IAA Health Section

Stochastic Modeling Webcast
Theory and Reality from an Actuarial Perspective

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Overview of Discussion

- General Concerns and Health Insurance Specific Issues
- Stochastic Modeling for Health Actuaries
- Examples
- Questions

Overview of Discussion

- Based on the recently published book by the IAA
- Stochastic Modeling: Theory and Reality from an Actuarial Perspective
- Available on the IAA website

General Concerns

- What factors should be stochastically generated?
- When should I be using stochastic models?
- When should I not use stochastic models?
- Are there alternatives to stochastic models?
- What are the disadvantages of stochastic models?

General Concerns

When should I be using stochastic models?

- Are these required by regulation or professional guidelines?
- Do we need a better understanding of the effects of extreme outcomes?
- Do we need a better understanding of the tail risk or risks in general?
- What is the probability of an event?
- What is the probability of ruin?
- Are certain risk measures needed for reporting?

General Concerns

When should I not use stochastic models?

- Can you calculate a probability distribution?
- Can you calibrate the model?
- Can you validate the model?

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General Concerns

Are there alternatives to stochastic models?

- Stress Testing/Scenario Testing
- Static Factors/PADs/MADs
- Range testing

General Concerns

What are the disadvantages of stochastic models?

- “Black Box”
- Inappropriate distributions
- Inappropriate parameters
- Improper calibration
- Validation – Beware of False Positives
- Model size
- Computer power

General Concerns

Model Calibration

- Two General Approaches
  - Calibration to Historical Experience
  - Calibration to Current Market Conditions
- Considerations
  - Does the model track to expected assumptions?
  - Reflect expectations today?
  - Experience period
  - Range of possible outcomes?
  - Extremes

General Concerns

Model Calibration

- Calibration to Historical Experience
  - Can you create a distribution?
  - Expected
  - Correlations
  - Volatility
  - Mean reversion

General Concerns

Model Calibration

- Calibration to Current Market Conditions
  - Observed market prices or conditions
  - Closed form formula
  - Market consistent results

General Concerns
Model Validation

- How do I validate a model?
  - Cellular checking
  - Reasonableness review
  - Assumption review
  - Formula testing
  - Calibration review
  - Distribution of outcomes

General Concerns
Model Validation

- Documentation Review
  - Source of data
  - Experience period
  - Testing
  - Audit/Checking/Peer Review

General Concerns

Model Peer Review

- Reviewed for Accuracy?
- Credible data?
- Model and parameter development?
- Correlations?
- Testing
- Validation

General Concerns

Model Communication

General Ideas

- Average
- Outliers – Worst Case and Best Case
- Specific Scenario
- Type of Audience

General Concerns

Model Audit

Parameter Development

- Source of Information
- Development of Assumptions
- Development of Correlations
- Expected Assumptions

General Concerns

Model Audit

Stochastic Generator Analysis

- Cell Checking
- Review of Distributions, range of outcomes, extreme cases
- Correlation checking
- New set of scenarios produce similar results

General Concerns

Health Specific Issues

- Identification of All Factors
- Identification of Key Material Factors

Example of Long Term Health Insurance Risk Model

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Risk Factors of Interest to Health Actuaries

General Categories of Risk Factors of Interest to Health Actuaries:

- Economic Factors
- Demographic Factors
- Morbidity/Claims Factors

All categories include assumptions that could be modeled stochastically

Risk Factors of Interest to Health Actuaries

- Economic Factors Include:
  - Interest Rates
  - Unemployment
  - Inflation

Risk Factors of Interest to Health Actuaries

- Demographic Factors Include:
  - Policy Terminations (Claim termination considered separately)
  - Shock Lapses
  - Mortality

Risk Factors of Interest to Health Actuaries

- Morbidity/Claims Factors Include:
  - Claim Costs (or Incidence and Severity)
  - Morbidity Improvement
  - Medical Trend

Economic Risk Factors

- Interest Rates
  - Many commonly used interest rate generators are available.
  - Approach to selecting an interest rate generator:
    - How will the projection be used?
    - Should scenarios be “real-world” or “risk-free”?
    - Is mean-reversion incorporated in the model?
    - Do you want to model stochastic volatility?
    - Evaluate arbitrage-free properties of the model
    - If appropriate, validate scenarios produced by the model against observable market prices or conditions.

