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# Optimisation of limit systems for investment risks in accordance with Solvency II

*Abstract*

In order to satisfy the requirements of Solvency II (e.g. Framework directive on the EU Solvency II Project on Safety Measures and its implementation according to § 64a German Insurance Law) -- insurance companies should implement an overall risk limit system. The starting point for developing this system is the entity's risk strategy and risk bearing capital approach based on economic principles. For life insurance companies, the dominant risk category is investment risk. Therefore the limit system should focus on such risks.

In practice there are multiple interactions between the core life insurance business and the asset side. Because of these interactions, a limit system for investment risks cannot be separated from life business risks. There is a particular need to integrate the entity's asset liability management approach into the risk limit system.

The regulatory requirements call for consistent integration of a top-down view with a bottom-up risk management perspective in the investment department. In creating an adequate system, the first step is to categorise the individual types of risk and the corresponding risk management approaches. It is most important to get clear definitions of the bottom-up and the top-down views in the context of life insurance investment risks, and to integrate these into the entity's overall solvency control regime.

The current financial crisis has revealed problems of valuation and an enormous and unprecedented increase in volatility in the capital markets. It is clear that an ongoing and effective analysis of these market developments and their impact on asset allocation and portfolio optimisation is necessary. The crisis also implies the need to think in detail about how to manage model risk implications.

In this paper we propose an integrated view of these issues as the basis for optimal design of the company's risk limit system.

# 1 Introduction

## 1.1 Characteristics of investment limits

Limits are an important instrument in controlling the investment process. The investment planning of the company should be the basis for the definition of limits, but the limit setting process should also take account of external (legal and regulatory) restrictions. Limits can focus on different parameters, especially exposures, expected incomes and risk measures. They can be defined absolutely (in the entity's currency) or in relation to other underlying indicators, e.g. the contribution of an investment in relation to the overall volume of the investment portfolio.

Because of the high importance of investment risks, most insurance companies have already implemented various types of limits in their investment systems, for example:

- Limits for transactions (e.g. the maximum volume of transactions in a defined period),

- Limits based on the objectives of the strategic and tactical asset allocation process, for example to control the timing of cash-flows and exposure in individual asset classes. These limits may consider valuation based on market values as well the accounting view (with focus on local GAAP or IFRS),

- Limits to control additional specific characteristics of the investment portfolio, for example limits for the maximum volume of individual investments and limits for counterparty risks,

- Limits based on risk model and stress test calculations (as restrictions on the investment strategy).

## 1.2 Implementation of limits

Several activities are necessary to implement limits. Initially, the company has to identify those indicators for which limits should be defined. These indicators should be chosen based on the objectives of the company's investment planning, management and control. From the risk management perspective, the indicators should focus on the entity's risk structure and especially on the identified key investment risks.

The next step is the definition of specific quantitative limits for these indicators. Besides these limits, additional alert levels are used to indicate the risk of a possible future breach of the limits. These limits (consisting of indicators and quantification of the current risk situation) should be integrated into the IT systems and processes of the company.

Finally the company should think about the activities necessary to apply the limits and about responsibilities in this process. One of the most important issues is the need for a separation of responsibilities, especially between asset management and the investment control function. The company has to implement two main processes:

- Activities and responsibilities for evaluating and reporting on the current risk situation in relation to the defined limits,

- Escalation procedures if limits have been broken (including processes that start detailed analysis of the current situation). The company also might have to reallocate the existing limits.

Therefore the implementation of limits requires the solution of various design issues and additional process issues (e.g. in relation to activities to apply the limits).

## 1.3 Requirements of Solvency II regarding solvency control

The key element of the solvency analysis according to Solvency II is the adequacy of the entity's own funds compared to its overall risk situation.[1]

The Solvency II framework contains specific requirements for the definition and classification of own funds into three different tiers of the so-called "Available Solvency Margin (ASM)".[2] This classification depends on defined criteria regarding the characteristics of the entity's own funds.

On the other side of the equation, risk modelling techniques are used to measure the aggregate risk situation of the company. The result of these analyses is the "Solvency Capital Requirement (SCR)" which means the capital needed to prevent the entity's default and to absorb significant losses within the next twelve months.[3]

For both ASM and SCR, the Solvency II framework requires a calculation based on economic valuation principles and assumptions.[4]

The ASM is calculated as the difference between the market value of assets and the market value of liabilities. Specific guidelines exist in the Solvency II framework for the evaluation of assets and liabilities.[5]

Risk modelling techniques using up-to-date assumptions on key risk drivers and adequate risk scenarios are the basis for estimating SCR. The risk capital requirement is calculated bottom-up in separate modules for individual risk drivers and then aggregated to the overall SCR by using assumptions on risk correlations. The evaluation of risk capital depends in particular on:

- The time horizon for solvency risk analyses and

- The risk measure and calibration used.

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[1] Framework directive on the EU Solvency II Project on Safety Measures / Solvency II Framework Directive, General Guidance (17)
[2] Solvency II Framework Directive, Art 17a
[3] Solvency II Framework Directive, Art 100
[4] Solvency II Framework Directive, General Guidance (27)
[5] Solvency II Framework Directive, Chapter VI

