%	355.7	220	141	69.8	93

Table: Quoted market price of iTraxx Europe tranches at 31/7/2008 (Source: www.creditfixings.com)

- Protection seller promises to cover percentage of defaults in exchange for premiums
- Similar to insurance contracts with deductible and policy limit (attachment and detachment points)

Credit Spread Risk

Tranches	0-3%	3-6%	6-9%	9-12%	12-22%	Index	DP
31/7/2008	31.48%	355.7	220	141	69.8	93	0.0154
31/1/2007	10.34%	41.59	11.95	5.6	2	23	0.0038

Table: DP = Default probability, calibrated from the Index spread (Source: www.creditfixings.com)

Default probability calibrated to index tranche prices - vary correlation to match default probability

Increase in spread = Increase in expected future default losses = write down in value (marked to market) or increased capital/loss provision

One Factor Gaussian Copula

Introduced by Li (2000), assumes firm defaults when it's asset value falls below a certain level Asset return of firm i as:

$$X_i = \rho_i Y + \sqrt{1 - \rho_i^2} Z_i$$

X_i, Y, Z_i are assumed to be standard Normals

Map the distribution of X_i to the distribution of default time τ_i on a percentile to percentile basis:

$$F_i(t) = P(\tau_i < t) = P(X_i < D_{i,t}) = \Phi(D_{i,t})$$

$$\Rightarrow D_{i,t} = \Phi^{-1}(F_i(t))$$

Homogeneous portfolio

$$P(\tau_i < t|Y) = P(X_i < D_{i,t}|Y) = P(\rho_i Y + \sqrt{1-\rho_i^2} Z_i < \Phi^{-1}(F_i(t)))$$
$$= \Phi(\frac{\Phi^{-1}(F_i(t)) - \rho_i Y}{\sqrt{1-\rho_i^2}})$$

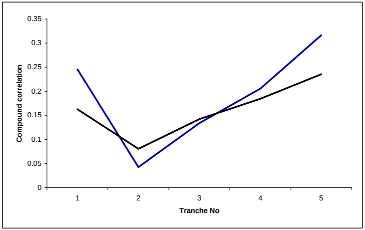
Homogenous portfolio. All correlations equal $\rho_i = \rho$ all i.

Conditional distribution of number of defaults for portfolio of M companies $N_t|Y \sim Binomial(M, P(\tau_i < t|Y))$.

Unconditional distribution (asymptotically Gaussian) is:

$$P(N_t = n) = \int_{-\infty}^{\infty} P(N_t = n|Y) \cdot f(Y) \cdot dY$$

Pricing with Compound correlation



Tranches	0-3%	3-6%	6-9%	9-12%	12-22%	Dp
31/07/2008	0.47	0.87	NA	0.14	0.25	0.0154
31/03/2008	0.47	0.85	NA	NA	0.22	0.0205
28/09/2007	0.25	0.04	0.13	0.21	0.32	0.006
31/01/2007	0.16	0.08	0.14	0.18	0.24	0.0038

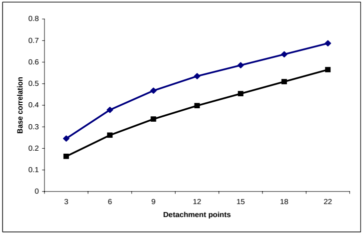
Table: Fitted compound correlation to market prices.

Pricing with base correlation

Selling protection on $a - b\%$ tranche can be replicated by selling $0 - b\%$ tranche and buying $0 - a\%$ tranche at the same time. (MacGinty et al (2004))

Base correlations are the correlations that calibrate to market prices for equity tranche spreads with different detachment points.

Pricing with base correlation



Tranches	0-3%	3-6%	6-9%	9-12%	12-22%	Dp
31/07/2008	0.47	0.61	0.69	0.77	NA	0.0154
31/03/2008	0.47	0.59	0.66	0.71	NA	0.0205
28/09/2007	0.25	0.38	0.46	0.53	0.69	0.006
31/01/2007	0.16	0.26	0.34	0.40	0.57	0.0038

Table: Fitted base correlation to market prices.

