A World of Mortality Issues and Insights Seminar
May 23, 2012

Session 12 – Mortality Uncertainty

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Mortality Uncertainty
How to get a distribution around the Best Estimate Mortality

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What are we going to do?

- This presentations contains
  - Definition of mortality risk
  - What are the parts of the distribution around the BE
  - Example
Definition Life Risk

- Life Risk relates to the deviations in timing and amount of cash flows due to incidence or non-incidence of death
  - Deviations relative to the Best Estimate Assumptions (US: Best = Current)
  - Overall mortality can be described by a probability function in which the Expected Value is the Best Estimate.

Definition Life Risk

- This probability function is not just a single distribution
- Following the IAA Book: “A Global Framework for Insurer Solvency Assessment” (also called “the Blue Book”) EVERY risk type should be analysed in 3 parts:
  - Volatility
  - Uncertainty (model + parameter)
  - Extreme events
Distribution around the BE mortality

- Best Estimate Mortality rates can be analysed into two parts:
  - Level
  - Trend
- The distribution is defined by the following sub-risks:
  - Volatility
  - Uncertainty Level
  - Uncertainty Trend
  - Extreme event risk (Calamity)

Question

Stochastic?

OR

Analytic?
Volatility

- Extreme Event (calamity)
  - Uncertainty Level
  - Uncertainty Trend

Volatility

- Risk of random fluctuations
  - frequency
  - important: variation of sum-at-risk across policies
  - also additional volatility because of external causes
    - like cold winters
    - influenza epidemics
    - severe accidents

<Mortality is not a full independent process>
Volatility

- Because of the (small) dependency between the several lifes the Compound Poisson distribution (instead of Compound Binomial) is used.

Volatility

- Via an analytical way this CP distribution can be estimated using the Normal Power Approach:
  \[ P\{\frac{[S - E(S)]}{\sigma_s} \leq s + \frac{1}{6} \gamma(s^2 - 1)\} \approx \Phi(s) = \alpha \%
  \]

- Or with x # standard deviations needed:
  \[ x = s + \frac{\gamma}{6}(s^2 - 1) \]

- The NP approach is based on the Cornish Fisher expansion only using the first 3 (or 4) moments.
Volatility

Standard deviation total claim level \((c(j))\) follows (for Compound Poisson):

- **\(X\) is capital at risk \((=\text{face value} - \text{reserve})\)**

\[
\sigma = \sqrt{\sum_i q_i(x)X_i^2}
\]

- And Skewness:

\[
\gamma = \frac{\sum_i q_i(x)X_i^3}{\sigma^3}
\]

\(\gamma\) can be fine-tuned based on observed volatility in the past
(Maximum likelihood)

Example Normal Power

Portfolio 1: Typical distribution of sum assured, typical age distribution
Portfolio 2: Distribution skewed to high sums assured.
Portfolio 3: Typical distribution, but compared to 1 a rather small portfolio

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Number insured</th>
<th>Max insured / average</th>
<th>Skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>125,970</td>
<td>11.6</td>
<td>0.13</td>
</tr>
<tr>
<td>2</td>
<td>60,777</td>
<td>40.3</td>
<td>0.77</td>
</tr>
<tr>
<td>3</td>
<td>24,570</td>
<td>14.7</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Results (% RP):

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Simulation</th>
<th>Normal Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22.7%</td>
<td>22.8%</td>
</tr>
<tr>
<td>2</td>
<td>69.9%</td>
<td>68.1%</td>
</tr>
<tr>
<td>3</td>
<td>57.2%</td>
<td>57.4%</td>
</tr>
</tbody>
</table>
Volatility
- Extreme Event (calamity)
  - Uncertainty Level
  - Uncertainty Trend

Extreme event risk
- Worldwide the worst thing related to mortality that can happen is a Pandemic.
  - Some events are simple too extreme to model like an impact of a large asteroid.
- Still every insurance company should analyse concentration risk within their portfolio’s. An incident can, in case of high concentrations, have a larger impact than a pandemic.
Extreme Events in mortality

![Graph showing deaths per 1000 age group 15-45 in the Netherlands, source CBS.]

