DYNAMIC ASSET LIABILITY MANAGEMENT

A METHOD FOR OPTIMISING INVESTMENT STRATEGY

Aldo Balestreri
Milliman Srl, Italy
aldo.balestreri@milliman.com

Jeremy Kent
Milliman Consulting Ltd, UK
jeremy.kent@milliman.com

Ed Morgan
Milliman AG, Switzerland
ed.morgan@milliman.com

1. Introduction

The objective of this paper is to consider the following question: “How can we use modern actuarial modelling to optimise the investment strategy of a life insurance company?” We will propose techniques for doing this, introduce some concepts behind these techniques and discuss some illustrative results.

In the experience of the authors sophisticated financial models have frequently been built for the purpose of calculating items such as Market Consistent Embedded Value (“MCEV”), Economic Capital and other related uses, but the focus has not been on operational Asset Liability Management (“ALM”).

In the majority of cases the investment strategy of the company is an INPUT to these models rather than an OUTPUT from them. By an input we mean that the investment strategy is one of the assumptions input into the model rather than something calculated from the model. If the model is used to guide investment strategy at all it is usually only for calculating high level indicators (such as the relative duration of assets and liabilities or the value at risk due to asset/liability mismatches) which are only one of several inputs to the investment management process.

The recent financial crisis has created significant issues for insurers in managing their ALM. Regulatory change towards Solvency II and a fair value/realistic approach also leads to greater transparency as regards risk management and the stability of the economic balance sheet. In this context it seems timely to discuss what advances can be made in the area of Dynamic Asset Liability Management.

We aim to show how sophisticated financial models can be used more directly to optimise investment strategy.

2. Optimising Investment Strategy

2.1 Market Value of Liabilities

To give a theoretical background, our starting point is to assume that the objective of a life insurer is to maximise value[1]. It may be observed that there are often subsidiary objectives such as managing risk or providing adequate investment returns to policyholders, but we believe that it is generally possible to integrate all of these within a single objective.

The economic balance sheet is a familiar concept used in modern insurance management techniques and is embedded in recent regulatory developments towards realistic solvency measures such as Solvency II. In the economic balance sheet the (tangible) economic value of the company or "Available Economic Capital" is the market value of assets less the market value of liabilities.

We assume that the market value of assets is simple to calculate if we hold assets for which there is a liquid market.

There has been on-going debate about the exact definition of the market value of liabilities ("MVL"), but we assume a definition in line with that of the Solvency II Framework directive[2], namely:

MVL = Best Estimate of Liabilities + Risk Margin[3]

where

(1) The Best Estimate of Liabilities ("Best Estimate") "*shall correspond to the probability-weighted average of future cash-flows, taking account of the time value of money (expected present value of future cash-flows), using the relevant risk-free interest rate term structure*". We assume that a probability weighted approach implies a stochastic approach to valuing future liability cash-flows, although of course approximations could also be used if appropriate.

(2) The Risk Margin is "*calculated by determining the cost of providing an amount of eligible own funds equal to the Solvency Capital Requirement ("SCR") necessary to support the insurance and reinsurance obligations over the lifetime thereof*" on a cost-of-capital basis.

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[1] For a proprietary company this would be taken to mean shareholder value
[2] Solvency II Framework Directive as adopted by European Parliament 22nd April 2009
[3] Where cash-flows can be replicated reliably using financial instruments this split into best estimate and risk margin is not required

CEIOPS Consultation Paper 42 covers the calculation of the risk margin. Its advice is that the amount of capital held for calculating the risk margin should exclude hedgeable risk, on the basis that, if the assets and liabilities were transferred to another undertaking, that undertaking would immediately de-risk the assets in order to minimise the market risk. As regards market risk only "unavoidable" (non-hedgeable) market risk would therefore be included for the risk margin calculation.

The CRO Forum paper "Market Value of Liabilities for Insurance Firms" July 2008 sets out examples of hedgeable and non-hedgeable risks:

	Hedgeable	Non-hedgeable
Financial	• 10-year USD, EUR or Yen cash flow or interest rate option • 10-year equity option	• 60-year USD, EUR or Yen cash flow or interest rate option • 15-year emerging markets cash flow • 30-year equity option
Non-financial	• Screen- or exchange-traded CAT risks • Actively traded securitised risks	• Most insurance risks, eg mortality, property, casualty • Policyholder behaviour

On the other hand, as regards the calculation of the Best Estimate, CEIOPS Consultation Paper 39 states that *"there are cases where the valuation of discretionary benefits depends intrinsically on the assets held by the firm. The assets assumed in such circumstances should be the assets held by the undertaking at the valuation date. Future changes in the asset allocation should be taken into account if the requirements on management actions are met."*

For profit participation business where policyholder benefits depend either directly or indirectly on the returns on the assets underlying the policies, the Best Estimate can be directly affected by the investment strategy adopted by the company. By investment strategy we mean how future sales and purchases of assets will be made. This is likely to be dependent on a variety of factors such as future interest rate conditions, the level of unrealised gains and losses etc, but also upon the actions of policyholders in different scenarios.