Commonly used Interest Rate Generators: Equilibrium Models

Commonly used Interest Rate Generators - Equilibrium Models

Model	Formulaic Description	Comments
Rendleman and Bartter Model	■ $dr = \mu r\, dt + \sigma r\, dz$	■ μ and σ constant ■ r follows geometric Brownian motion ■ μ is a drift component. σ is a volatility parameter. ■ No mean reversion ■ Negative interest rates are possible ■ Drift is time-independent
Vasicek Model	■ $dr = a(b-r)\, dt + \sigma\, dz$	■ Incorporates mean reversion through the term a(b-r) ■ The σdz term incorporates stochastic element ■ Negative interest rates are possible ■ Drift is time-independent
Cox, Ingersoll and Ross Model	■ $dr = a(b-r)\, dt + \sigma \sqrt{r}\, dz$	■ Very similar to Vasicek model, except for the presence of $\sqrt{r}$ in the second term ■ The $\sqrt{r}$ exists in order to preclude negative interest rates ■ Drift is time-independent

Note: dz is a Wiener process satisfiying the following two properties:

(1) $dz = Z\,(dt)^{(1/2)}$, where Z is standard normally distributed
(2) Values of dz for separate time intervals are independent

Commonly used Interest Rate Generators: No-Arbitrage Models

Commonly used Interest Rate Generators - No-Arbitrage Models

Model	Formulaic Description	Comments
Ho-Lee Model	■ $dr = \theta(t)\,dt + \sigma\,dz$ ■ $\theta(t) = F_t(0,t) + \sigma^2 t$	■ σ constant ■ θ(t) term exists to calibrate model to initial term structure ■ Drift is time-dependent ■ Ft(0,t) is the partial derivative with respect to t of the forward rate for a maturity t. This term helps ensure that future short rates move in line with intial term strucutre
Hull-White Model	■ $dr = [\theta(t) - ar]\,dt + \sigma\,dz$ ■ $\theta(t) = F_t(0,t) + aF(0,t) + (\sigma^2/2a)(1-e^{-2at})$	■ Effectively, a Ho-Lee model with mean-reversion

Note: dz is a Wiener process satisfiying the following two properties:

(1) $dz = Z\,(dt)^{(1/2)}$, where Z is standard normally distributed
(2) Values of dz for separate time intervals are independent

Economic Risk Factors

- Inflation/Unemployment Models
  - Purchasing Power Parity
  - ARIMA (Autoregressive Integrated Moving Average Models
  - Phillips Curve Models

Economic Risk Factors

- Purchasing Power Parity
  - In a no-arbitrage world, forward exchange rates are completely determined by:
    - The current foreign exchange rate
    - Projection of US-denominated risk-free interest rates
    - Projection of Fx-denominated risk-free interest rates
  - Using this fact, we can model inflation stochastically without adding additional complexity to the model.
  - Key advantage: your model will be internally consistent and arbitrage-free

Economic Risk Factors

- Purchasing Power Parity
  - Key Point:
    - Arbitrage is avoided if and only if:
      - Investing $1 in USD for 5-years at the risk-free rate is expected (on a risk-free basis) to be equivalent to:
        - Converting $1 to GBP
        - Investing for 5-years at the risk-free rate
        - Converting the cumulative value to USD at the end of 5-years

Economic Risk Factors

- Purchasing Power Parity
  - Example

Time	US Dollar Risk-Free Forward Rates	GBP Risk-Free Forward Rates	Spot Exchange Rate (USD/GBP)	No-Arbitrage Forward Exchange Rate (USD/GBP)
0			1.4000	
1	0.75%	5.00%		1.3433
2	1.00%	5.00%		1.2922
3	1.25%	5.00%		1.2460
4	1.50%	6.00%		1.1931
5	1.50%	6.00%		1.1425

Economic Risk Factors

- Purchasing Power Parity
  - Conclusions:
    - Over 5-years, USD has appreciated relative to GBP
    - This is due to relatively high GBP risk-free rates relative to USD risk-free rate
    - USD Inflation has been low (relative to GBP)
    - The implied forward exchange rate can be used to develop inflation relativities
    - Doing so requires some professional judgment, but this technique can be quite useful if you compare your currency to a currency where the central bank is known to target a certain inflation range (and does so credibly).

Economic Risk Factors

- ARIMA Models
  - Many different variations of this approach using time-series/regression models
  - Both continuous-time and discrete-time models exist

Economic Risk Factors

- Phillips Curve Models
  - Basic Idea: there is a trade-off between inflation and unemployment (i.e., the “Phillips Curve”)
    - If your model already incorporates a stochastic projection of unemployment, you can make use of a Phillips curve to extract the implied level of inflation given the projected unemployment
    - Alternatively, your inflation model (using either Purchasing Power Parity or an ARIMA model) can be used to project unemployment (again through use of the Phillips curve)