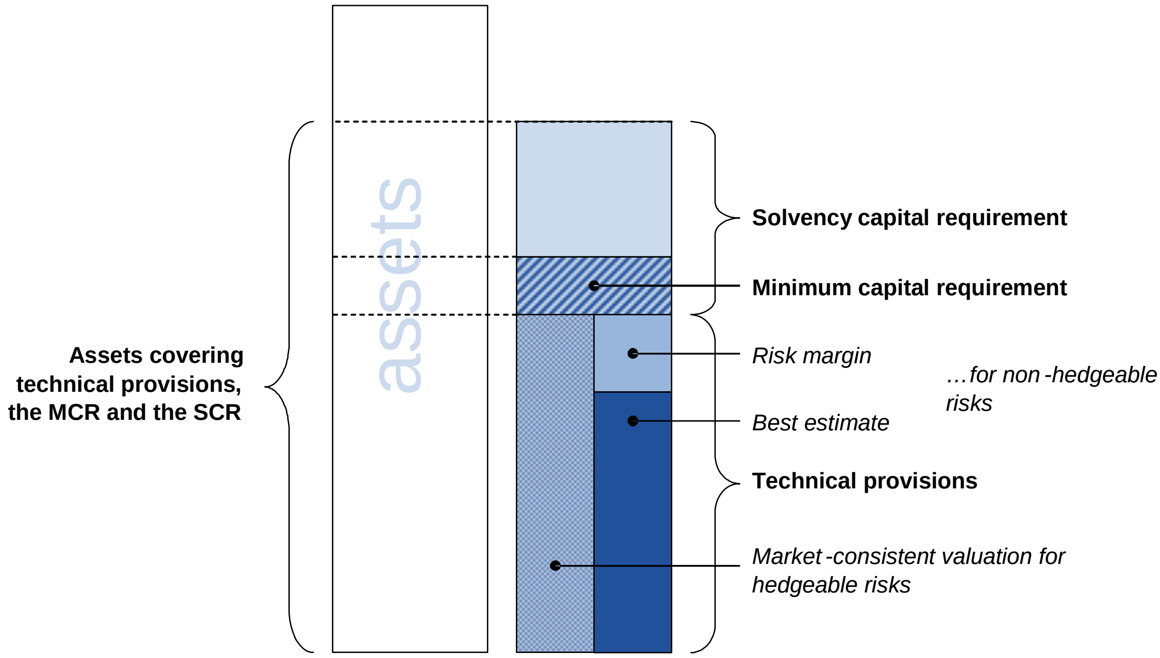
According to the Solvency II Framework Directive, the time horizon to apply to the entity's default risk is 12 months. In the Quantitative Impact Studies (QIS 4), Solvency II currently requires the use of Value at Risk (VaR) as the risk measure, because this measurement is consistent with the approach used in the banking sector. (From the theoretical point of view other more appropriate measures like Tail Value at Risk or Expected Shortfall might exist. These are currently used in the Swiss Solvency Test and have been proposed in former QIS studies.)

At the moment a confidence level of 99.5% is demanded for VaR measurement.[6]

The horizon of one year for solvency measurement does not mean that future risk scenarios should only focus on short time events. Long term risk scenarios (e.g. regarding changes in the interest rate curve up to the expiry of life insurance policies) are necessary to measure the volatility of market values of insurance liabilities and underlying investments.

Solvency analyses also should take into account existing guarantees (e.g. interest rate guarantees incorporated into insurance contracts) and the impact of deferred taxes and policyholders' participation in future profits of the company. These requirements increase the complexity of the calculation of SCR and of the valuation of the entity's assets and liabilities.

The following illustration summarizes the Solvency II control concept:[7]




[6] Under the simplifying assumption of a normal risk structure the VaR rises with the square root of the underlying time horizon, which means that doubling the horizon from one to two years would lead to an approximate increase in VaR by a factor of 1.4.

[7] *Source: CEIOPS-CP-09/06, p.9, *MARKT/2515/04*

## 1.4 Requirements of Solvency II regarding the implementation of risk limits

As yet, the Solvency II Framework Directive does not include specific requirements regarding the implementation of a risk limit system. But according to Article 43 of the directive, "Insurance and reinsurance undertakings shall have in place an effective risk management system comprising strategies, processes and reporting procedures necessary to identify, measure, monitor, manage and report, on a continuous basis the risks, on an individual and aggregated level, to which they are or could be exposed, and their interdependencies. That risk management system shall be well integrated into the organisational structure and in the decision making processes of the insurance or reinsurance undertaking. (…)".

At the moment, the requirements for aggregate solvency control should be the basis for the structure of an entity-wide limit system. This system should be located at the interface between the control of overall solvency (as described in the previous section) and the risk management activities in the departments of the company.

This limit system focusing on the company's solvency should have a transparent structure reflecting the interactions between the major risk categories and all important risk drivers for the overall financial situation. It has to be an "early-warning system", used to control critical risk situations.

When implementing the limit system according to Solvency II, the first step is to review the existing limits in the entity and to integrate them into the limit system. The following questions arise from the overall risk perspective:

- Which additional risk indicators have to be covered in the limit system?
- Is it possible to find indicators which directly influence the overall solvency situation?
- For individual risk indicators already used in the company: Is it necessary to adjust the defined limits and alert levels from the overall solvency perspective, e.g. because of stronger requirements resulting from the solvency perspective than from other restrictions relevant to the company?

## 1.5 Optimization of the risk limit system for investment risks

We recommend integrating the risk limit system into the existing investment management and control processes. The following objectives are of high importance for the optimal structure of the limit system:

- Minimize the number of indicators in the overall limit system:

  The company should ensure transparency and include only those limits which are relevant to the control of its aggregate solvency position. Further limits in relation to investment risk (which appear less important from the overall solvency perspective) could be integrated into investment guidelines and into the investment control process of the company.

- Optimize the limit system in response to the dynamics of investment markets:

  The limit system should include "early-warning" indicators which focus on the volatility of the investment market and on inefficiencies in financial markets. Market frictions and market shocks are important risk drivers which impact on risk capital and own funds.

- Increase efficiency and consistency of the limit system:

  The limit system should take into account the company's investment strategy. Internal models and methods used for asset allocation and asset liability management should be integrated into the limit system.

We will discuss these objectives in our paper. We aim to provide an overview of approaches currently under discussion and challenges of implementation.

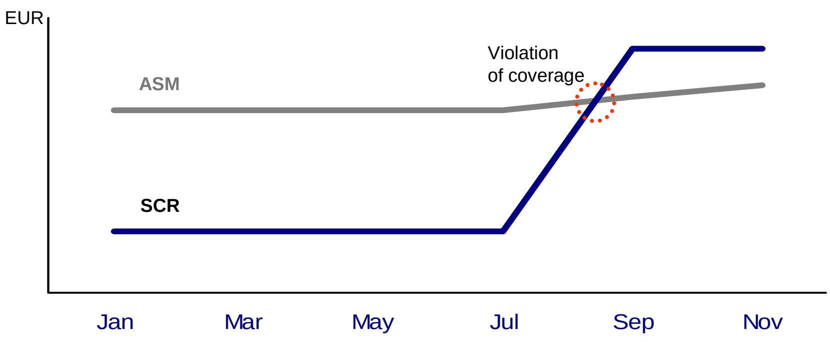
# 2 Structuring the limit system

## 2.1 Focus on the aggregate risk situation

As illustrated in section 1.4, insurance companies have to define a method of integrating overall solvency control into the limit system.