Correlation mappings

- No mapping: Assume same base correlation or compound correlation for bespoke and standard portfolio. Correlation does not depend on changed default probability.

- ATM (At-the-Money) mapping: If the ratio of default probability of bespoke and standard portfolio is a then the $0 - X\%$ tranche of bespoke portfolio is valued with the same correlation as the $0 - aX\%$ tranche of the standard portfolio.

- TLP (Tranche Loss Proportion) mapping:

$$\frac{ETL_S(K_S, \rho(K_S))}{EPL_S} = \frac{ETL_B(K_B, \rho(K_S))}{EPL_B}$$

Equity tranche with detachment point K_B valued with same correlation as equity tranche of standard portfolio with detachment point K_S based on equating expected tranche loss of equity tranches as proportion of expected portfolio loss.

Assessing the risk

Assume a set of scenarios for future default probabilities

Determine credit spread of CDO tranches based on market methods under each scenario.

Given the default probability, which method/model prices the CDO tranche spread most effectively?

This is also closely related to the issues of:
-Hedging CDO tranches with the Index.
-Pricing CDOs on bespoke portfolios.

Price data and methodology

Data used: iTraxx Europe tranche spreads of 101 dates from 22/09/07 to 12/09/08 (source: Bloomberg)

Models/methods are fitted to market prices as at date 1/1/08 CDO tranches are priced with the fitted model for the next 71 dates assuming the index tranche spread is known. These are compared with the actual spreads.

Method similar to that proposed in Finger (2008) to test the ability of a model to hedge CDO tranches with the index.

Basis error

Market prices for CDO tranches differ through time even when the index spread is the same (which implies the same probability of default).

Group data with the same index spread and assume the best a model can achieve by pricing CDO tranches using default probability as the only variable input is the average of the tranche spreads within the same group:

Tranches	0-3%	3-6%	6-9%	9-12%	12-22%
Basis Error	4.56%	3.77%	4.31%	5.19%	5.84%

Table: Average absolute pricing errors as percentage of actual spread.

Proportionate to spread

Tranches	0-3%	3-6%	6-9%	9-12%	12-22%	Index
1/1/2008	22.03%	156.07	89.5	57.13	31.99	50
Modeled	$\beta_1 50$	$\beta_2 50$	$\beta_3 50$	$\beta_4 50$	$\beta_5 50$	

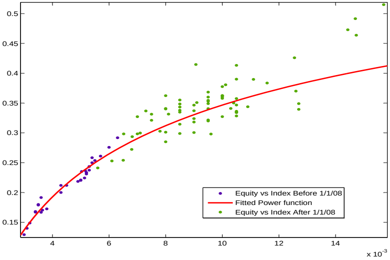
Table: Calibrate betas on market prices at 1/1/08

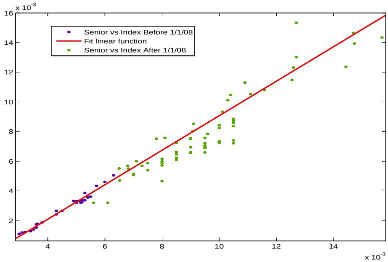
Tranches	0-3%	3-6%	6-9%	9-12%	12-22%	Index
31/7/2008	31.48%	355.7	220	141	69.8	93
Predicted	$\hat{\beta}_1 93$	$\hat{\beta}_2 93$	$\hat{\beta}_3 93$	$\hat{\beta}_4 93$	$\hat{\beta}_5 93$	
	37.45%	265.3	152.2	97.1	54.4	

Table: Predict 31/7/2008 Tranche prices, use calibrated betas

Regression on past spreads

Regress tranche spreads on index spread, using data from 22/09/07 to 1/1/08. Use fitted model to predict the tranche spreads after 1/1/08, given the index spreads.