Demographic Risk Factors

- Policy terminations and Shock Lapses
  - Generally modeled using commonly available distributions:
    - Normal distribution is the most commonly used
      - Establish a deterministic lapse assumption, which may vary by duration
      - Model additions/subtractions to the lapse rate as a stochastic, normally distributed random variable with mean 0 and standard deviation, (Sigma)
      - Sigma generally estimated based on historical data (for the block of business being modeled or from general industry experience), but may involve element of judgment or provision for adverse deviation
    - Shock Lapses
      - The shock is sometimes modeled as a stochastic, normally distributed random variable (similar to routine lapses)
      - Other models may express shock lapse as a randomly-generated percentage of some other observed value (e.g., x% of rate increases)

Demographic Risk Factors

- Mortality
  - Often a smaller concern for health products (relative to traditional life products)
  - Several approaches exist:
    - Normal distribution probably the most widely-used
      - May fail to capture "regime changes" (e.g., pandemics, terrorist attacks or other "1 in x-hundred year events")
    - IAA Stochastic Modeling text presents a sophisticated 3-component approach
      - Baseline Model
      - Disease Model
      - Terrorism Model

Morbidity/Claims Factors

- Claim Incidence
  - Commonly-used approaches:
    - “Pure” Monte Carlo
    - Normal Distribution
    - Log-Normal Distribution
    - Poisson Distribution

- Claim Severity
  - Stochastic models are generally constructed off of pre-existing continuance curves/tables
    - Stochastic generation of curves is generally complex and (often) unnecessary
  - Approach:
    - Generate random number from some distribution (e.g., uniform)
    - Use the random number to “choose” the severity of a particular claim from the appropriate continuance curve
    - Calculate total claim based on “chosen” severity

Morbidity/Claims Factors

- Comments on Medical Trend
  - Medical trend may be linked to inflation
  - Can model medical trend stochastically as:
    - Inflation + “Real Cost Spread”
  - Some choose to model the “Real Cost Spread” stochastically, as well
  - Advantage: Model is internally consistent
  - Disadvantage: May desire a more “scientific” approach to determine the “Real Cost Spread.”

Example: Long Term Care Product

- Illustrative Example for "Long-Tailed" Health Product:
  - Long Term Care Policy
  - Policy Characteristics:
    - 5-year Benefit Period
    - $100/day benefit
    - No inflation protection or other special riders
    - Annual premium = $1,000
  - Policyholder Data:
    - Female
    - Attained age 70 as of the valuation date
    - Purchased the policy 10 years ago (i.e., is in policy duration 10)
  - Other Information:
    - Policy is expected to have a rate increase of 50% in projection year 3
  - We will do a 5-year projection of premium and incurred claims for a single cell, under a single scenario

Claims Modeling - Incidence

- Claim Incidence
  - Example (Poisson Distribution for Claim Incidence)
    - Estimated parameter for the Poisson Distribution
      - Lambda = 0.3

Probability of a Claim (Poisson)		**0.2222**
Projection Year	**Random Number Generated**	**Claim Incidence**
1	0.901	0
2	0.930	0
3	0.164	1
4	0.363	0
5	0.007	1

Claims Modeling - Continuance

- Claim Severity (Continuance)
  - Actuary has estimated claim continuance curve as follows:

Illustrative Claim Continuance Curve

Claim Duration (Years)	0.5	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
Probability of Survival	98.00%	90.00%	80.00%	60.00%	30.00%	15.00%	7.50%	3.75%	0.00%

Claims Modeling - Continuance

- Claim Severity (Continuance)
  - Actuary selects a uniform distribution to stochastically model claim continuance
  - Following considers claim continuance only; we will combine incidence and severity next:

Projection Year	Random Number Generated	High	Low	Modeled Claim Severity (Years)
1	0.213	30.00%	15.00%	3.29
2	0.001	3.75%	0.00%	4.99
3	0.699	80.00%	60.00%	2.25
4	0.920	98.00%	90.00%	1.25
5	0.048	7.50%	3.75%	4.35

  - For random numbers in between points on the claim continuance curve, we use linear interpolation

Claims Modeling – Incurred Claims

- Incurred Claims
  - To model total claim costs, we combine claim incidence and claim severity from the previous slides, as follows:

Projection Year	Claim Incidence	Claim Severity (Years)	Annual Benefit	Total Incurred Claim (Undiscounted)
1	0.000	3.29	36,500	-
2	0.000	4.99	36,500	-
3	1.000	2.25	36,500	82,191
4	0.000	1.25	36,500	-
5	1.000	4.35	36,500	158,947

Claims Modeling – Incurred Claims

- Policy Termination Rates
  - We use the “stochastic addition” method discussed earlier
  - The additive term is normally distributed with mean = 0 and standard deviation = 5%

Sigma 5.00%

Policy Year	Projection Year	Deterministic Voluntary Lapse	Random Number	Stochastic Adjustment to Lapse	Total Policy Termination Rate
10	1	5.0%	0.317	-2.4%	2.6%
11	2	5.0%	0.663	2.1%	7.1%
12	3	5.0%	0.812	4.4%	9.4%
13	4	4.0%	0.213	-4.0%	0.0%
14	5	3.0%	0.049	-8.3%	0.0%

Note: The Voluntary Lapse is assumed to include mortality; Mortality is not modeled stochastically in this example.