The first option is to refer directly to the SCR as a key risk indicator and to limit the maximum volume of risk capital (e.g. in Mio €), which should not be exceeded. Based on the modularized evaluation of SCR, the company could implement a predefined top-down structure for deriving limits reflecting the acceptable level of risk capital required for individual risk categories (e.g. risk capital requirement for market risks). The following two examples illustrate the advantages and problems of this approach:

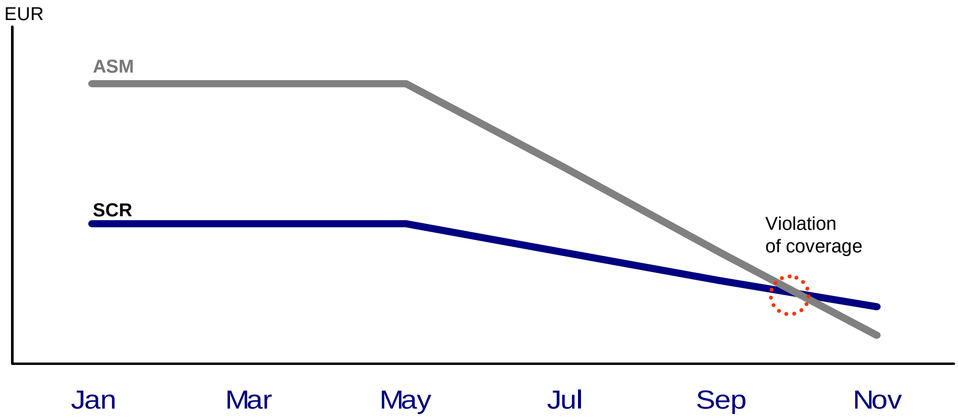
*Example 1: Change of investment strategy to a riskier strategy*




This example illustrates how a change of investment strategy increases expected future returns and therefore strengthens the company's own funds and ASM. However, the changed asset allocation (and therefore the increased investment risk) immediately impacts on the company's risk capital requirement. This might cause a violation of the company's coverage as shown in the illustration; it therefore appears sufficient to focus directly on SCR as a key risk indicator and to define a maximum limit (in Mio EUR) for solvency purposes.

But the next example illustrates that this is not always an appropriate approach:

*Example 2: Decrease of the market value of equity investments*




In this example we assume that SCR is driven by the risk capital requirement for market risks only. The SCR depends on the market value of investments and will be reduced if the market value of investments decreases.

An assumed market value shock (starting in May) reduces the company's SCR and decreases the ASM at the same time. The result is that the company experiences a violation of coverage, because the ASM decreases at a faster rate than SCR. In this situation the coverage of SCR by ASM would be an adequate key risk indicator for solvency purposes, but the absolute volume of SCR would not.

*Conclusions from these two examples:*

From the solvency perspective it is necessary to control simultaneously changes in SCR and ASM in the risk limit process. For some critical risk scenarios (especially an adverse investment market development) it is necessary to focus on coverage of SCR by ASM as the key risk indicator in the limit system.

There are two options for controlling coverage. First, to target the absolute volume of excess capital (in Mio EUR) as

$$Coverage = \quad ASM - SCR$$

Or alternatively, to focus on the coverage ratio (ASM in relation to SCR):

$$Coverage \quad Ratio \quad (CR) = \frac{ASM}{SCR}$$

The Coverage Ratio appears more appropriate in practice because it allows direct comparison of the solvency position of two companies of different sizes.

Starting with the Coverage Ratio as a key solvency indicator at the entity level, the company should implement a process to break this limit down to individual major risk categories and to control them directly at the risk level. Then the company can define the "Coverage Ratio on Risk Level" as a key risk indicator in the limit system, e.g. for investment risks ("IR"):

$$CR_{IR} = \frac{ASM_{IR}}{SCR_{IR}}$$

While $SCR_{IR}$ can normally be estimated based on a well-defined bottom-up risk model approach, the problem is to define $ASM_{IR}$ (own funds available for the coverage of investment risks) in an appropriate manner for the purpose of solvency control.[8]

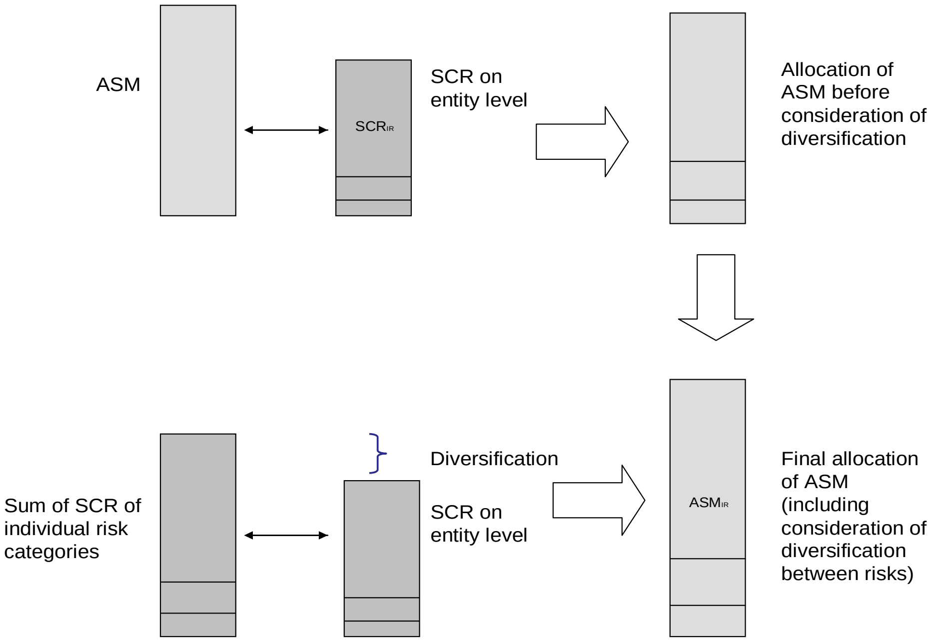
Specific approaches for the allocation of own funds using assumptions on the entity's risk structure and correlations between risk categories are known.[9] But the problem of allocation of ASM for solvency control is more complex because the impact of diversification in the SCR has also to be taken into account.[10]

The following illustration describes the necessary steps in the allocation process:

[8] From a legal and accounting point of view the allocation of ASM to individual risk categories has no validity, because own funds are defined on the entity level only (as the difference between the value of assets and the value of liabilities).

