

## Longevity: A "simple" stochastic modelling of mortality

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### International Actuarial Association – Life Section Colloquium – Munich 2009

**Scientific Theme:** Stochastic modeling of biometric risks  
**Keywords:** Longevity Risk, Mortality Projection, Annuities, Model Risk  
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### Paper submission

All UK insurers exposed to longevity risk need to perform stress tests for their Individual Capital Assessment (ICA). Some have put in place deterministic models which are arguably too simple; others have developed stochastic models that can be demanding and complex.

Partner Re has developed a simple stochastic model easy to explain and to implement, which could be an alternative to some of the well known models. The model can be applied to company specific best estimates for future mortality rates.

We will present the method and some outcomes on sample in payment annuity portfolios.

We use the following model to describe how longevity behaves around the projected expected values. Assume the best estimate mortality is given by  $q_{x,t}$ . Then we use the following stochastic model for the mortality process:

$$\hat{q}_{x,t} = q_{x,t} \times C_t + \varepsilon_{x,t}$$

where  $x$  represents the age of the insured and  $t$  the period in time.  $q_{x,t}$  is the expected mortality and  $\hat{q}_{x,t}$  the "real" mortality. The stochastic process  $(C_t)$  is described by:

$$\begin{cases} C_t = \exp(X_t) \times C_{t-1} & C_0 = 1 \\ (X_i)_{i \in \mathbb{N}} & iid \rightarrow N(\mu, \sigma) \\ (C, \varepsilon) & independant \end{cases}$$

The aim of this study is to focus on the non diversifiable risk, represented by the  $(C_t)$  distribution.

Based on the England and Wales data we estimated the underlying parameters  $\sigma$  and  $\mu$  using various smoothing methods for particular age and calendar year ranges and tested their validity.

Using sample portfolios and the stochastic process  $(C_t)$ , we can simulate cash flows to determine the distributions of the net present values (NPV) of annuity outgo at outset.

Results on sample portfolios are presented and we examine the impact of limitations on the total annuity outgo in a given period on the distribution of results and hence on capital needs.

We will observe:

- the NPV distribution and the impact of introducing limitations (caps) on total annuity outgo
- some statistical measures of the distribution of NPV's of capped cash flows and discuss the implications for determining the best estimate level of annuity outgo, as well as capital needs.
- the implied percentile of certain deterministic scenarios, e.g. as stress on mortality tables (as Solvency II's QIS4) and/or a fixed percentage per annum in addition to assumed mortality improvements.