Claims Modeling – Incurred Claims

- Rate-Increase Shock Lapses
  - Modeled as follows:
    - Deterministic assumption that shock lapse = 10% of Rate Increase Amount
    - In the stochastic projection, the deterministic assumption is multiplied by a stochastic adjustment that is normally distributed with mean = 1, standard deviation = 25%

Projection Year	Rate Increase	Deterministic Shock Lapse	Random Number	Stochastic Adjustment to Lapse	Projected Shock Lapse
1	0.0%	0%	0.317	0.881	0.0%
2	0.0%	0%	0.508	1.005	0.0%
3	50.0%	5%	0.632	1.084	5.4%
4	0.0%	0%	0.618	1.075	0.0%
5	0.0%	0%	0.832	1.240	0.0%

Claims Modeling – Incurred Claims

- Cash Flow Projection Detail
  - Based on the components developed on the previous slide, we have the following cash flow projections:

Time	Policy Count	Paid Premium	Incurred Claims
0	1.000		
1	0.974	987	-
2	0.905	939	-
3	0.775	1,260	82,191
4	0.775	1,162	-
5	0.775	1,162	158,947

  - Example ignores accounting implications, expenses, etc.

Example of Short Term Medical Insurance Risk Model

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Stochastic Risk Models for Short Term Medical Insurance

- What is Stochastic Modeling Used for in Short Term Medical Insurance?
  - Claim Level Estimation
  - Surplus Requirements (Economic Capital)
  - Distribution of Medical Loss Ratios
  - Stop Loss Rating
  - Other

Stochastic Risk Models for Short Term Medical Insurance

- What are Major Risks to Insurance Companies Selling Short-Term Medical Insurance Products?
  - Rating Parameter Adequacy
  - Regulatory Issues / Delays
  - Catastrophic Events
  - Expense Recoupment
  - Other

Stochastic Risk Models for Short Term Medical Insurance

- Is Stochastic Modeling Necessary to Establish Surplus Requirements?
  - No
  - However, it is superior to deterministic models that involve projection of a limited set of likely scenarios
  - Also, it is superior to peer group analysis
    - Where conclusions are drawn from companies with “similar” characteristics
  - Stochastic models allow for simultaneous consideration of multiple risk factors and ranges of possible outcomes

Stochastic Risk Models for Short Term Medical Insurance

- What are some of the considerations in developing a Stochastic Model?
  - Establish risk level – high likelihood, sufficiency, virtual certainty – corresponding to 90^{th}, 95^{th}, 98^{th} percentiles
  - Determine risks to include in model
  - Develop distributions of outcomes for each risk, based on ranges of potential outcomes
    - Some risks can be easily measured and parameterized
    - Other risks may be more subjective and harder to define
    - Interdependent risks need to be evaluated

Stochastic Risk Models for Short Term Medical Insurance

- How are Stochastic Models Tested to Ensure Meaningful Results?
  - Sufficient number of iterations are run to ensure stability of result
  - Underlying distributions are calibrated to observed data history
  - Model results are validated by comparison to other independent approaches or results

Stochastic Risk Models for Short Term Medical Insurance

- Examples of Stochastic Modeling Used in Short Term Medical Insurance:
  - Risk Associated with Catastrophic Events
    - Already established claim risk model
    - Need to evaluate additional risk due to pandemics
  - Develop Optimal Surplus Target Range
    - For non-profit Short Term Medical policies sold by a stand-alone health insurance company

Risk Associated with Pandemics

- Claims Trend

Risk Associated with Pandemics

- Pandemic Cost Component

Event	Probability	Outcome
No Pandemic	98.0%	0% Additional Cost
Mild Pandemic	1.0%	2% Additional Cost in Year
Major Pandemic (duration 1 year)	.75%	14% Additional Cost in Year
Major Pandemic (duration 2 years)	.25%	14% Additional Cost in 1st Year and 2% Residual Cost in 2nd Year