[9] We refer to Tillmann, M. (2006): *„Allokation von Risikokapital im Versicherungsgeschäft"*, Risikomanager ERM 05/2006.

[10] Note in addition that in a stochastic risk model approach the impact of diversification on risk capital evaluation is not stable but depends on the evolution of the risk situation.




This illustration shows that the process of allocating ASM depends on the company's risk structure (exposure to risks and correlations between risks: both can change during the year) and the allocation methods used by management. Thus, for limit setting it is important for the company to

- Ensure consistency between the assumptions used for bottom-up aggregation of risk capital (SCR) and top-down allocation of own funds (ASM), because both indicators should be considered simultaneously,
- Understand the impact of changes in the risk structure on the allocation of ASM,
- Control and update the assumptions underlying the allocation process on a regular basis because the investment risk situation, in particular, can change very quickly.

## 2.2 Dynamics in the risk limit process

Insurance companies have to ensure continuous compliance with solvency requirements. Therefore they should set up risk processes which integrate solvency control into the overall business planning and control processes. The following questions arise from the perspective of integrated planning:

- Is the company's business strategy (including investment strategy, asset allocation and planning of interest income) adequate from an overall risk perspective?

- Which solvency regime (including the development of ASM, SCR and coverage) does management require for the planning period? Is this expectation consistent with the company's overall business plan?

- How fast could the current solvency position change in the future? What impact would follow an unexpected adverse change in capital markets on the company's business plan and – simultaneously – on the development of the solvency position?

From the planning perspective, changes in SCR, ASM and coverage are uncertain stochastic processes. These stochastic processes depend on

- The current position,

- The uncertain future development of key risk drivers,

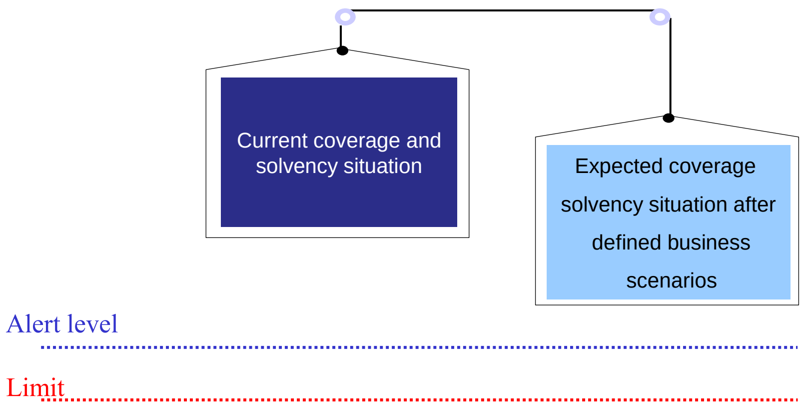
- Appropriate current and future assumptions for valuation (-> ASM) and for modelling (-> SCR), and on

- Management rules (e.g. dividend and investment policy) resulting from the company's business strategy.

For life insurance companies these uncertainties are mostly driven by changes in investment risk. The company's risk management processes should be able to understand these risks and be able to estimate the impact on the entity's solvency planning.

The next illustration describes the challenges of implementing the process of risk planning and limit setting:

## How fast can the solvency situation change?




- Setting limits (and alert levels) based on the current situation and the entity's business (investment) planning
- Critical issues: Capital market shocks, valuation and model risks

## 2.3 Structuring the process of limit setting

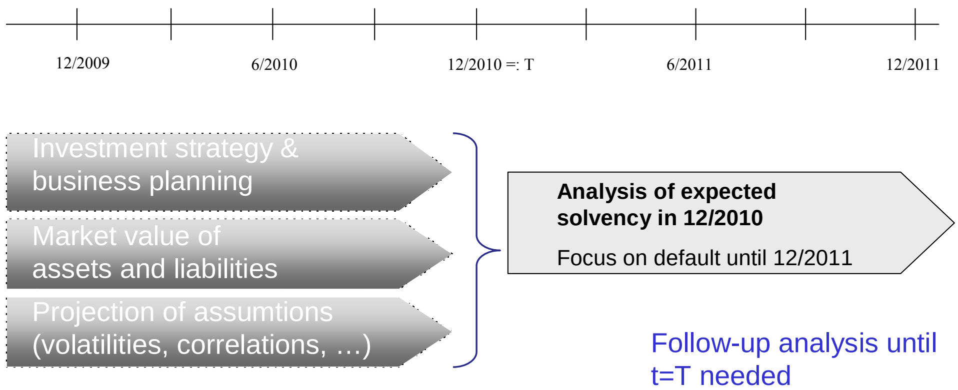
### 2.3.1 Strategic limit setting

The next flow-chart describes the requirements for projections of ASM and SCR based on the company's strategic planning process. Here, simultaneous stochastic projections of ASM and SCR are based on:

- The company's business planning and investment strategy,
- Analyses of market dynamics and volatilities and
- Additional assumptions (e.g. management rules for planned dividends)

These are necessary to evaluate the expected coverage ratio for future planning (at date T in the following illustration). In addition, further scenario analyses have to be performed to estimate the impact of uncertainties underlying key risk drivers (especially market risk volatilities).

**Strategic planning and projection of ASM and SCR**




$$E(CR_T \mid A_{t \le T}) = E\left( \frac{ASM_T}{SCR_T} \mid A_{t \le T} \right)$$

Stochastic dependencies between ASM and SCR processes

Management should initiate these projections of the expected development of CR in combination with additional sensitivity analyses (as described in the illustration in section 2.2) for setting strategic solvency limits.