For such business the Risk Margin will also depend on the investment strategy to the extent that it impacts non-hedgeable risks. Non-hedgeable risks which would be impacted by investment strategy could include, in particular, long term cash-flows and dynamic policyholder behaviour.

This can be illustrated by the following example:

- If an investment strategy involving a high proportion of equities is adopted for assets covering a portfolio of participating products with onerous interest rate guarantees then we might expect more economic scenarios in which the investment return is less than the guaranteed minimum interest rate compared with a strategy whereby a matching bond portfolio is held. This will increase the probability weighted value of future liability cash-flows and hence increase the Best Estimate.

- Such a strategy could increase the impact of dynamic lapses, as there are more scenarios where additional assets need to be sold when market values have fallen, which in turn would increase the Risk Margin.

Therefore both the Best Estimate and the Risk Margin can be viewed as functions of the assumed investment strategy. This can be expressed as follows:

BE(IS) = Best estimate assuming investment strategy "IS".
RM(IS) = Risk margin assuming investment strategy "IS".

It could therefore be argued that MVL is also a function of the investment strategy adopted. However this seems to be in contradiction with the principle that there exists a market value of the liabilities. In fact we could argue that if the current investment strategy ($IS_{current}$) was less than optimal then there would be an arbitrage opportunity to purchase the company and improve the investment strategy and create value. Therefore we would argue that implicitly:

$MVL = BE(IS_{optimal}) + RM(IS_{optimal})$

Where $IS_{optimal}$ is the optimal investment strategy, i.e. that which leads to the lowest market value of liabilities.

However there may be reasons why the optimal investment strategy is not expected to be adopted by management such as commercial considerations related to policyholder returns or internal rules constraining investment strategy.

From a regulatory and accounting point of view it would probably be difficult to justify calculating the MVL on a different investment strategy than that which management are expecting to follow.

In fact in determining the technical provisions for Solvency II purposes, the advice in CEIOPS Consultation Paper 32 requires, amongst other things, in respect of future management actions that:

- *the (re)insurance undertaking considers it both possible and realistic that they will carry out such actions in the circumstances being considered*
- *assumed future management actions should be consistent with the (re)insurance undertaking's current principles and practices to run the business unless there is*

sufficient current evidence that the insurer will change its practice and has taken the necessary steps to implement this change

Thus the optimal investment strategy should only be assumed in modelling for external reporting purposes if this strategy is to be followed in practice.

Therefore it is more correct to define

$MVL = BE(IS_{current}) + RM(IS_{current})$[4]

and to define

$MVL_{optimal} = BE(IS_{optimal}) + RM(IS_{optimal})$

where the cost of a sub-optimal investment strategy = $MVL - MVL_{optimal}$

Whether it is followed or not it is clearly very useful for an insurer to be able to determine the optimal investment strategy since this will allow informed decisions about whether it makes sense to continue adopting any other strategy.

We note as an aside that the optimal investment strategy is a related but not identical concept to the Replicating Portfolio, but further discussion of this is beyond the scope of this paper.

2.2 Determination of Optimal Investment Strategy

The discussion above focuses on trying to maximise **Available Economic Capital** by minimising the MVL. In practice companies will be interested in **Required Economic Capital** as well as just Available Economic Capital. The Required Economic Capital will include all market risks, and not just be limited to non-hedgeable market risks as in the case of the Risk Margin calculation.

We refer to taking account of these different factors in assessing different investment strategies as **"optimising the Economic Balance Sheet"**. In order to find an investment strategy to optimise the Economic Balance Sheet the key measures we would look at could be:

(a) The Best Estimate of Liabilities

For the purpose of our examples below we will consider the closely related measure of the mean of the Present Value of Future Profits (PVFP) over a set of

[4] Noting that the Risk Margin is only impacted by actual investment strategy to the extent that this impacts non-hedgeable risks

stochastic economic scenarios[5]. We will refer to this value as **"Stochastic PVFP"** in this paper.

(b) Variability

Solvency II requires that the Solvency Capital Requirement should be determined as the *"Economic Capital to be held by insurance and reinsurance undertakings in order to ensure that ruin occurs no more often than once in every 200 cases or, alternatively, that those undertakings will still be in a position, with a probability of at least 99.5%, to meet their obligations to policyholders and beneficiaries over the forthcoming 12 months"*.