Risk Associated with Pandemics

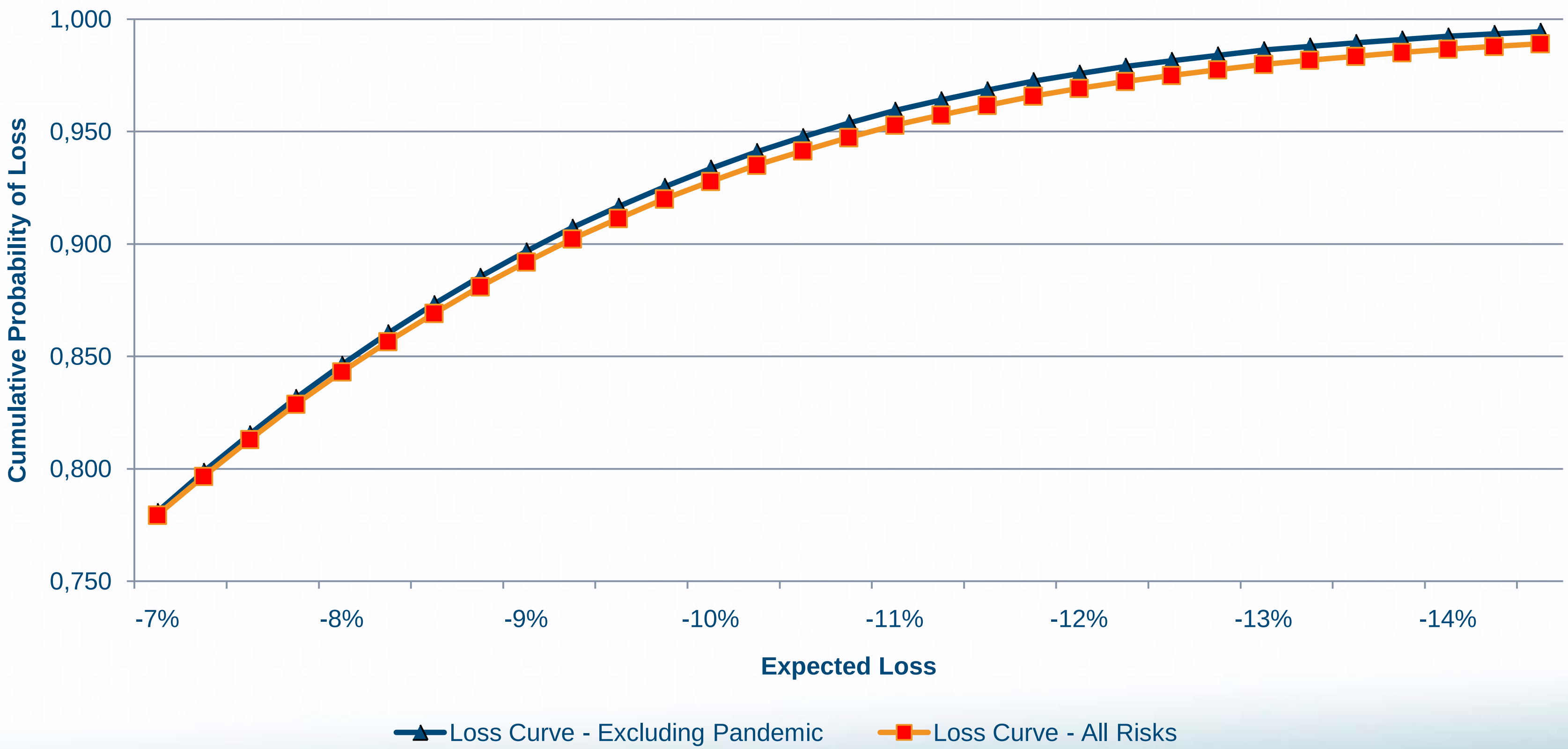
- Premium Development

Risk Associated with Pandemics

- Recovery of Losses from Pandemic
  - Recovery of Loss, if any, over 5 years after event
  - Potential for not being able to recover losses due to MLR limitations under Federal Health Care Reform
- Distribution of results with and without pandemic risk

Risk Associated with Pandemics

- Illustrative Distribution of Losses



Optimal Surplus

- Analytical Approach
  - Approach has been refined over time
  - Best described as a variant of Enterprise Risk Management
  - Involves the use of a model to reflect company-specific financial situation and risk exposure:
    - Forward looking projection of potential outcomes
    - Based on historical and anticipated loss distributions
    - Detailed approach to development and documentation of assumptions
  - Externally peer reviewed
    - Approach found to be reasonable and acceptable

Optimal Surplus

- Analytical Approach
  - Basic Approach to the Evaluation:
    - Analysis of historical underwriting loss cycles
      - Company-specific loss history
      - Comparison (by relative size) set of Insurers
    - Identify and evaluate major risks
      - Develop risk and contingent distributions
      - Monte Carlo simulation of loss cycles
    - Identify reserve target thresholds and criteria
      - Surplus target goals
      - Loss cycle provisions to satisfy goals