Extreme event risk

- Although often a model for extreme events is connected to some confidence level, like 1 in 200 for Solvency II, in my opinion this is wrong.
- A pandemic is a conditional event.
  - The 1 in 200 connected to the 0.15% is valid in case we are in a WHO phase 1 situation.
  - We never were!
Better to talk about a scenario, like the Spanish Flu: combined with the 1 in 200 VAR of the other risks you should be able to survive a certain scenario.

- Beside this “conditional” nature of a pandemic it is also a fact that nobody really knows the real pandemic distribution.
- This is also the case for other types of extreme events
Question / Discussion

- Now we start calculating the uncertainty.
- It should at the end result in an uncertainty around the BE.

- Should it be a one-year approach or a multi-year approach?
  - Solvency II Standard Model is based on a one year approach...

Multi-year

- Long term guarantees are given
- So liabilities also contain uncertainty in the future (e.g. medical developments)
  - This uncertainty doesn’t appear within one year
  - Random walk is no reason to adjust BE assumptions (it is modelled in volatility)
  - Uncertainty has to deal with the BE
Multi-year

- In the internal model I’m using the uncertainty is based on Multi-year
- Volatility and calamity on one-year.
- Volatility
- Extreme Event (calamity)
  - Uncertainty Level
  - Uncertainty Trend

**Uncertainty Level**

- The best estimate is based on a factor times the population mortality (Basis Best Estimate)
- The factor in formula:

\[
\text{factor} = \frac{\mu_{\text{obs}}}{\mu_{\text{pop}}}
\]
Uncertainty Level

- Because the $\mu_{\text{obs}}$ is the result of a random process (=volatility in the past!) We are not sure that this number represent the real expected claim level.
  - It could have been an observation in the (wrong) tail.
  - To be sure for $\alpha\%$ that the liabilities are sufficient we need to add a capital.

Uncertainty Level

- Recalculate the liabilities based on mortality rates on "$\alpha\%$ level", this is

$$q_{EC}(x; t) = \text{factor}_{EC} \times q_{pop}(x; t)$$

$$\text{factor}_{EC} = \frac{\mu_{\text{obs}} + (-)unc_{\alpha\%}}{\mu_{\text{pop}}}$$
Uncertainty Level

To find the uncertainty (unc) around the Best estimate we use like in Volatility the Compound Poisson distribution

- Poisson instead of binomial because there is some dependency in mortality
- Compound because of the spread of the insured sums
- Like in volatility a correction can be made to the skewness based on historical volatility

Uncertainty Level

- The Compound Poisson distribution can be calculated in a rather simple way using the normal power approach similar to the volatility calculations
- The distribution is translated into a normal distribution using the first three moments
  - -> NP(3)
Uncertainty Level

- The economic capital “level” follows:

\[ EC_{level} = LIAB_{EC} - LIAB_{BE} \]

Liabilities based on \( q_{EC}(x;t) \)

Liabilities based on \( q_{BE}(x;t) \)

Volatility
- Extreme Event (calamity)
  - Uncertainty Level
  - Uncertainty Trend
Uncertainty Trend

- It is impossible to predict a future trend.
- Medical development, environment, new diseases and resistance against a medical cure can change trend (drift).
- Also volatility in the observations will cause uncertainty (random walk).
- Are we using the right model?
- We must try to say something about the uncertainty

Uncertainty Trend

- In Lee Carter the stochastic part is based on
  - The random walk around the drift
  - Mistake in estimation of the drift because of volatility in used data
  - The drift is rather linear (in standard LC model)
  - This means that changes in drift are modeled as random walk
  - Often the Normal Distribution is used.
Uncertainty Trend

- The real uncertainty is the uncertainty in the drift. But exactly this part is hard to model.
- Other than assumed under Lee Carter is the drift not a straight line and for sure not Normal Distributed.
- It is influenced by several factors like medical developments, resistance against medicines, climate change.
- Even extreme events can occur like a cure against important causes of death (cancer).

