Proposal
to present a paper called

Risk Analysis and Valuation of Life Insurance Contracts: Combining Actuarial and Financial Approaches

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Abstract

Interest rate guarantees are a very common product feature within traditional participating life insurance contracts in many markets. There are two major types of guarantees: One type is a simple point-to-point guarantee where a guaranteed maturity value communicated at outset is not changed during the contract term. Another type is a cliquet-style guarantee. Here, every year a certain rate of return has to be credited to the policyholder's account (which includes previous years' surplus).

There are financial and actuarial approaches to assess financial guarantees within life insurance contracts. The financial approach is concerned with risk-neutral valuation. The actuarial approach focuses on quantifying risk with suitable risk-measures under an objective 'real-world' probability-measure. In the present paper we introduce a methodology (based on an idea by Barbarin and Devolder (2005))[1] that allows for an integration of financial and actuarial approaches for both types of guarantees. In particular, we propose a methodology that allows us to find parameter combinations that minimize the real world risk for a given fair value of the contract.

We use an asset model with stochastic interest rates and a liability model following Kling, Richter and Russ (2007).[2] In particular, we analyze several different surplus participation models

We derive a theoretical result which proves that – as long as there are no arbitrage opportunities (e.g. by providing guaranteed rates above risk free rates) – for any two different contracts that may differ in asset allocation, guaranteed rate, etc., there exist two terminal bonus rates that make the contracts' values coincide. Alternatively, for any arbitrage-free contract, there exists a terminal bonus rate that makes the contract 'fair', i.e. the contract value coincides with the premium paid by the client.

Based on this theoretical result and the fact that the terminal surplus rate influences only the value but not the real world risk of the contract, we analyze the following strategy for product design: First, minimize the contract's real-world risk under some given constraints. Then choose a terminal bonus rate that makes the contract fair. In order to minimize the risk, we implement an optimization algorithm based on evolution strategies which is able to cope with non-linear, non-convex target functions and which is used for the derivation of e.g. asset allocations which yield minimal shortfall probabilities. This allows us to determine e.g. the 'optimal' asset allocation that minimizes the insurer's risk given some guaranteed rate of interest and other parameters. Another example would be the determination of the highest possible interest rate guarantee for a given shortfall probability level.

Considering the point-to-point model, we find that legal constraints e.g. with respect to admissible asset allocations may pose additional pressure on the companies' risk exposure, in particular for rather high guaranteed interest rates. Additionally, by comparing the risk-return profile of different contracts, we identify risk-minimizing strategies as being beneficial for the insurance clients as well.

Regarding cliquet-style guarantees, we find – even assuming the insurer credits only the minimum surplus participation required by German law – that shortfall probabilities generally increase significantly compared to the point-to-point model. Whereas the shortfall risk within the point-to-point model can be controlled by using an appropriate risk-minimizing asset allocation, there always remains a substantial shortfall risk for cliquet guarantees even under an optimal strategy. Furthermore, risk-minimizing strategies tend to improve the risk-return profile from a client's perspective, as well.

Assuming surplus participation according to a management rule that mirrors what we observe in the German market, we find that prevailing surplus participation rules in general create (ceteris paribus) a higher contract value than the required minimum surplus participation although the increase of the insurer's risk is marginal. However – according to our theoretical result stated above – by adjusting terminal surplus rates, a fair contract value can be achieved again.

Finally, we briefly examine the effect of the new German insurance contract law which became effective January 2008. This law forces the insurers to provide additional participation in so-called hidden reserves to the policyholders. We can show that the insurer's risk increases substantially under the new law. Under certain circumstances, this law even forces insurers to provide contracts with a value exceeding the premium. In other words, we are able to show that arbitrage opportunities are generated by this law. Hence, insurance companies might be forced to apply different surplus participation rules in the future in order to eliminate such arbitrage opportunities. Further, risk-minimizing strategies also significantly change under the new law.

Summarizing, our paper provides insight into the key risk factors of German life insurance contracts, i.e. guaranteed interest rates and obligated participation in hidden reserves and – based on the methodology proposed by Barbarin and Devolder (2005) combined with our theoretical result – proposes a pricing methodology that combines actuarial and financial approaches and should therefore be of interest to insurers as well as regulators.

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[1] The Interaction of Guarantees, Surplus Distribution, and Asset Allocation in With Profit Life Insurance Policies. *Insurance: Mathematics and Economics* 40(1): 164-178

[2] Risk measure and fair valuation of an investment guarantee in life insurance. *Insurance : Mathematics and Economics* 37(2): 297-323