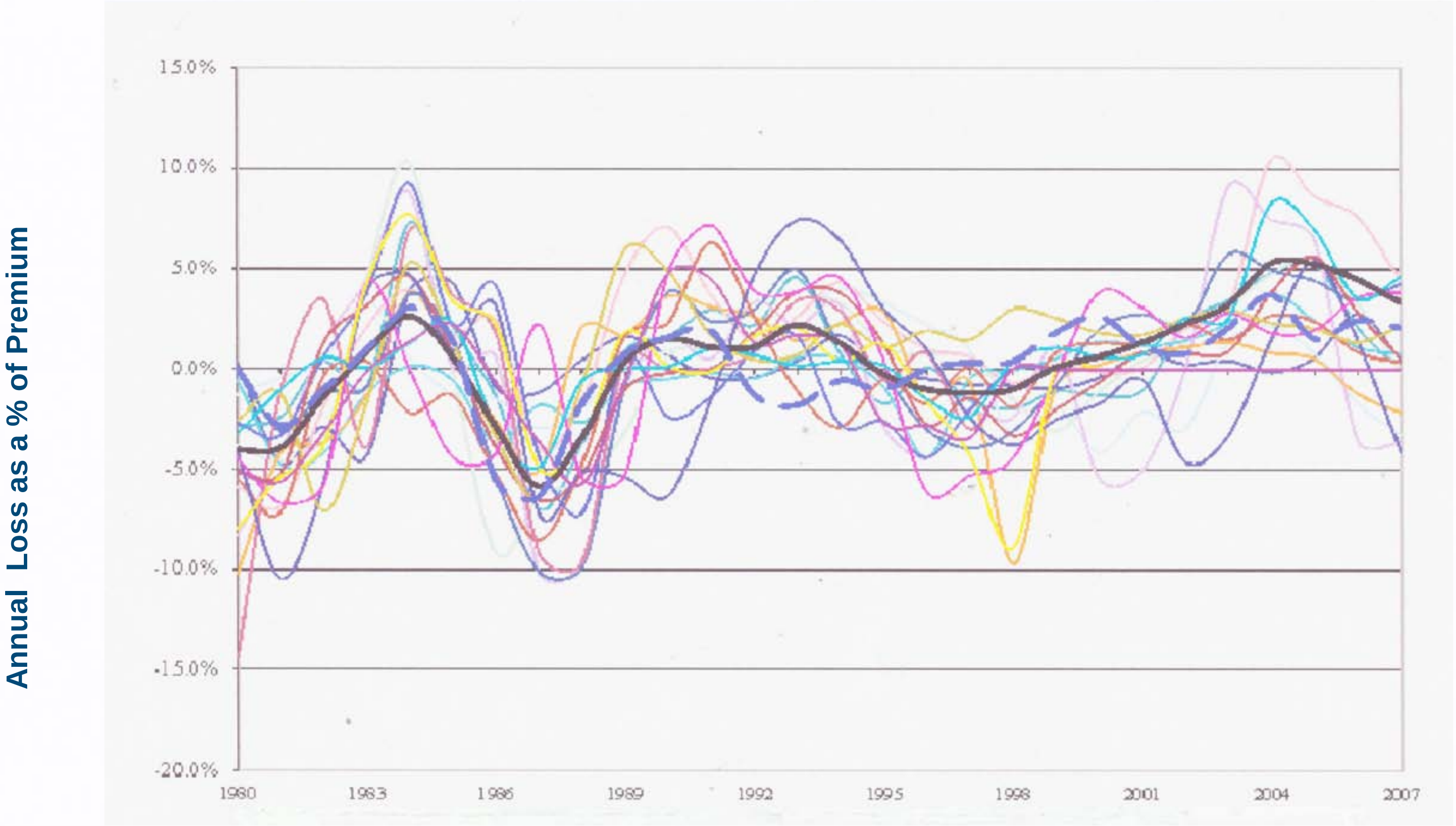
Optimal Surplus

- Analytical Approach
  - Basic Approach to the Evaluation:
    - Develop and initialize pro forma projection model
      - Pro forma assumptions for company
    - Develop reserve target range
      - "Stress test" loss cycles against reserve thresholds
      - Develop reserve levels necessary to withstand cycle losses

Optimal Surplus

- Analytical Approach
  – Basic Approach to the Evaluation:
    - Analysis of historical underwriting loss cycles