For the purpose of our examples below we will consider:

> Stochastic PVFP *minus* 0.5^{th} percentile lowest value of PVFP over the stochastic scenarios

as an indicator of economic capital in respect of financial and related risks (e.g. policyholder behaviour). To the extent that these risks are not hedgeable this will also impact the Risk Margin.

Since we are considering investment strategies, other non market risks are only of second order importance for this purpose (i.e. the variability due to most of the non-financial risks should not be dependent on the investment strategy).

We will refer to this value as **"VAR"** in this paper

We would need to decide how to weight these two values in order to determine a single measure which we wish to minimise or maximise. There is no unique solution to this and different companies will have different criteria depending on the importance they attach to the available and required economic capital (this could depend, for example, on their level of capitalisation).

We assume that some weighted measure of the following type can be determined to represent the measure to be considered by a given company:

Measure = X * Stochastic PVFP - Y * VAR

where X and Y are weightings.

By running our cash-flow model with a variety of investment strategies (IS_x) we can determine the investment strategy which leads to the highest value of this weighted measure.

[5] This forms a component of Market Consistent Embedded Value (MCEV). Another way of approaching this would be to maximise MCEV.

2.3 Investment Strategies to Consider

Since a wide variety of different parameters can be set to vary in the investment strategy, finding the optimal strategy is likely to be an iterative process. Factors which could vary in the investment strategy include:

- types of non-derivative assets to purchases (e.g. government and corporate bonds, equities etc)
- use of derivative assets
- activeness of investment strategy (e.g. whether to invest or disinvest only when there is spare cash available or when cash is required to pay policyholder benefits)
- relationship between the duration of assets and liabilities

Investment strategies could be dynamic to varying degrees. For example the investment strategy could be more or less active depending on the size and sign of unrealised losses and gains, interest rate levels etc. In reality we would expect the investment strategy to vary according to such factors and if our model is able to reflect the expected reality of dynamic management behaviour it will produce more meaningful results. The types of dynamic interactions which may be modelled are described in section 3.2 below.

We can conveniently distinguish between types of investment strategies (e.g. "invest in a mixture of government bonds and equities") and parameters (e.g. the 20% in the strategy "invest 20% in equities and the remainder in government bonds"). Of course these are very simple examples and real dynamic investment strategies may be much more articulated than this.

The process of determining the optimal investment strategy will then involve running our model starting with a range of investment strategies which we consider to be representative of those which we may realistically wish to consider, and then running the model iteratively varying the parameters of these strategies in order to determine the combination of strategy and parameters which gives the optimal result.