Uncertainty Trend

Lee Carter model

\[
c = \frac{1}{T} \sum_{t=1}^{T} [k(t) - k(t - 1)] = \frac{k(T) - k(0)}{T}
\]

Standard error \(e(t)\sigma\):

\[
se(e) = \sqrt{\frac{1}{T-1} \sum_{t=1}^{T} [k(t) - k(t - 1) - c]^2}
\]

Error in \(c\):

\[
se(c) = \sqrt{\frac{\sigma^2}{T}} \approx \frac{se(e)}{\sqrt{T}}
\]
Uncertainty Trend

- In my opinion this method is underestimating the risk around trends.
- The random walk part of a trend analysis is better separate modelled under volatility.
  - It contains for example impacts of cold winters, flu epidemics etc.
- Future uncertainty (e.g. medical developments) are not included

Uncertainty Trend

- The uncertainty is not only based on a statistical error but also:
  - Model choice
  - Used data
  - Unexpected developments (extreme events)
    - E.g. a medical cure against an important cause of death.
Uncertainty Trend

- Therefore uncertainly trend should be partly based on information and ideas out of the medical world to get an insight of the extreme events related to longevity.
- Also historical changes in trend can help to get information about model choices and data used.

Uncertainty Trend

- The trend uncertainty is analysed in two part:
  - Statistical Uncertainty in used trend data
  - Uncertainty is long term projection, also includes Expert Judgement on
    - Medical developments
Uncertainty Trend

- For the uncertainty in the long term life expectancy the goal table we use a recent study of the Dutch CBS. In that study also several international conclusions are used
- It is stated that a 95% confidence interval means an increase of plus / minus 5 in the projected life expectancy in 2060.
  - This is based on both statistical and medical information and medical expert ideas
- Using this, by adjusting the mortality rates of the BE long term projection in such a way that the life expectancy is adjusted with 5 years a shocked projection table (both upper and lower) can be created

Playing with a goal table

- The uncertainty can be included in the projection using the following formula:
  \[ q(x; j + t) = q(x; t) \times f'(x; j)^t \times e^{\frac{a(x)(t+1)}{2}} \]

  Original trend

For \( j+t \) is final year of projection the adjusted goal table this formula can be solved
Playing with a goal table

- Solving $\alpha(x) \leq j + t$ is the last year of the projection:

$$
\alpha(x) = \frac{\log q(x; j + t) - \log q(x; t) - t \times \log f(x; j)}{\frac{1}{2} \times t \times (t + 1)}
$$

Uncertainty Trend (goal table)
Uncertainty Trend

- Based on the history it can be analysed how different 10 year trends look like
  - I used here Dutch figures
- This gives an idea how also in the future short term trend can change or what the impact can be for other assumptions
- By setting all those different trends at the beginning of the prognosis some conservatism will be introduced
  - With each trend observed in the past a separate generation table can be calculated
  - I used 12 trends
- See next graph
How to use this?

- An insurance company and pension funds need to know what the impact is on the Best Estimate liabilities of the uncertainty around the trend.
- Because a complex dependency between the development for the several ages it is not advised to calculate liabilities directly with the at a certain confidence level shocked mortality rates.
- Better is to use each of the generation tables (11 for start trend and 1 for the shocked goal table) plus the BE generation table to calculate liabilities.

Uncertainty capital starting trend

With each of the 12 historical 10 year trends a generation table is derived and with each of them liabilities can be calculated:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.09420843</td>
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<tr>
<td>2</td>
<td>13.33246787</td>
</tr>
<tr>
<td>3</td>
<td>13.066988</td>
</tr>
<tr>
<td>4</td>
<td>13.17006177</td>
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<tr>
<td>5</td>
<td>13.33750594</td>
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<tr>
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<td>9</td>
<td>13.48099007</td>
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<tr>
<td>10</td>
<td>13.09073366</td>
</tr>
<tr>
<td>11</td>
<td>13.25972385</td>
</tr>
<tr>
<td>12</td>
<td><strong>14.0581669</strong></td>
</tr>
</tbody>
</table>

Standard deviation: 0.293
Distribution: Student with 11 DoF
97.5% upper conf. level: 2.2*0.293
= 4.9% of the mean
Uncertainty trend

- Assuming 100% correlation between the uncertainty goal table and uncertainty start trend the total capital can be set at the sum.
- The results will be highly dependent on the discount rate!
- Most likely the ratio’s mentioned can be used for other countries with not enough data available to make own calculations.

Standard model Solvency II
Standard Model Solvency II

- For longevity just a simple shock of 20% on the mortality rates
  - Not duration dependent
  - One year time horizon

Comparison S II with internal model
Standard model

IMO:

- Standard model less suitable for risk management
- Perhaps OK for SCR calculations for average portfolio’s