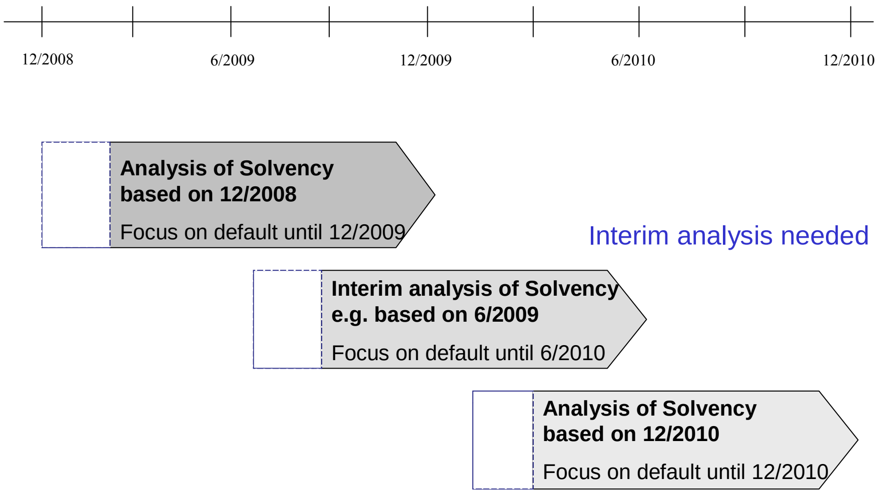
### 2.3.2 Roll forward process for limit setting and controlling

In addition to strategic solvency planning, a roll forward analysis for interim control of the solvency situation can be used to ensure continuous solvency control. In this way risk managers can set up a process to adjust solvency limits based on the latest evaluation of the solvency situation.

The company can reallocate the interim limits for major risk categories based on

- The current ASM,
- The defined coverage ratio limit at the entity level (maximum accepted coverage ratio, defined by management in its strategic planning process) and
- The process of allocating ASM based on the current risk structure.

## Roll-forward process for limit setting and controlling




Continuous update of SCR and ASM evaluation (or approximation based on analysis of sensitivity) is necessary because of the uncertainties resulting from the company's investment portfolio. Thus, besides the illustrated half-year analysis further interim analysis should be implemented, e.g. based on information in the first and third quarters.

# 3 Risk indicators for investment risks

## 3.1 Relevant criteria

In addition to limits based on analysis of the solvency position, the company should implement further limits to control major investment risks. The basis for identifying these limits is:

- Analysis of the company’s investment risks including correlations between these risks,
- Integration of risk management requirements resulting from the overall asset liability management strategy (e.g. the planned profits for policyholders) and
- Understanding of model risks and the limitations of solvency models used (e.g. limitations regarding extreme values, market frictions and model path-dependencies).

We will discuss these three criteria in the next sections. Some basic requirements should be taken into account:

- Transparency of the underlying risk indicators to ensure reliability of the process of limit setting and of control of these indicators,
- The ability to promptly measure these indicators timely enough to use them as an early-warning function in the process of solvency control,
- Materiality and consistency of the implemented limits to ensure efficiency and transparency of the whole limit system.

## 3.2 Analysis of the company’s investment risk structure

The limit system should cover all important risk drivers for the company’s investment risk position.

The first step is to structure the investment risk environment and to develop clear definitions for risk categories and sub-categories. The company can refer to an internal risk model or to the model approaches used in the Solvency II QIS studies. The major risk categories are market, credit, concentration and liquidity risks. The exposure in these risk categories depends on the company’s investment strategy and the current structure of the investment portfolio. But the company also has to take into consideration problems with determining adequate market values of investments in relation to the estimation of risk capital and own funds.

Typical examples for key risk indicators for investment risks included in the limit system are:

- Market value exposure and volatility of investments in asset classes,
- Ratings (with focus on changes of ratings) and the spread structure of the portfolio,

- Indicators focussing on the risk of asset liability mismatch and on requirements resulting from the insurance business of the company (e.g. from interest rate guarantees).

The relevance of these indicators depends on the individual investment structure of the company as is illustrated by the following table:

## Examples for risk indicators (--- Illustration---)

| | Indicator | | | | | | |
|---|---|---|---|---|---|---|---|
| | Market Value (Exposure of asset classes)<br><br>Market and credit risk | Volatility of investment portfolios<br><br>Market risk | Rating structure<br><br>Credit risk | Credit Spread<br><br>Market and credit risk | Duration<br><br>ALM risk | Convexity<br><br>ALM risk | Net interest income<br><br>ALM risk (comparison to the guaranteed interest rate in the insurance contracts) |
| Overall | X | X | n/a | n/a | n/a | n/a | X |
| Equity | X | X | n/a | n/a | n/a | n/a | X |
| Fixed Income | X | X | X | X | X | X | X |
| Real Estate | X | X | n/a | n/a | n/a | n/a | X |
| Other | X | X | n/a | n/a | n/a | n/a | n/a |

We will discuss further details with respect to credit risks based on a simplified model approach in section 3.6.

## 3.3 Model risks and limitations of risk model approaches

Risk models play a key role in the Solvency II framework. But the current financial crisis has also shown their limitations in the event of market turbulence. One significant problem in these cases is insufficient (and partially inadequate) historical experience, especially in terms of volatility information on benchmarks. In addition, the economic environment in the last decades changed significantly, e.g. by the introduction of new and complex investment products (like credit default swaps and ABS/MBS). Therefore the interactions between the insurance company's key investment risk drivers have also changed, and behave completely differently from what is usually expected in normal periods. All these circumstances lead to limited reliability of volatility based VaR approaches for measuring risk capital requirements, not to mention the basic problems of estimating reliable market values of investments (using mark-to-market and mark-to model approaches) necessary for the analysis of ASM.

Therefore the limit system needs to include both risk indicators and limits[11]

- For handling "everyday" investment risks and

- For controlling the impact of unusual extreme events in the financial markets.

As a result, VaR based risk model approaches using historical market experience and volatilities are only appropriate for controlling ordinary risk situations. Insurance companies may need to implement further stress test approaches to estimate the impact of unusual shocks on the overall stability of the company.

The effectiveness of these stress test approaches depends on the adequacy of the scenarios analyzed for the company's individual investment structure (also taking into account hedging and portfolio insurance strategies in place). They are always subjective, and depend critically on judgments of the external regulator (for stress tests prescribed by legal or regulatory authorities) and the company's investment and risk control departments.