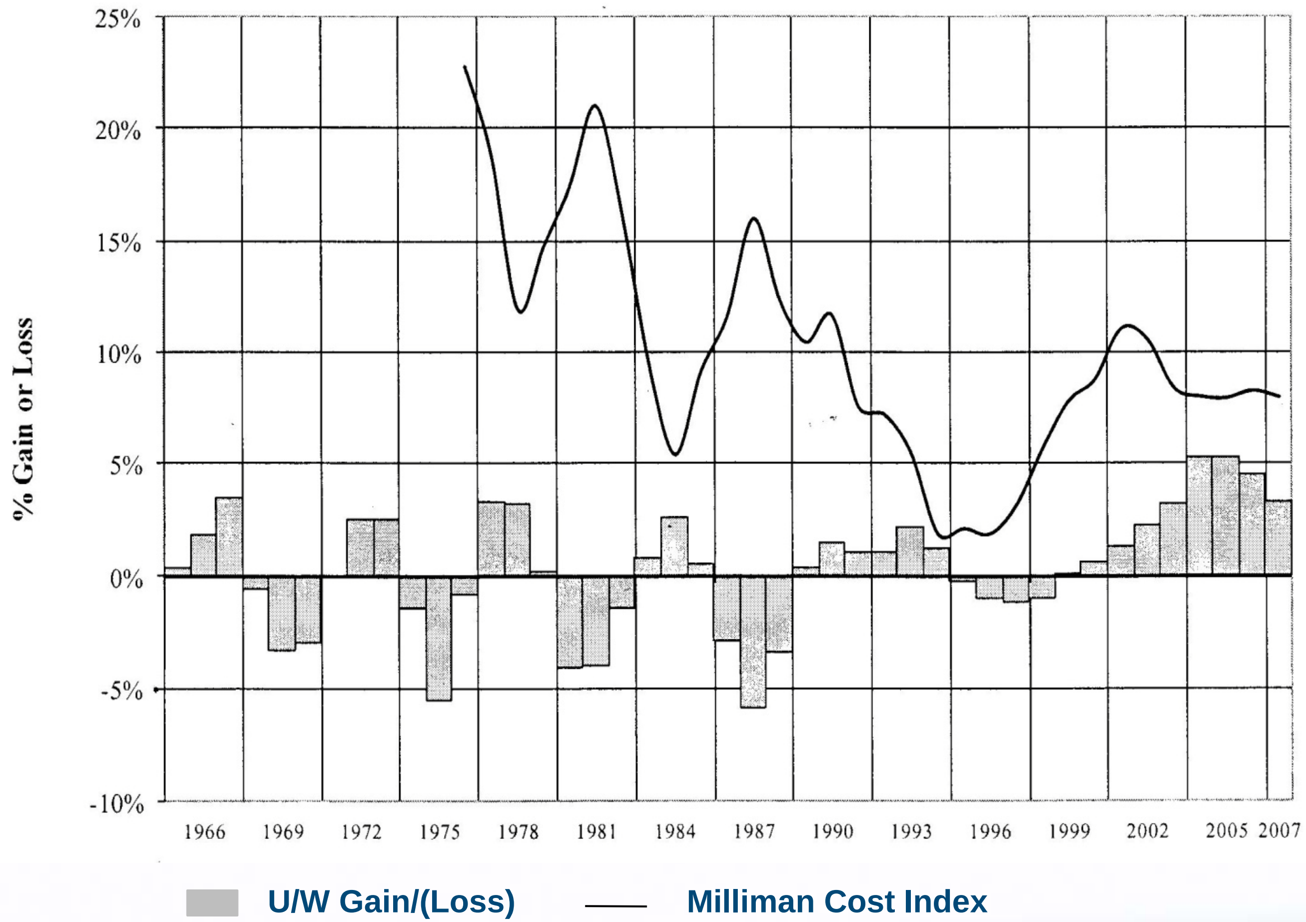
Underwriting Gain/(Loss) for Selected Insurers



All Plans

Other Lines = Comparison Set of Plans

Average Underwriting Gain / (Loss) for a Sample of Health Insurers



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Optimal Surplus

- Analytical Approach
  - Basic Approach to the Evaluation:
    - Identify and evaluate major risks
      - Develop risk and contingency distributions
        - Rating adequacy and fluctuation
        - Unpaid claims and other estimates
        - Portfolio risks (equity market, interest rates, impairment)
        - Overhead expense recovery and fee income risk
        - Other business risks, including self-funded
        - Catastrophic events, including Health Care Reform
        - Provision for unidentified development and growth
        - Other factors

Optimal Surplus

- Analytical Approach
  - Basic Approach to the Evaluation:
    - Identify reserve target thresholds and criteria
      - Surplus target goals
        - > Early warning monitoring threshold avoidance (375% RBC)
        - > Company Action Level (200% - 300% RBC)
        - > Loss of trademark avoidance (200% RBC)
      - Loss cycle provisions established to satisfy goals – for example:
        - > Early warning – 90^{th} to 95^{th} percentile of Monte Carlo simulated loss cycles
        - > CAL – 95^{th} percentile of simulated loss cycles
        - > Loss of trademark – 98^{th} percentile of simulated losses

Optimal Surplus

- Analytical Approach
  - Basic Approach to the Evaluation:
    - Develop and initialize pro forma projection model
      - Pro forma assumptions for company
        - Generally based on company forecast
        - Normalized assumptions to reflect longer term view of operations
        - GAAP to Stat adjustments