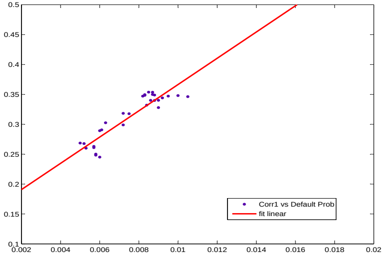


$$Y = 1 - aX^b$$

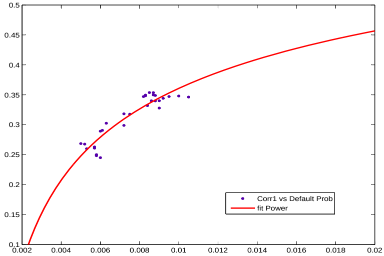
$$Y = aX + b$$

Past correlations

Regress base correlations on default probabilities from 22/09/07 to 1/1/08. Use fitted model to calculate correlation parameters corresponding to level of default probabilities after 1/1/08.



$$Y = aX + b$$



$$Y = 1 - aX^b$$

Results - spreads and correlations

Tranches	0-3%	3-6%	6-9%	9-12%	12-22%
Ccc	19.62%	25.43%	54.01%	34.18%	36.43%
Cbc	19.62%	7.31%	9.14%	10.02%	13.28%
ATM	9.03%	19.29%	31.89%	64.17%	195.28%
TLP	7.71%	15.35%	9.14%	15.53%	11.92%
Reg1	21.08%	22.05%	28.11%	30.29%	20.9%
Reg2	6.67%	17.82%	22.45%	10.51%	10.65%
CrL	14.17%	48.62%	39.8%	22.35%	22.63%
CrP	7.89%	34.6%	24.52%	10.16%	13.98%

Table: Average absolute pricing errors as percentage of actual spread.

Implied Copula

$$P(\tau_i < t|\lambda) = P(1 - exp(-\lambda t)|\lambda)$$

Given λ, the default of all firms are independent, assume homogenous portfolio, the conditional distribution of number of defaults for a portfolio of M companies is

$$N_t|\lambda \sim Binomial(M, P(\tau_i < t|\lambda))$$

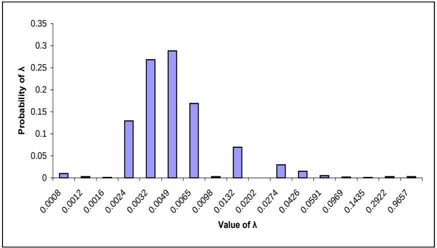
The unconditional distribution is therefore:

$$P(N_t = n) = \int_{-\infty}^{\infty} P(N_t = n|\lambda) \cdot f(\lambda) \cdot d\lambda$$

Hull & White (2006) determine λ using a distribution that fits market prices of all tranches - "perfect copula".

Fitted Copula - Hazard Rates

Implied copula models assume a recovery rate assumption and use constraints to smooth the distribution. Implied copula model with a 17-point discrete distribution fitted to iTraxx tranche spread as at 1/1/08.



Hull & White (2006) assume that hazard rate of bespoke portfolio is related to standard portfolio by: $\lambda_B = \beta\lambda$

Results - Implied Copula

Tranches	0-3%	3-6%	6-9%	9-12%	12-22%
Ccc	19.62%	25.43%	54.01%	34.18%	36.43%
Cbc	19.62%	7.31%	9.14%	10.02%	13.28%
ATM	9.03%	19.29%	31.89%	64.17%	195.28%
TLP	7.71%	15.35%	9.14%	15.53%	11.92%
IC	45.3%	17.36%	15.7%	13.89%	19.95%

Table: Average absolute pricing errors as percentage of actual spread.

Conclusions

- Spread risk for CDO tranches assessed using various market methods for pricing/valuation based on default probability.
- Current market methods to hedge CDOs hold base correlation constant (no mapping). Method reasonably accurately prices CDO tranches (except equity) given a change in default probability.
- Bespoke CDOs priced with correlation mapping methods. TLP mapping methods perform reasonably. ATM mapping method performs poorly.
- Implied copula models did not perform as well compared with market models for hedging.
- Current methods of pricing and hedging may benefit from incorporating past spread data particulary for equity tranche.