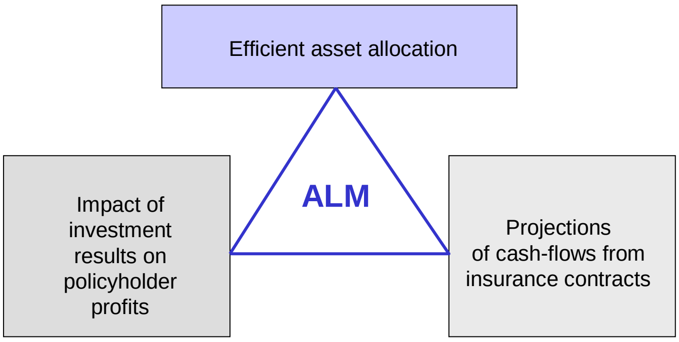
## 3.4 Integration of asset liability management (ALM) approaches

The ALM process has to control mismatches between the structure of assets and liabilities which could have significant impacts on the company's financial stability. The risk of such mismatches impacts the entity's risk situation. Therefore the company's asset liabilty management approach should be integrated into the risk limit system to supplement the focus of solvency control.

The graphic below describes the complex interactions in the ALM process with relevance to the planning and control processes of the company[12]:

[11] Kevin Dowd (1998): "*Beyond Value at Risk*", John Wiley and Sons.
[12] Different kind of deterministic and stochastic ALM approaches are described in the literature.

## Interactions in the ALM processes




From the perspective of risk management and solvency control, ALM approaches are important because:

- ALM approaches reflect both economic valuation principles and the existing legal and regulatory requirements which impact on future cash-flows.
- ALM calculations can be implemented as deterministic or stochastic (dynamic) approaches to analyze the risk of an asset liability mismatch over multiple periods (e.g. mid-term projections based on plan balances or long-term stochastic analyses up to the expiry of the insurance policies in the portfolio).
- ALM projections focus on the evolution of economic surplus and Net Asset Value. These projections can also be used to estimate and project assets and liabilities in the process of limit planning and solvency control (see also section 2.3).

ALM approaches can be integrated directly into the limit system, e.g. using them for planning and controlling the solvency position (ASM, SCR and coverage ratio). But they can also be integrated by defining risk indicators and limits for the specific risk of an asset liability mismatch. For example, the risk management department can define limits on interest rate sensitivities for Net Asset Value or on interest rate shifts on the company's assets and liabilities.

In addition to ASM/SCR solvency analysis based on the one year horizon focusing on the company's default, long-term stochastic ALM analyses are of high importance for the company's risk management. The results of these analyses depend especially on

- The approaches used to model options and guarantees included in insurance contracts,

- The appropriateness of assumed management rules and business plans and assumptions on policy holder behavior.

- The current market environment and assumptions about its future development.

When using modelling techniques for long term stochastic ALM analyses the company also has to be aware of the existence of model risks which may impact significantly the reliablity of results (see also section 3.3).

Two important indicators resulting from the ALM process are the duration gap and – in more sophisticated approaches – the convexity gap between the expected cash-flows of assets and insurance liabilities[13]. In this context estimating the cash-flows of insurance liabilities should consider:

- Expected guaranteed payments to policyholders and additional profits already proposed to policyholders and

- Behaveior of policyholders, options and guarantees included into the contracts, e.g. expected payments in the event of lapse.

For cash-flow projections of insurance liabilities, different kinds of methods can be used. Currently the so-called replicating portfolio approach is discussed as an alternative to direct stochastic simulations[14].

Additional profits proposed to policyholders and expected payments for lapses reduce the duration of expected cash-flows of insurance liabilities. These two components of the cash-flow projection are again influenced by the company's investment risk position and overall changes in financial markets (as described in the illustration at the beginning of this section). So in the context of managing ALM risks, it appears reasonable to include the volume of lapses and the volume of profits proposed to policyholders in the overall solvency system.

## 3.5 Consideration of the company's asset allocation process

For long-term strategic planning, insurance companies usually define benchmark portfolios assuming the existence of an efficient frontier based on the theory of portfolio optimization according to Markowitz. The key assumptions in this investment planning process (forecasts for expected investment returns, risk free interest rates, volatilities and correlations between the asset classes) should be consistent with the assumptions used for projections of ASM and SCR in the process of strategic solvency control and limit setting (see also section 2.3.1).

Mid-term and short-term asset allocation takes into account especially the latest information and expectations of economic cycles and changes of financial market prices and risks. In this process it is the task of the company's investment control to consider also changes in relation to:

[13] Convexity is a measure for the curvature or second derivative how the price of a bond varies with interest rate.
[14] We refer to Thorsten Wagner's presentation „*Replicating portfolios*" in the 2009 AFIR/Life colloquium.

- Key risk drivers (like inflation, the risk-free interest rate and also the average guarantee interest rate resulting from the life insurance portfolio) and

- Benchmark portfolios in the investment market, including deviations between the company's investment structure and the defined benchmarks.

These overall key risk drivers, including the development of benchmark indicators defined for asset allocation, can be integrated into the limit system. The company should use scenario analyses in the process of limit setting (as described in sections 2.2 and 2.3) to analyse the sensitivity of the company's solvency to changes in these indicators.

## 3.6 Scenario analysis and dynamic limit setting in the investment management process

In this section we propose a model for limit setting which responds to dynamic changes in selected risk factors and their corresponding impact on solvency capital.

In relation to solvency issues, an important question to be answered is how the dynamics of the available solvency margin (ASM) and the solvency capital requirement (SCR) are linked to each other, as we have seen in section 2.2 and 2.3.

We propose a simplified dynamic limit-setting model taking into account the different effects of risk factor evolution on ASM and SCR which can be seen as a first step in developing such a model. We will solely focus on credit risk, as the importance of counterparty risk has significantly risen since the credit crunch. We assume that an insurance company holds a portfolio of several bonds. The required risk capital for each bond depends on the creditworthiness of its issuer. Consequently the risk arising from a bond investment concerns downgrade as well as default risks. In the case of a downgrade, there is an adverse effect on ASM and SCR. This is due to a drop in market value - leading to declining ASM (everything else equal), and an increase of SCR on the other hand. Given these movements, the entity faces the risk of running at least into technical insolvency in case of ASM falling below SCR.[15] Therefore the question arises how a limit setting process taking these effects into account can be structured. To analyse this problem we make the following assumptions:

i. The insurance company solely invests in bonds with ratings from Aaa to C and maturity in T. The rating is the only attribute to distinguish the bonds, i.e. there is no distinction between industry sector, country etc. Consequently only rating migrations will affect the bond price. This also implies that interest rates whose changes naturally affect bond prices are treated as constant[16].

ii. In $t_0$ SCR and the associated capital allocation (portfolio structure) are given.

iii. The risk capital to be held for each bond is solely determined by its rating class. We assume the following capital requirements:

[15] Similar to the illustrations in section 2.1
[16] This is done since in this paper we assume the technical provisions as constant. Otherwise we would asymmetrically account for interest rate effects as they only influence the asset side.

