# CORRECTIONS TO THE GREGORIAN CALENDAR

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## 1. INTRODUCTION

1.1 This exercise all started out of a harmless puzzle in the July 1993 issue of the Staple Inn Actuarial Society's publication "The Actuary". The puzzle was suggested by P. R. Watson and required the solver to discover on which day of the week the 13th of the month was most likely to fall on the assumption that the Gregorian calendar continued indefinitely. The cycle of 400 years includes 97 leap years and thus consists of an integral number of weeks; or put another way, the average length of the Gregorian year, 365.2425 days is 7 times 52.1775, giving exactly 20,871 weeks in 400 years. The solution appeared in the August 1993 issue, together with a subsidiary puzzle which gave rise to the mathematical exercise described in section 3.

1.2 This led to the appreciation of the fact that the number of days in the true solar or tropical year to four decimal places is 365.2422, or to eight decimal places 365.24219878, and a new puzzle was set to devise a modification to the Gregorian system to reduce the error of about 3 days in every 10,000 years.

## 2. HISTORY (THE JULIAN AND GREGORIAN CALENDARS)

2.1 The Gregorian calendar is a modification of the Julian calendar, which itself set out to correct the accumulated errors in the Roman calendar. Full details of the changes in that calendar before the time of Julius Caesar are hard to come by, but much information can be obtained from Ovid [(2)] and Sandys [(3)]. Initially the Romans adopted a lunar system (hence months) and were apparently inhibited by a superstition against even numbers, so the months were alternately of 29 and 31 days, and the Calends (1st), the Nones (5th or 7th) and the Ides (13th or 15th) all fell on odd days of the month. Sometimes additional months were inserted (intercalation) but by the time Julius Caesar amended the calendar in 46 B.C. a year of 450 days was required to bring the seasons back to the right place in the year. But the pre-Julian months, as amended by Julius, form the basis of the months we still use; his plan was for months to be alternately of 31 and 30 days (the superstition about even numbers having apparently been forgotten or ignored) and Julius was careful to ensure that the month of Quintilis (which was renamed after the Emperor himself) should be one of the greatest months (31 days). Julius Caesar's astronomers calculated the length of the year as 365.25 days, so February (only) was left with 29 days, with a 30th day in every fourth year, giving the pre-Gregorian system of 365, 365, 365 and 366 days in the year. It was not until 7 B.C. that the month of Sextilis was renamed after Augustus Caesar (although he had become emperor in 43 B.C.), and to make August a 31 day month a further day was taken from February. The counting of the A.D. years from 31st December in the year 1 B.C. was not adopted until the 6th Century A.D.

2.2 The months in which the Nones fell on the 7th and the Ides on the 15th (March, May, July, October) are all now 31 day months, but I have found no reference to the fact that if Julius had alternate 31 and 30 day months right through from March to January, October would only then have been a 30 day month. Unfortunately Ovid [2], who analysed each important day of each month, stopped when he reached the end of June; but the lengths of the months are not really relevant to the current exercise.

2.3 When Pope Gregory XIII realised that the seasons were no longer exactly where they should be, he took his astronomer's advice that the length of the year was really 365.2425 days, so he introduced the system still in force today, of making the century years ordinary (non-leap) years unless they were multiples of 400. The astronomer also advised that there should then (in the 16th century) be an adjustment of 10 days to correct the accumulated error. Why it was 10 days and not 12 (for the A.D. years 100, 200, 300, 500, 600, 700, 900, 1000, 1100, 1300, 1400 and 1500) is not clear. Although Ovid [2] put the summer solstice at June 26th, Frazer's [2] notes say it was June 24th, whereas now it falls on June 21st or 22nd. We shall probably never know whether Pope Gregory (or his astronomer) deliberately altered the seasons from the way they were in the days of Julius Caesar, whether the 10 day adjustment was a mistake, or whether it was an intentional device to ensure that the seventh day A.D. would have fallen on the Christian Sabbath (Sunday).