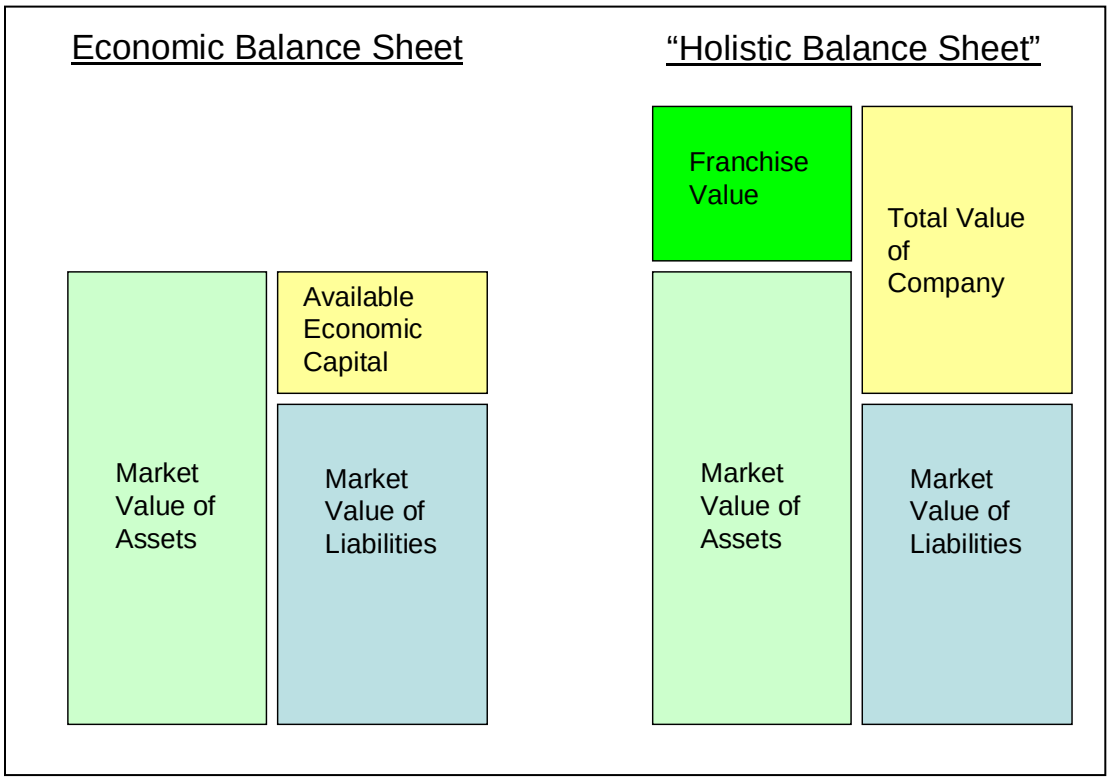
2.4 The Holistic Balance Sheet

In the discussion above we have assumed that our aim is to optimise the value of the economic balance sheet by maximising a measure of the type set out above.

However in practice an insurer may well have other objectives in mind. For example, it may wish to achieve rates of return for policyholders relative to market interest rates or competitors' returns which are judged to be sufficient to generate certain new business volumes and hence new business values. Another criterion could be to have a sufficiently aggressive investment strategy to attract distributors who want to recommend companies selling products which they believe can offer attractive returns to their clients.

Such objectives may be defined in a variety of different ways, but they will tend to be linked to protecting or maximising the franchise value of the insurer.

We can think about this in terms of what we have termed the **"Holistic Balance Sheet"** which adds intangible elements such as the franchise value to the tangible assets and liabilities of the economic balance sheet. This is illustrated below.



If this dimension is ignored in our model we will tend to optimise investment strategies based on the situation of a company closed to new business. Commonly closed companies will tend to follow prudent investment strategies and these are the strategies which will normally prove to be optimal if we are only trying to optimise the economic balance sheet and ignoring aspects impacting franchise value.

Indeed, CEIOPS Consultation Paper 32 relating to the calculation of technical provisions advises that:

- *For a given scenario, the assumed management actions should reflect an appropriate degree of competitiveness of the (re)insurance undertaking. The degree of competitiveness should be consistent with corporate planning.*

3. Modelling

In this section we discuss in more detail the issues that need to be considered when setting up a model to carry out calculations to determine optimal investment strategies for profit participation business.

3.1 Basic Features of Model

There is a well established methodology for building asset-liability cash-flow models. Typically the model projects asset and liability cash-flows based on data files of the assets and liabilities at the valuation date. The liability cash-flows depend upon various projection assumptions (such as lapse rates) and asset values and returns depend upon a file of economic scenarios containing details of interest rates, equity returns etc for each scenario.

The asset and liability cash-flows interact in that:

- Based on the investment returns achieved on the assets, profit sharing is added to the policies which affects the liability cash-flows
- The liability and asset cash-flows combine together to give net cash-flows in each time period, which determine whether sales or purchases of assets need to occur

The model also contains rules for sales and purchases of assets (investment strategy).

3.2 Dynamic Interactions in the Model

It is important that the model we are using realistically reflects all material interactions between assets and liabilities as well as the impact of the actions of policyholders and the management of the insurer which could impact operational ALM. This becomes especially important under extreme market conditions which drive economic capital calculations.

Such interactions should be considered under all scenarios being modelled.

We must consider the impact and interaction of the following:

(1) Market and other external conditions
(2) Actions by policyholders
(3) Actions by management

Certain movements in (1) can cause certain actions by management and/or policyholders, but the actions of management and policyholders can also be dependent on one another.

(1) Market and other external conditions

A model will consider a range of different economic and other external assumptions. This could typically be via a set of stochastic scenarios, or by individual shock scenarios, possibly combined with a set of stochastic scenarios under each shock.

Such assumptions could reflect the level and shape of interest rates, market values and returns on equities and other real assets, credit spreads, defaults and downgrades of corporate bonds, inflation etc

It is important that our set of economic scenarios is internally consistent. For instance in a scenario in which equities fall in value we might also expect to see increased corporate bond defaults and downgrades.

It is also possible that other assumptions could be correlated to economic conditions, such as the level of expenses.

(2) Actions by policyholders

Dynamic policyholder behaviour arises particularly when an option to a policyholder becomes more or less valuable depending upon economic or other external conditions. For instance, a surrender value capital guarantee can become more valuable at a time of depressed market values of assets.

Other policyholder options can include:

- Take-up of guaranteed annuity options (GAOs)
- Extending the original term of the policy on guaranteed terms
- Take-up of various benefits or fund switches under Variable Annuity products

The assumptions we make about how policyholders may react will depend upon the extent to which we believe they will behave in an economically rational way.

Clearly the actions of policyholders will have an impact on financial results and on the investment strategy.

