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On the Pricing of Inflation-Indexed Caps

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Extended Abstract

The recent evolution on stock and bond markets has shown that neither stocks nor bonds inherit an effective protection against the loss of purchasing power. Declining stock prices and a comparatively low interest rate level were and are not able to offer an effective compensation for the existing inflation rate. One characteristic of inflation is the lasting and overall rise in prices leading to a loss in the purchasing power of the money. Inflation is usually measured by a consumer price index (CPI) which is reflecting the actual price level of a basket of typical consumer goods. In the eurozone this is a harmonized consumer price index (HCPI) determined by EUROSTAT. EUROSTAT's Monetary Union Index of Consumer Prices (MUICP) is the weighted average of the specific nationals HCPIs of the eurozone where the weights rely on the national proportion of the overwhole expenses on consumption. Inflation itself is then defined as the percentage change of the inflation index, the so-called inflation rate. No matter what causes inflation – a rise in prices in foreign countries, rising costs, increasing government debt or a growing demand on consumption goods – central banks play a major role in stabilization policy. Furthermore investors ask for a compensation to cover inflation when investing into bonds - hence there is an obvious connection between the interest rates on the market and expectations on the future inflation rate. One theory reflecting this relation dates back to Fisher (1930) who describes

the relation between market interest rates and inflation by the so-called Fisher Equation

$$r_N(t,T) = r_R(t,T) + \mathbb{E}[i(t,T)] \tag{1}$$

where $r_N(t,T)$ is the today's cumulative nominal interest rate from t up to T on the market, $r_R(t,T)$ is the real cumulative interest rate, the gain in wealth an investor wants for an investment up to maturity T excluding the loss of purchasing power since time t and $\mathbb{E}[i(t,T)]$ is the expected overall inflation rate from t up to time T. We note that in the presence of continuous annual interest rates the Fisher equation rewrites as follows

$$e^{(r_N - r_R)(T-t)} = \mathbb{E}\left[\frac{I(T)}{I(t)}\right] \tag{2}$$

where r_N is the constant continuous nominal interest rate, r_R the constant continuous real interest rate and $I(t)$ the inflation index at time t taking into account that the relation between the inflation index I and the inflation rate i is given by

$$i(t,T) = \frac{I(T) - I(t)}{I(t)}. \tag{3}$$

On the background of the latest issues of inflation-linked government bonds in the eurozone[1] and a growing number of inflation-linked life insurance products the market for inflation-linked derivatives is expected to grow. These derivatives are designed to protect investors against the changes in purchasing power of a certain country or a certain economic region such as the eurozone. In addition to inflation-linked bonds these derivatives cover up the risk of inflation – or even deflation – and can be used to hedge future cash flows against inflation risk.

Inflation-linked derivatives are usually priced using a foreign currency analogy. Nominal values can be transfered into real values in purchasing power by using the value of the quotient of the two relevant inflation indices as an exchange rate. In these frameworks the nominal interest rates refer to the "domestic currency" while the real can be concerned as the "foreign currency" or vice versa. We note that this analogy does not take into account, that today's real rates on the market differ from the actual later realized gain in purchasing power.

Hugston (1998) introduced a Heath-Jarrow Morton (HJM) framework modelling the nominal and real bond prices and achieves a closed-form solution for vanilla options on the actual inflation. Jarrow and Yildirim (2003) also used a HJM framework to derive the price of a plain vanilla call on the inflation index. Jarrow and Yildirim assumed the nominal and real rates both to follow one-factor Gaussian processes along with a lognormal CPI preserving the Fisher equation. A simple model introducing a similar setting to Jarrow and Yildirim (2003) can be found in Korn and Kruse (2004) also preserving the Fisher Equation. Alternative approaches using lognormal forward CPI have been introduced

[1] Germany has issued its first inflation-linked bond with maturity in 2016 in March 2006 and a second one with maturity 2013 in October 2007.

by Kazziha (1999), Belgrade et al. (2004) and Mercurio (2005). These authors assume that the forward CPIs follow driftless Brownian motions under the relevant forward measures. Mercurio and Moreni (2006) extend these lognormal forward CPI models to stochastic volatility. The macroeconomic concept of Fisher is not reflected by those lognormal forward CPI models.

We consider the problem of pricing vanilla options on the actual inflation rate as well as the pricing of inflation caplets. Based on the Black-Scholes type model of Korn and Kruse (2004) we derive the price of a caplet on the future inflation and extend our market model to stochastic volatility as in Heston (1993). Other well known models of stochastic volatility can be found in Hull and White (1987) and Stein and Stein (1991), a good overview is given in Fouque et al. (2000). Since we believe the Heston model to be of significant importance to practioners we chose this framework to introduce our new model for inflation. Critical discussion of Heston's model as in Quesette (2002) seem to have been overcome. Kruse and Nögel (2005) show that it can be satisfactorily fitted to market data and very well reproduces implied Black-Scholes volatilities and how one can avoid critical implementation problems. We note that a detailed description of the implementation procedure can be found in Mikhailov and Nögel (2003).

The main contributions presented are the following:

1. In the simple Black-Scholes framework as in Korn and Kruse (2004) we derive the price of a caplet on the inflation over a future time interval.

2. By extending the Black-Scholes framework for the inflation index to the presence of stochastic volatility we introduce a new model for inflation preserving the Fisher equation.

3. By using Heston's work on stock options we state the price of a plain vanilla call on the actual inflation rate.

4. We derive a closed-form solution to the pricing problem of options on the future inflation rate by using the analogy to the pricing of forward starting options in the presence of stochastic volatility.