2.4 At first the Gregorian calendar was adopted only in Roman Catholic countries, but other countries followed throughout the centuries (some not until the current 20th century) with the adjustments for the surplus days increased from Gregory's 10 days according to the century of adoption (11 days for Britain in the 18th century, 13 for Russia in the 20th).

## 3. A MATHEMATICAL EXERCISE

3.1 My first solution to the puzzle described in 1.2 was to convert to non-leap years those which are divisible by 3,200, but to reinstate as leap years those divisible by 80,000, to reconvert to non-leap years those divisible by 800,000, and to re-instate as leap years those divisible by 4 million. This would give an average of 365.242199 days in the year (ie: accurate to 6 decimals). I subsequently realised that if this system of exceptions stopped at 800,000, with no change at 4 million the result would be *more* accurate at 365.24219875 (accurate to 7 decimals), while to obtain accuracy to 8 decimals would require the reinstatement as leap years those divisible by 32 million!

3.2 A.B. Pepper came up with an elegant alternative, correct to 8 decimals. After converting to non-leap years those divisible by 3,200, the reinstatement as leap years would not occur until those divisible by 86,400, with reconversion to non-leap years then divisible by 3,369,600.

3.3 G.P. Mann suggested a sequence with multiples of 2,800 as non-leap years, but multiples of 14,000 as leap years, but multiples of 70,000 as non-leap years, and multiples of 980,000 as negative leap years (ie: 364 days).

3.4 A further suggestion by both Pepper and Mann was to dispense with the 400 year cycle in favour of a 500 year cycle, so that after every 4 leap but every 100 non-leap the sequence continued: every 500 leap but every 5000 double leap (367 days), but every 1 million leap.

3.5 I suggested a modification to this. Consider a watch which gains 1 minute per week (as does mine); when it has gained half a minute I move the large hand back one whole minute (thus keeping it synchronised with the supplementary hand which makes a revolution every minute), and by this means my watch is never more than half a minute out plus or minus. So I felt that the system in 3.4, rather than going out in the same direction for 5,000 years, might have an intermediate stage at 2500 years, thus: every 500 leap, but every 2,500 double leap, but every 5,000 leap, but every million non-leap. (A similar half way modification to the other suggestions would not be appropriate as they would require a retrospective change in the years 1600 or 1400!).

3.6 However, none of these suggestions is really practical, which is why I have labelled this section as a mathematical exercise. With an average year of between 365 and 366 days there is never likely to be acceptance of a system requiring any year to be 364 or 367 days; and that only leaves the systems described in 3.1 and 3.2 which can involve years which are somewhat awkward numbers (albeit multiples of 400). So in a later section I will make a more practical suggestion.

## 4. MODIFICATIONS ALREADY SUGGESTED

4.1 It was at this stage that I acquired a copy of Aveni[1] and I can do no better than quote from the book:-

*Minor reforms have taken place since the time of Pope Gregory. By agreeing to convert A.D. 4000, 8000 and 12000 to common years we have reduced the difference to 1 day in 20,000 years. Finally, at an Eastern Orthodox congress held in Constantinople in 1923, yet another rule was adopted in the parade of legislation engendered by the drive for accuracy. It stated that century years divisible by 900 will be leap years only if the remainder is 200 or 600. The resulting calendar is accurate to 1 day in 44,000 years.*

I had not seen in any book of reference that years which are multiples of 4,000 should be non-leap years (more of this later). And the Eastern Orthodox rule seems to me to be written in mathematical gobbledygook; century years divisible by 900 are not leap years anyway, unless they are also divisible by 3,600, while (in my understanding of the word "remainder") if they are divisible by 900 there is *no* remainder. It has been suggested that the Eastern Orthodox rule may have been mistranslated, and that it intended to change to an ordinary year any fourth century year which, when divided by 900, left a remainder of 200 or 600. But this would mean, in the first ten millennia A.D., reducing to non-leap years the years 2000, 2400, 5600, 6000, 9200 and 9600, which would be too many reductions; and far too many if the intention was to treat 4000 and 8000 A.D. in the same way.