Another policyholder action which needs to be considered is the level of future new business written. This will have an impact on financial results and the ALM position of, say, a segregated fund where new business is pooled together with existing policies and shares in investment returns. One approach could be to model dynamic volumes of future new business which depend on policyholder returns projected on policies. A simpler approach is to give a weighting to policyholder returns when considering the optimum investment strategy, as described in section 2 above.

Generally we consider policyholder behaviour to be unhedgeable in the sense that it is not possible to find hedges against uncertainties in the level of policyholder behaviour.

(3) Actions by management

Management will have discretion in certain areas, which may include:

(a) Investment strategies. This would include deciding which assets to buy and sell in different scenarios and could depend on, for example, cash-flow requirements, the requirement to cover policyholder guarantees and the level of unrealised gains/losses in the fund

(b) Discretionary bonus rates or profit sharing. In some countries bonus rates are prescribed by formulae, but a degree of control could be available, for instance by controlling the rate of realisation of gains and losses

(c) Application of a Market Value Adjustment (MVA) where the insurer has the right to adjust the policy value paid on early surrender when the market value of assets has fallen

(d) Control of the level, type and destination of new business written.

For instance, writing new business in a particular segregated (pooled) fund could improve or worsen the ALM position of that fund, so management could decide to write new business in an existing fund, or open a new fund depending on conditions. Writing new business in a fund could, for instance, avoid having to sell assets to pay claims as new premiums bring a positive cash-flow to the fund.

(e) Ability to default on certain policyholder obligations – eg indexed products with 3rd party guarantees, if policy conditions allow this.

Management will have certain restrictions in terms of its actions. For instance, some countries may have limits on the amounts which can be held in certain asset types.

It can be seen that the various actions in (2) and (3) can interact with one another.

For instance, management could react in a certain way in order to mitigate the impact of mass surrenders by policyholders when assets are depressed, perhaps by selling short assets.

However, policyholders could also be expected to react in a certain way to particular management actions, for example:

- A reduction in discretionary bonus rates could result in increased surrenders or a reduction in future new business volumes

- Mass surrenders at a time of increased market interest rates could cause management to sell shorter assets in order to avoid having to realise losses on

longer assets. This, however, could worsen investment returns (for instance compared with a new fund), as the possibility of reinvesting at increased yields has been reduced. This may in turn reduce policyholder returns and drive further surrenders, with a further requirement to sell assets. In this way a "negative feedback loop" can occur.

- Adding new business to an existing pooled fund at a time of reduced market interest rates could dilute policyholder returns for existing policyholders, causing higher surrenders.

Management actions could also have an impact on market and other factors:

- Impact on market prices of certain assets (including derivatives) resulting from significant sales or purchases

- Impact on expense levels due to the level of new business affecting the degree to which economies of scale are achieved

When we are considering sales and purchases of assets we also need to consider:

- Liquidity issues. It may not be possible to buy or sell a particular quantity of a particular class of asset without causing a significant movement in the market price

- Dealing costs. These should be taken into account at a reasonable level based on the amount of trades being made.

Market Consistent Embedded Value (MCEV) Principles© [6] require that, for example:

"Where management discretion exists, has passed through an appropriate approval process and would be applied in ways that impact the time value of financial options and guarantees, the impact of such management discretion may be anticipated in the allowance for financial options and guarantees but should allow for market and policyholders' reaction to such action".

CEIOPS in Consultation Paper 32 in respect of future management actions for the calculation of technical provisions for Solvency II advises that:

"the insurer should also estimate the time taken to implement actions, for any costs associated with these actions and for any changes to policyholder behaviour following these management actions. The cash-flows included the technical provisions should reflect this accordingly"

[6] The European Insurance CFO Forum Market Consistent Embedded Value Principles. Copyright © Stichting CFO Forum Foundation 2008

Considering optimum management actions can also be complicated by accounting, for instance in Italy where unrealised losses need to be taken through the profit and loss account on some classifications of asset, but are not considered when determining investment returns for the purpose of calculating policyholder profit sharing.

4. Example Model

4.1 Basic Features of the Model

We now discuss an example model which has been built to illustrate the application of these techniques.

Our example model is based on Italian profit participation business. There are two simple products, both whole of life, one with single and one with recurrent premiums, and both contain an interest guarantee of 3% pa. The benefit payable on surrender or death is equal to the mathematical reserves.

The assets backing the policies are invested in a single "segregated fund", and additional policyholder benefits ("profit sharing") can be added so that the annual return to policyholders is:

MAX (MIN (R * 80%, R – 1%) , 3%)

where R is the % investment return on the segregated fund during the 12 months up to the end of November each year, with profit sharing being added in December. R is based on investment income (coupons, dividends and interest), realised gains/losses on investments and amortisation of bonds.