*Table 1*

| RATING CLASS | REQUIRED RISK CAPITAL (% OF NOTIONAL AMMOUNT) |
|---|---|
| Aaa | 5 |
| Aa | 10 |
| A | 20 |
| Baa | 50 |
| Ba | 80 |
| B | 90 |
| Caa-C | 100 |

iv. In case of a downgrade the fair value of the respective bond declines in a predefined manner, also reflecting possible liquidty deductions. The fair value is assumed to change as follows as a result of rating migration:

*Table 2*

| FROM/ TO | Aaa | Aa | A | Baa | Ba | B | Caa | Ca-C | Default |
|---|---|---|---|---|---|---|---|---|---|
| Aaa | 0.0% | -4.0% | -8.8% | -14.3% | -20.3% | -26.7% | -52.3% | -61.9% | -70.0% |
| Aa | 4.2% | 0.0% | -5.0% | -10.7% | -17.0% | -23.6% | -50.3% | -60.3% | -68.7% |
| A | 9.6% | 5.3% | 0.0% | -6.0% | -12.6% | -19.6% | -47.7% | -58.2% | -67.1% |
| Baa | 16.6% | 12.0% | 6.4% | 0.0% | -7.0% | -14.4% | -44.4% | -55.5% | -65.0% |
| Ba | 25.4% | 20.4% | 14.4% | 7.5% | 0.0% | -8.0% | -40.2% | -52.2% | -62.3% |
| B | 36.3% | 30.9% | 24.3% | 16.9% | 8.7% | 0.0% | -35.0% | -48.0% | -59,1% |
| Caa | 109.7 % | 101.4 % | 91.3% | 79.8% | 67.2% | 53.8% | 0.0% | -20.0% | -37.0% |
| Ca-C | 162.2 % | 151.7 % | 139.1 % | 124.8% | 109.0% | 92.3% | 25.0% | 0.0% | -21.3% |
| Default | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |

As we take the interest rate as constant and furthermore consider a portfolio of bonds with identical maturity this matrix implicitly reflects the change in spreads due to rating migrations. Other spread effects e.g. altering spread distances for certain rating classes due to market fluctuations or volatility are subsumed under market risk.

v. The rating migration follows a Markov Process and is characterised by the following transition matrix ∏[17].

| FROM/TO | Aaa | Aa | A | Baa | Ba | B | Caa | Ca-C | Default |
|---|---|---|---|---|---|---|---|---|---|
| Aaa | 88.44 | 10.98 | 0.00 | 0.58 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Aa | 0.13 | 84.28 | 14.14 | 0.40 | 0.26 | 0.00 | 0.14 | 0.14 | 0.53 |
| A | 0.00 | 1.46 | 92.80 | 5.06 | 0.17 | 0.09 | 0.09 | 0.00 | 0.34 |
| Baa | 0.00 | 0.19 | 2.26 | 91.99 | 4.61 | 0.28 | 0.19 | 0.00 | 0.47 |
| Ba | 0.00 | 0.00 | 0.18 | 4.60 | 81.61 | 9.92 | 1.11 | 1.47 | 1.11 |
| B | 0.00 | 0.00 | 0.17 | 0.19 | 2.81 | 78.10 | 15.57 | 1.08 | 2.08 |
| Caa | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 3.51 | 73.79 | 9.46 | 13.24 |
| Ca-C | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 6.25 | 31.26 | 62.50 |
| Default | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 |

vi. Technical provisions are constant

We start in $t_0$ with the calculated SCR and the prevailing portfolio structure given by the vector $\alpha_0 = (\alpha_0^1,...,\alpha_0^9)^T$ with $\alpha_0^i$ representing the portfolio weight of the i-th rating class in $t_0$ e.g. $\alpha_0^1 = 0{,}4$ states that 40% of the portfolio is invested in Aaa rated bonds.

We now consider the evolution of the portfolio as a result of short term rating migrations. As the transition matrix ∏ contains one year transition probabilities we first have to translate these probabilities to shorter time intervals.

To accomplish this we adopt an intensity approach, meaning that there exists an intensity "process" describing rating probabilities in a manner consistent with ∏. This process is described by a generating matrix Λ. The i-j-th element $\Lambda_{ij}(t)$ describes the intensity at time t to

[17] Moody's Investor Services, February 2009. The matrix was slightly modified since rounding errors led to row sums unequal to one. Furthermore we distributed the column "not rated" to the remaining rating classes according to their respective weights and added a default row to obtain a 9x9 matrix. Since the rating matrix in $t_0$ is also used for calculating the risk capital it is assumed to be consistent with the SCR model assumptions.

jump from rating class i to rating class j. If such an intensity generator can be found the connection between the transition probabilities and the intensity process is as follows:[18]

$$\Pi(s;t) = e^{\Lambda(t-s)}, \; s<t$$

Thereby $\Pi(s;t)$ represents a matrix containing transition probabilities from time s to t, e.g. the i-j-th element in $\Pi(s;t)$ describes the probability of a rating migration from rating class i to rating class j within the time intervall [s,t]. By constructing the intensity matrix one can calculate the transition and default probabilities for time steps smaller than one year. The analysed case will be a quarterly view.

To determine the generating matrix we use the results obtained by *Israel et al.*[19] From their results we find that for the given transition matrix an exact generating matrix is not available. This is due to the fact that in $\prod$ we can find states i and j which show a direct migration probability of zero. However there exists a path of subsequent rating migrations which leads from i to j with positive probability.[20] For example the probability for a migration from Aaa to Ba is zero but the probabilities for a downgrade to Aa and subsequent downgrade from Aa to Ba are both greater than zero. We can therefore find an approximate matrix $\Lambda$ for which $\Pi(0;t) = e^{t\Lambda}$ nearly holds. This is also based on the ideas given in Israel et al.