4.2 One of the interested actuaries had already communicated with a representative of the Royal Greenwich Observatory, Cambridge, who at first had seemed encouraging, but later lost his enthusiasm; he said the matter had already been "done to death", civilisation will have changed markedly in the long term, and any adjustments will be made as and when necessary. So far as being "done to death" is concerned, he offered no book of reference to confirm this. Civilisation will no doubt have changed somewhat in 1,000,000 years or even in 100,000 years, but can we be so sure it will have changed by 20000 A.D? And when it changes, are we right to assume there will be no *homo sapiens* (or perhaps some being even more *sapiens*) who will be interested in counting time in solar years? And as for adjustments "as and when necessary", what will happen if the astronomers (or governments) of different nations cannot agree when they are necessary? Surely it is better while a United Nations Organisation is in existence to try to get some agreement to correct the error which is already accumulating?

4.3 I was not to be put off by the R.G.O's lack of enthusiasm, and after reading Aveni [1] I wrote to the same official. In his reply he said that so far as he knew there had been no official adoption of the 4000, 8000, 12000 rule. Also he agreed that the Eastern Orthodox rule, as quoted, was gobbledygook. He commented that any possibility of defining a Universal Calendar receives short shrift from all participants at major conferences. It sounds as though the use of astronomical knowledge is too important to be left to the astronomers!

4.4 I would not dream of advocating a Universal Calendar! I believe both the Jewish and the Moslem calendars deal with leap years by the intercalation of additional months from time to time, with lengths of year varying by considerably more than one day. But I would advocate endeavouring to obtain agreement by governments of all countries which have adopted the Gregorian calendar. Indeed, the fact that one modification has apparently been adopted in the U.S.A. (see Aveni's "royal we") and another by many countries of Eastern Europe and of Asia, indicates how necessary it is for such agreement to be obtained.

## 5. GENERAL CONSIDERATIONS

5.1 The foregoing is merely a preamble to interested readers in the subject! It seems desirable that (weather permitting) the seasons should come round the same time each year. Why the solstices and equinoxes should ever have been fixed by the pre-Julian, Julian and Gregorian calendars in the months of March, June, September and December, and not at the beginning or end of these months - nor even in the middle - I have been unable to discover, but now users of the Gregorian calendar have become used to them they should stay that way.

5.2 The precession of the equinoxes causes variations which, however, "return to base" every 25,800 years but, as these movements are slow and, ultimately, self-correcting, I propose to ignore them.

5.3 The solar year is gradually shortening. In the London "Sunday Telegraph" of January 29th, 1995 it was reported that Dr George Williams, a geologist at the University of Adelaide, had proved that 620 million years ago the length of the solar year was between 393 and 407 days. This represents a reduction, on the average, of between .00009 and .00014 of a day (or between 8 and 12 seconds) every 2,000 years, due to the spin of the earth being slowed by the drag of the ocean tides, the earth's orbit around the sun having remained essentially unchanged . And Whitrow [(4)] states that the solar year decreases by about .00006 days every 1,000 years, having been approximately 365.24232 days when the Julian calendar was introduced in 45 B.C.; this coincides with Dr Williams' conclusion. So, as the number of days in the solar year is decreasing, the tendency will be for more and more leap years to need to be "suppressed" or converted to non leap years, thus *increasing* the existing calendar errors. This seems to make it imperative to provide for the errors of which we are already aware.