We assume we are modelling from 31.3.09 with economic conditions at that time. The asset mix assumed at this date is shown in the following table (these are intended to be "realistic", but the exact numbers do not have a particular significance):

CASH	4%
FIXED RATE GOVERNMENT BONDS	62%
FLOATING RATE GOVERNMENT BONDS	31%
EQUITIES / FUNDS	3%
TOTAL	**100%**

Lapse rates of 10% pa and premium suspension rate of 4% pa are assumed. Mortality is 60% of the SIM/F 92 Italian table.

A dynamic policyholder behaviour rule is considered as follows:

If market rate[7] >= policyholder return + 1% then:
Lapse rate = base lapse rate * [1 + 50 * (market rate – (policyholder return+1%))]

If market rate <= policyholder return – 1% then:
Lapse rate = base lapse rate * [1 + 25 * (market rate – (policyholder return-1%))]

[7] Market rate is the 5 year market interest rate

4.2 Investment strategies

We assume that the "base case" is the current investment strategy of the hypothetical company. This is a passive strategy in which assets are only bought/sold when there is a net positive/negative cash-flow (on a monthly basis):

Asset purchases:
48% fixed rate government bonds, term 5 years
48% floating rate government bonds, term 5 years
4% equities

Asset sales:
"Circulating bonds" [8] are sold first, starting with those with shortest duration
When all these are sold, "immobilised bonds" [9] are sold, starting with those with shortest duration

The above is subject to the following maximum proportions in each category:

80% fixed rate government bonds
40% floating rate government bonds
5% equities

Cash is maintained at between 1 and 3% of total assets

Investment strategies – alternatives

A number of alternative investment strategies were also considered:

"Buy short" strategy

This is the same as the "base" strategy except that bonds purchased have a term of 1 year rather than 5 years.

"Buy long" strategy

This is the same as the "base" strategy except that bonds purchased have a term of 10 years rather than 5 years.

"Duration matching purchase" strategy

This is the same as the "base" strategy except that, when asset purchases are made, they are selected from a list of possible fixed interest bonds and equities in order to try to

[8] i.e. those which the company would consider for sale before maturity
[9] i.e. those which the company intends to hold to maturity

bring the total duration of the assets in line with the liabilities. Note that the asset purchases are only made when required by the conditions described for the base case above.

"Buy floors" strategy

This is the same as the "base" strategy except that floors are bought as follows:

- Nominal value EUR 2.5bn
- Maturity 5 years
- Strike rate 2.5%
- Index interest rate 5 year government bond yields

The premium for the floor is EUR 23.8m spread over 5 years.

"Buy caps" strategy

This is the same as the "base" strategy except that caps are bought as follows:

- Nominal value EUR 2.5bn
- Maturity 5 years
- Strike rate 4.5%
- Index interest rate 5 year government bond yields

The premium for the floor is EUR 19.6m spread over 5 years.

"Buy swaps" strategy

This is the same as the "base" strategy except that a swap is entered into as follows:

- Nominal value equivalent to nominal value of fixed rate bonds (1.6bn)
- 10 year term
- Company pays 3.37% (the 10 year swap rate) and receives 6 month government yields

"10% Equities" strategy

This is the same as the "base" strategy except that:

- Equities are not sold, unless the total proportion of equities exceeds 10%
- When assets are purchased, 20% are in equities, until equities reach 10% of total assets

Clearly in practice other investment strategies could be conceived of, including more sophisticated dynamic strategies, but to keep this example manageable we have limited ourselves to the strategies above.

5. Model Results

We now consider the results of the model for the investment strategies described above.

5.1 Optimisation of Economic Balance Sheet

We will focus initially on the following indicators used to optimise the economic balance sheet:

- Average of Present Value of Future Profits ("PVFP") over all stochastic scenarios ("Stochastic PVFP" as defined in section 2.2 above)
- Stochastic PVFP *minus* 0.5th percentile lowest PVFP over the stochastic scenarios, as an indicator of economic capital requirements ("VAR" as defined in section 2.2 above)

The table below shows the results for these indicators and for a combined measure which we are using as an indicator to optimise the economic balance sheet, as described in section 2.2 above.

The combined measure is defined as:

Stochastic PVFP – 20% * VAR

The combined measure reflects the weight we choose to give to the importance of the "best estimate" and the "variability". This weighting could vary from company to company and will depend on the importance given to maximizing the available economic capital, minimizing the required economic capital and other considerations.

The combined measure used here is merely an example.