Optimal Surplus

- Analytical Approach
  - Basic Approach to the Evaluation:
    - Develop reserve target range
      - “Stress test” loss cycles against reserve thresholds
        - Use Pro Forma model to project company balance sheet
        - Based on various starting surplus amounts
        - Tested under a range of forecast projection assumptions
        - Ending surplus target goals
      - Develop reserve levels necessary to withstand cycle losses
        - Expressed as ranges of RBC

Optimal Surplus

Recap of Resulting Surplus Ranges

- 11 Distinct Studies in Recent Years
- Circumstances of Individual Companies Different
- Range of Results 650% - 1250% of ACL-RBC

IAA Health Section

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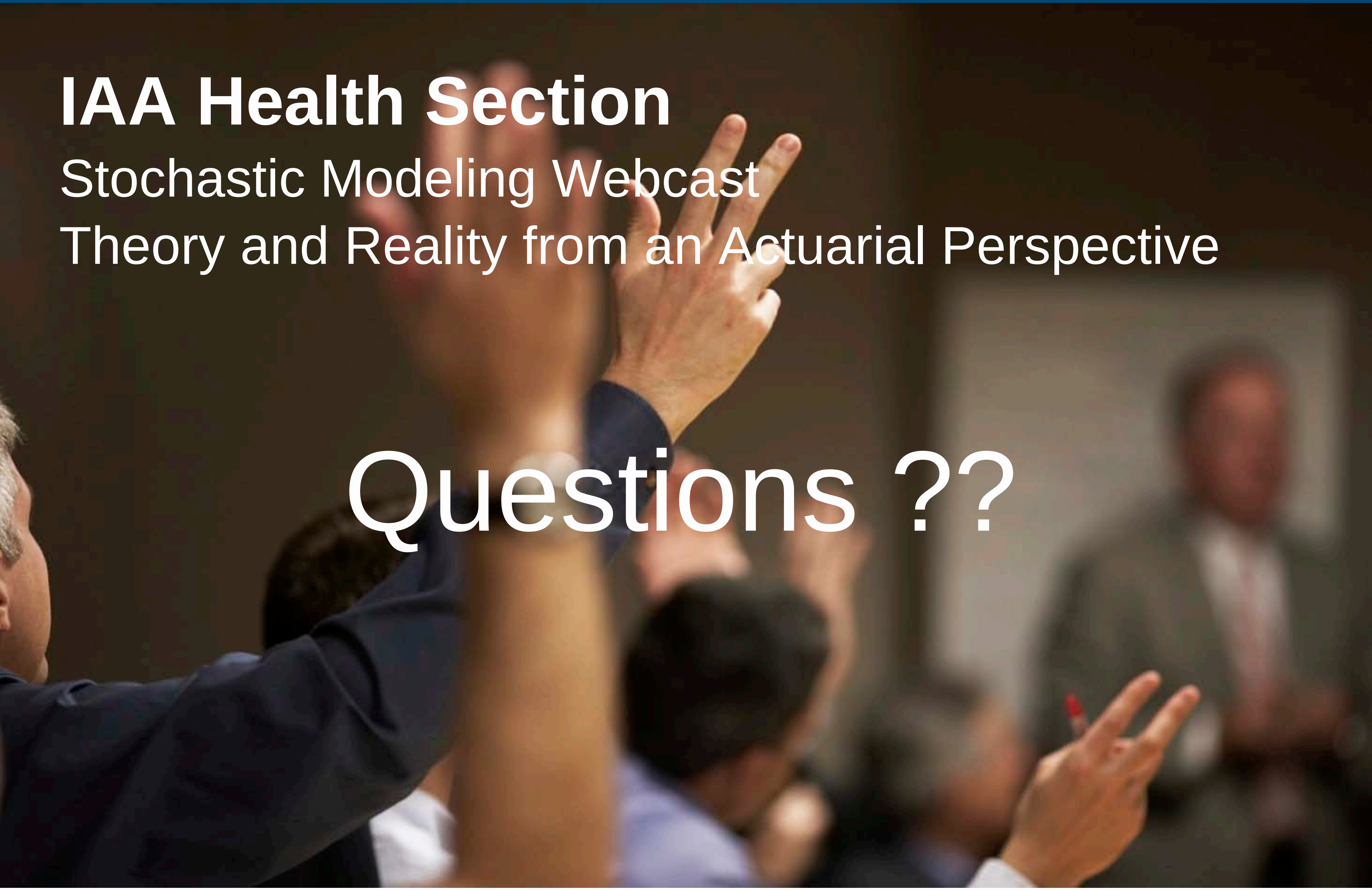
Theory and Reality from an Actuarial Perspective



Questions ??

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