5.4 The slowing of the spin of the earth causes another complication. From time to time an additional second (a "leap second") is needed; one such was added to midnight on 31st December, 1987. As the slowing continues, more and more leap seconds will be required. In due course one will be needed every year, and when this occurs it would be desirable for a "new second" to be used, of such length of time that a year of new seconds will be equal to a year of old seconds plus one old second! Aveni [1] states that the variable length of the day is irregular, but the calculations of Whitrow [4] and of Dr Williams seem to indicate that the second needs redefining about once in every 200 years. This is however irrelevant, although related, to the problem under discussion.

## 6. RECOMMENDATION

6.1 Had it not been for the information in 5.3, my recommendation would have been to avoid a complicated system by keeping to simple and easily remembered numbers; this could have been achieved by suppressing the leap year (i.e.: to substitute a non-leap or ordinary year) whenever the year A.D. is a multiple either of 4,000 or of 10,000 (or of both). This would give an average calendar year of 365.2422 days and a remaining error of about one day in 800,000 years.

6.2 However, the continual reduction in the length of the solar year (see 5.3), although apparently minute, has quite a sizeable effect even in what I would regard as the medium term (say up to the year 20000 A.D.). If y is (year - 2,000) ÷ 2,000 then 10y is the number of seconds reduction after the year 2000 (ignoring fractions) and the accumulated reduction (in days) by any year after 2000 is $10{,}000\ y^2 \div 86{,}400$, resulting in the following table:

| Year A.D. | y | Accumulated reduction in days (to nearer one decimal place) |
|---|---|---|
| 4000 | 1 | 0.1 |
| 6000 | 2 | 0.5 |
| 8000 | 3 | 1.0 |
| 10000 | 4 | 1.9 |
| 12000 | 5 | 2.9 |
| 14000 | 6 | 4.2 |
| 16000 | 7 | 5.7 |
| 18000 | 8 | 7.4 |
| 20000 | 9 | 9.4 |

Thus, even if the years 6000, 14000 and 18000 A.D. are converted to non-leap years, (ie: all multiples of 2,000 remaining in 6.1 apart from 2000 itself) there will still be an error of over 6 days by the year 20000 A.D. (This paragraph assumes the rate of reduction in the solar year will remain unchanged; and it is desirable for posterity to know the sixth decimal figure in the rate of .00006 per 1,000 years; the figure of 9.4 in the above table may correctly be any number of integral days from 8 to 11 depending upon the sixth decimal figure).

6.3 So it will be necessary not only to agree that all years *after* 2000 A.D. which are multiples of 2,000 should be ordinary (or non-leap) years, but also a note should be left for posterity in the medium term that one further leap year should be suppressed in the 11th, 13th, 15th, 16th (or 17th for the sake of smoothness), 19th and 20th millennia, subject to the caveat in brackets at the end of 6.2. (The additional information in 5.3 does, of course, make nonsense of the modifications to the Gregorian calendar suggested by Aveni [1] to have been agreed.)

6.4 It is too soon to make any recommendation for the longer term. But if the rate of reduction in the number of days in the solar year remains the same, then some time between the 36th and 42nd millennia the solar year will be 365.2400 days, and by then all century years should be non-leap years. While after about 4 million years the 366 day year should disappear altogether, and distant posterity will need to decide which years should have only 364 days! So it will be a very long time before actuaries constructing mortality tables will need to ask themselves "when is a year not a year?"

6.5 I would suggest that the IACA, through its membership in many countries, should pass the recommendations made in 6.3 above to the U.N., to be adopted by all member countries using the Gregorian calendar.

## 7. ACKNOWLEDGEMENTS

7.1 My thanks to Katie Bedford of Cheltenham Ladies' College for sending me extracts from Sandys [3] and Ovid [2]; to my daughter, Fiona Wild, for following this up by presenting me with a copy of Ovid [2]; and to Nick Anderton, Brian Clark, Rod Marshall and Tony Pepper for suggesting improvements to earlier drafts of the paper.

**References:**

(1) Anthony F. Aveni.

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(2) Ovid.

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(4) G.J. Whitrow.

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