EUR m	Stochastic PVFP	VAR	Combined measure
Base	95,612	253,299	44,952
Buy short - 1 year	94,474	262,953	41,883
Buy long	85,979	258,148	34,349
Duration matching purchase	76,860	248,704	27,119
Buy floors	81,334	270,888	27,156
Buy caps	102,637	136,207	75,396
Buy swaps	69,918	320,511	5,816
10% equities	87,037	271,379	32,761

Current ('Base') Strategy

To understand the results is it is useful to understand first some of the risks which exist with the hypothetical company's current investment strategy (the 'base' strategy). In the base strategy the key risks are:

(a) a rise in interest rates in the early years, giving rise to losses on assets when they need to be sold. As assets are essentially accounted for at book value, unrealized gains and losses are not brought into the investment return. This effect is exacerbated by the presence of dynamic lapses, which increases lapses when 5 year interest rates rise, hence increasing the necessity for asset sales. The policyholders are assumed to be rational in this sense since they are exploiting the fact that the surrender values in these periods represent better value than can be expected to be earned through holding the policy.

(b) low interest rates in the later years, giving rise to lower reinvestment rates and hence a shortfall with respect to the guarantees in some years.

However, the impact of (a) is the more significant factor in the base strategy and it is this factor which drives the VAR result.

"Optimal Strategy"

'Buy caps' provides the best result in terms of both Stochastic PVFP and VAR and hence the combined measure.

The cap provides benefits in those scenarios which produce low fund yields in the early years (scenarios where interest rates rise), and since these fund yields tend to be below 4%, excluding the effect of the cap, the company takes much of the benefit of the resulting increase in fund yields. (The company takes all the benefit for any increases up to 4% since, for yields up to 4%, the policyholder always receives the guaranteed rate of 3%.)

In particular the VAR result is greatly reduced, as the scenario driving the VAR is one where interest rates have risen sharply in the early years.

Therefore the benefit from the cap outweighs its cost.

We refer to this strategy as "optimal" since it is the best strategy of those considered so far, but of course we cannot exclude other better strategies which may exist or the fact that this result could be improved by further iterations.

Alternative "non-optimal" strategies

We give below some comments on the other alternative investment strategies which gave results which were not as good as the "optimal" one.

'Buy short – 1 year' reduces the Stochastic PVFP and increases the VAR slightly compared with the base strategy due to the increase in risk (b) arising from this strategy outweighing the reduction in risk (a). The combined measure is therefore reduced compared with the base strategy.

The two strategies which involve buying assets longer than in the base strategy **'Buy long'** and **'Duration matching purchase'** both produce a combined measure which is worse than base, as the impact of risk (a) is increased and outweighs the reduction in risk (b). Note, however, that 'duration matching purchase' produces a small decrease in VAR compared with base.

The increased variability associated with the **"10% equities"** strategy reduces Stochastic PVFP and increases VAR, resulting in a lower combined measure than base.

Of the strategies not involving derivative instruments tested so far, the 'base' strategy is actually the best.

We now discuss the other alternative strategies involving derivatives:

'Buy floors' reduces the Stochastic PVFP and increases VAR and hence produces a combined measure lower than base.

Falls in interest rates in the early years will tend to increase investment returns and hence fund yields. In such scenarios the floor will provide a benefit, but much of this benefit will pass through to the policyholder as additional profit sharing if the resulting fund yield exceeds 4%. For fund yields up to 4% the payment to the policyholder is 3% (the guaranteed rate), since the margin to the company is a maximum of 1% and 20% of the fund yield. For each 1% increase in fund yield above 4% the policyholder will receive an additional profit sharing of 0.8%.

Therefore the benefit derived from the floor is outweighed by its cost.

In respect of the VAR for 'buy floors', the 0.5^{th} percentile worst PVFP (ie Stochastic PVFP – VAR) is greater than base by an amount which is approximately equal to the cost of the floor. This is because the scenario driving VAR is one of rising interest rates for which the floor gives no benefit; the cost of the floor will directly impact investment returns, most of which come through as losses to the company as the fund yields are below 4%.

'Buy swaps' produces the lowest Stochastic PVFP and highest VAR and hence the worst combined measure.

The initial yield curve slopes sharply upwards up to around 10 years meaning that, on a deterministic basis, short term yields are very low in the early years and higher in the latter part of the first 10 years. This means that, on the deterministic basis, the swap

produces large negative costs in the first 4 years, and positive benefits in years 5-10. This pushes fund yields below 4% in the first 4 years, and above 4% in years 5-10, meaning that most of the cost is borne by the company, but much of the positive impact of the floor is passed to policyholders. This means that the deterministic PVFP is much lower than the other strategies, and this effect also impacts the stochastic PVFP and VAR.