With the generating matrix at hand one can obtain the transition matrix for $\Delta t = 0{,}25$ by calculating $\Pi(0;0{,}25) = e^{\Lambda(0{,}25)}$. Based on this modified transition matrix we can examine the portfolio evolution. To do this we apply the respective probabilities drawn from $\Pi(0;0{,}25)$ to each rating class to decide whether the investments are facing a downgrade or not. The result of this procedure is a new portfolio vector $\alpha_{0,25}$.

Given this new portfolio rating structure in $t_{0,25}$ the change in fair value has to be calculated. To achieve this the fair value effect due to rating migration is taken from table 2 for every asset where $\alpha_0^i - \alpha_{0,25}^i \neq 0$. For example if a Aaa rated bond with a fair value of EUR 100 in $t_0$ happens to receive a downgrade to A the fair value has to be adjusted according to table table 2 to EUR 91.

In the second step one has to recalculate SCR. This is done by increasing the SCR prevailing in $t_0$ as specified in table 1. In our example above the downgrade of two steps to A leads to an SCR increase amounting to 20% of the bond's notional amount. For upgrades the SCR is reduced analogously. As long as the portfolio fair value guarantees an ASM above the SCR the company has no problem in $t_{0,25}$ . The question arises whether the remaining own funds

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[18] The matrix exponential is defined as follows $e^{t\Lambda} = I + t\Lambda + \frac{(t\Lambda)^2}{2!} + \frac{(t\Lambda)^3}{3!} + \ldots$, I represents the identity matrix of the same dimension as $\Lambda$.

[19] Israel et al (2001): "*Finding Generators for Markov Chains via empirical transition matrices, with applications to credit ratings*", Mathematical Finance Vol.10, No.2, 245-265.

[20] This criterion is presented in Israel et al. as Theorem 3.1.

will still suffice to meet the capital requirements under a stress scenario for the remaining period.[21]

To analyse the solvency position we define a worst case stress scenario as follows:

50 % of all bonds no matter which rating class they belong to will face a downgrade within the remaining period e.g. after the first calculation 50% of the portfolio will receive a downgrade. This leads to a hypothetic portfolio structure $\alpha_{0,25}^{stress}$ for which the same calculations as described above are carried out (fair value and SCR calculation). Now there are two possible results:

i) The fair value of assets net of technical provisions is still greater than SCR. In this case the company will survive the worst case scenario and still has enough equity to cover its risks.

ii) The fair value of assets less technical provisions is smaller than SCR. In this case the company needs to take action as it is not able to survive a worst case scenario. The own funds are not sufficient to cover the company's risks.

In the second case the company will try to reduce its risks. As we assumed technical provisions to be constant, the only parameter for risk reduction is the asset structure. Apart from real estate assets are easy to sell and it is a reasonable assumption that the company will reduce its exposure to risky assets. We assume the company will reduce exposure in such a way as to restore the original portfolio structure. This means it would start selling downgraded investments and investing the proceeds in bonds of the rating class the downgraded bonds belonged to before the downgrade. In terms of $\alpha$ this means turning $\alpha_{0,25}$ back to $\alpha_0$.

This results in an unchanged portfolio value in $t_{0,25}$ (with the original portfolio structure) and a reduced capital requirement (based on table 1) which is consistent with the portfolio structure based on the capital allocation relating to SCR in $t_0$.

After altering the portfolio structure we again stress the portfolio against the worst case scenario. If this still leads to case ii) we proceed by selling the riskiest asset class and investing the proceeds in one rating class higher.[22] By proceeding like this we sucessively reduce the SCR until the portfolio value in $t_{0,25}$ meets the capital requirement in order to survive the defined worst case scenario.

Of course the model analysed is a simplification in many ways. First it assumes the company only invests in bonds, which definitely violates the diversification. Secondly it assumes that bond fair values are solely affected by changes in credit rating. Even if credit quality has become more important since the credit crunch, interest rates, liquidity constraints etc. are pricing parameters with considerable impact. In practice, changing interest rates do have a considerable effect on bonds as well as technical provisions. Nevertheless our approach shows that dynamic limit-setting taking capital requirements into consideration can be designed in a

[21] The remaining period is defined as the original period the SCR was calculated for, less the simulation period, e.g. for a one year SCR the first calculation in a quarterly simulation leads to a remaining period of 9 months.
[22] This implicitly takes return aspects into account as investing just one rating class higher leads to higher returns than investing in Aaa bonds.

way which is consistent with the global solvency model used within the company. As the proposed approach solely focuses on one parameter it can be taken as a first order approximation to a sophisticated dynamic limtit setting process.

Therefore this approach should be the starting point in developing dynamic limits. In practice, actual dynamic limit setting has to take further aspects into account e.g. ALM aspects like duration and convexity gap and further relevant parameters such as volatility.[23] But this is not a major drawback, since these aspects can be added to the model.

[23] See also sections 3.2 and 3.4.

# 4 Summary

According to the Solvency II Framework Directive (Art. 43) an effective risk management system comprising strategies, processes and reporting procedures is necessary to identify, measure, monitor, manage and report, on a continuous basis, the risks, on an individual and aggregated level, to which insurers are or could be exposed, and their interdependencies. That risk management system should be integrated into the decision making processes of the insurance or reinsurance undertaking. This is the basis for the implementation and construction of an entity-wide limit system which focuses on solvency control.

In this paper we illustrated how the limit system should control simultaneously the evolution of risk capital requirements and own funds, because each of them is dependent on the other and both are influenced by the volatility of the investment market. We also demonstrated that risk models are of high importance for measuring the overall risk position and for constructing the overall limit system. The current financial market crisis has showed the limitations of these approaches and the need to integrate additional risk control concepts into the limit system, e.g. stress test techniques and continuous control of key investment risk drivers on the overall risk position.

The main challenges for the implementation of the limit system for life insurance companies are the need to integrate the company's ALM approach, adequate consideration of interactions between key risk indicators (especially for investment risks) and an effective approach to the consideration of investment market dynamics.

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