Note in particular that the VAR for 'buy swaps' is driven by low interest rates throughout, as low interest rates in the early years produce a negative impact of the swap, and low interest in the later years produce low reinvestment rates. This is in contrast to the other strategies where high interest rates in the early years, in some cases combined with low interest rate in later years, are driving VAR.

It can be informative to look at results with and without the impact of dynamic policyholder behaviour (DPB) or with alternative DPB rules. DPB has a significant negative impact for all strategies both in respect of Stochastic PVFP and VAR. Increases in 5 year yields drive higher dynamic lapses and hence higher asset sales, and this is just at the time when asset values will have fallen. However, the assumptions and parameters underlying DPB models are often highly subjective, particularly with respect to extreme scenarios, and hence the sensitivity of results to different rules should be considered.

5.2 Further Steps

5.2.1 Further iterations

Informed by the first set of results, we can prepare a modified set of results by varying the parameters of some of the more promising investment strategies to see if we can further improve the best results. This process should be iterated until we are reasonably confident that we have identified the optimal strategy.

For instance we could consider variations on the "buy short" and "buy caps" strategies as follows:

"Buy short – 3 year" strategy

This is the same as the "buy short" strategy except that bonds purchased have a term of 3 years rather than 1 year.

"Buy caps – reduced amount" strategy

This is the same as the "buy caps" strategy, except that the nominal value of caps purchased is only EUR 1.6bn rather than EUR 2.5bn

The results are shown below:

EUR m	Stochastic PVFP	VAR	Combined measure
Base	95,612	253,299	44,952
Buy short - 3 year	97,728	254,743	46,779
Buy caps - reduced amount	101,682	138,677	73,947

'Buy short – 3 year' increases stochastic PVFP and reduces VAR compared with base, as the increase in risk (b) is outweighed by the reduction in risk (a). This gives a combined measure better than base and represents an improvement over 'buy short – 1 year'.

'Buy caps – reduced amount' produces results which are not as good as 'buy caps'

After this iteration we still have "buy caps" as the best strategy, but "buy short – 3 year" is now the best strategy tested which doesn't involve derivatives. Further iterations of the model could be made to determine the optimal "shortness" of this strategy.

5.2.2 Internal Discussions

The implications of the calculations should be discussed with relevant internal parties such as members of the investment committee, the CFO etc. For example it should be ascertained:

- whether there are any practical obstacles to implementing the identified optimal strategy
- whether there are any risks which may not have been modelled in implementing the strategy (e.g. unmodelled counterparty risks, liquidity considerations, regulatory difficulties)
- whether there are any other strategies which should be considered

5.2.3 Other Indicators – The Holistic Balance Sheet

We can also use our model to calculate further measures such as the expected return for policyholders, as discussed in section 2.4 above. This can then be combined with an appropriate weighting to give a revised combined measure which we then try to optimise in the same way as above.

The weighting would have to reflect the expected impact of this return in generating additional new business sales and hence adding additional franchise value to the company.

6. Conclusions

The results shown in the previous section are for illustrative purposes only. The model is simplified and is intended to illustrate the techniques rather than give results which will hold generally. In other situations other investment strategies may be optimal.

The exercise in commenting on the results is to show, at a high level, the type of analytical process which is necessary in order to interpret the results properly. Stochastic ALM models are inherently complex and it is not advisable to trust results without being able to gain confidence that they have been properly understood. Counter-intuitive results should not be accepted until they can be understood.

A useful technique can be to study particular scenarios (e.g. high and low interest rate scenarios) to understand what underlies the results in these cases. It is also informative to look at the spread of stochastic results.

The authors believe that the techniques described in this paper can add significant value to the asset liability management of a company. For example they can:

(1) help to understand better the extent to which various aspects of the economic balance sheet (e.g. required economic capital, market value of liabilities etc) are influenced by the investment strategy

(2) help to inform investment management decisions to identify the best investment strategies for the company

(3) improve Enterprise Risk Management

It will be important to have robust ALM tools which can produce the full range of calculations based on cash-flow projections for purposes such as MCEV, risk management calculations and dynamic ALM as well as Solvency II. Since the cash-flow model is likely to be an important component of the model for companies which decide to adopt the internal model approach for Solvency II, it is worth pointing out that this type of application of the model is entirely consistent with the principles of Solvency II. For instance the Solvency II Framework Directive as adopted by European Parliament 22nd April 2009 states that:

"Insurance and reinsurance undertakings shall demonstrate that the internal model is widely used in and plays an important role in … their system of governance"

This method is a practical use of the internal model in the management of a life insurance entity.

7. References

The European Insurance CFO Forum Market Consistent Embedded Value Principles. Copyright © Stichting CFO Forum Foundation 2008 June 2008

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