VaR and other Risk Measures

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Outline

1. What is Risk?
2. Risk Measures
3. Methods of estimating risk measures
Risk is not equal to the size of loss or the size of a cost. Risk lies in the unexpected losses.
In Basel III, there are three major broad risk categories:

- Credit Risk:
Types of Financial Risk

- Operational Risk:
Types of Financial Risk

- Market risk:

Each one of these risks must be measured in order to allocate economic capital as a buffer so that if a catastrophic event happens, the bank won’t go bankrupt.
Def.

A risk measure is used to determine the amount of an asset or assets (traditionally currency) to be kept in reserve in order to cover for unexpected losses. It can be seen as the minimum extra cash that has to be added to the risky position and invested in a risk free asset to make the risky position acceptable.
A coherent risk measure satisfies the following properties:

Let $X$ and $Y$ represent any two portfolios set of returns and let $\rho(\cdot)$ be a measure of risk over a horizon, then

- **Monotonicity:** If $Y \geq X$ then $\rho(Y) \leq \rho(X)$
- **Subadditivity** $\rho(X + Y) \leq \rho(X) + \rho(Y)$
- **Positive homogeneity:** $\rho(hX) = h\rho(X)$ for $h > 0$
- **Translational invariance** $\rho(X + n) = \rho(X) - n$ for $n$ number.
A VaR of $d$ days at a $\alpha\%$ confidence level means that on $\alpha\%$ of $d$ days, we won’t see a loss higher than the VaR, but for the $(1 - \alpha)\%$ of times the loss will be higher.

For example, a 1 day VaR at the 99% confidence level of 5% means that on only 1 of every 100 days we will see a loss higher that 5%.
Is VaR coherent?

Ex: Assume we have two identical bonds A and B, each with a default probability of 4%, a notional of 100 and we assume no recovery. As the default probability is 4% this means that the 95% of times we will not lose anything, so the 95% VaR of each bond is 0, so \( \text{VaR}_{95\%}(A) + \text{VaR}_{95\%}(B) = 0 \).

\[
P[\text{Loss}_A = x] = \begin{cases} 0 & p = 0.96 \\ 100 & p = 0.04 \end{cases}
\]
Is VaR coherent?

**Ex:**

Now, we suppose that defaults are independent, so in a portfolio holding both bonds we will have

\[ P[\text{Loss}_{A+B} = x] \begin{cases} 0 & p = 0.9216 \\ 100 & p = 0.0768 \\ 200 & p = 0.0016 \end{cases} \]

So \( \text{VaR}_{95\%}(A + B) = 100 > 0 = \text{VaR}_{95\%}(A) + \text{VaR}_{95\%}(B) \) that is, by holding both bonds at the same time our portfolio is riskier than by holding each bond separately.

We are punishing diversification.
Expected Shortfall

The Expected Shortfall is an average of the worst 100(1 - \( \alpha \))% of losses

\[
ES = \frac{1}{1 - \alpha} \sum p\text{-th highest loss} \times \text{probability of } p\text{-th highest loss}
\]

It can be shown that this measure is coherent.
Expected Shortfall

The ES is a better risk measure than the VaR because:

- While VaR only tells us the maximum threshold for losses at a given confidence level, it doesn’t tell us how much we lose when a bad day occurs. However, by definition the expected shortfall tells us how much we expect to lose once a catastrophic loss happens.
- ES always satisfies subadditivity.
Spectral Risk Measures

So, what has the ES that doesn’t have VaR?

We can define more general risk measures $M_\phi$ that are weighted averages of the quantiles of the loss distribution:

$$M_\phi = \sum_p w(p) \times p\text{-th highest loss} \times \text{probability of } p\text{-th highest loss}$$

With $w(p)$ the weight of the $p-th$ highest loss. The weights must comply with being non negative, summing up to 1 and with being risk averse, i.e. that is a bigger loss must be at least as important as a lesser loss.
Spectral Risk Measures

We can see that the ES is a special case of a spectral risk measures with

\[ w(p) = \begin{cases} 
  0 & p \leq 1 - \alpha \\
  \frac{1}{1-\alpha} & p > 1 - \alpha 
\end{cases} \]

Also, if we try to write the VaR as a Spectral Risk Measure, it’s easy to see why it isn’t coherent.

\[ w(p) = \begin{cases} 
  1 & p = 1 - \alpha \\
  0 & p \neq 1 - \alpha 
\end{cases} \]

That is, we don’t care about losses higher than the VaR. This is why for Basel III, we are changing from a VaR basis market risk capital to a ES based capital.
The core of estimating a risk measure is managing to describe the P&L distribution of the portfolio. There are many ways to estimate this but we can encompass them in three categories:

1. **Non Parametric**

2. **Parametric**

3. **Monte Carlo**
Non Parametric Methods

The Non Parametric Methods are based on the historical simulation approach, in its most basic forms this is the simplest and most straightforward VaR method.
To make this calculation, you simply order observations from largest to smallest. The observation that follows the threshold level denotes the VaR limit.
Disadvantages of Basic Non Parametric Approach

- Implicit assumption that you expect future performance to follow the same pattern as in the past.
- Is unable to adjust for changing economic conditions.
- Observations that happened a long time ago have the same weight as a recent one.
Improvements to Basic Non Parametric Approach

- **Age-Weighted Historical Simulation:** We weight recent observation more and distant observations less.

  \[ w(i) = \frac{\lambda^{i-1}(1 - \lambda)}{1 - \lambda^n} \]

  With \( 0 \leq \lambda \leq 1 \)

- **Volatility-weighted Historical Simulation:** The intuition is that if recent volatility has increased then using historical data will underestimate the current risk level.

  We adjust the daily return \( r_{t,i} \) on day \( t \) upward or downward based on the then-current volatility forecast relative to the current volatility forecast on day \( T \).

  \[ r_{t,i}^* = \left( \frac{\sigma_{T,i}}{\sigma_{t,i}} \right) r_{t,i} \]
They consist on adjusting a probability distribution to the return distribution, most commonly a normal or a $t$ distribution.

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<th>Advantages</th>
<th>Disadvantages</th>
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<td>Not limited to only past scenarios</td>
<td>Model Risk</td>
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<td>Fast and easy to calculate</td>
<td>Less accurate for non-linear portfolios</td>
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Monte Carlo

This method consists in describing the asset prices diffusion in order to get the simulated P&L instead of trying to describe the P&L distribution.

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<td>Accurate for non-linear</td>
<td>Takes a lot of computational</td>
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<td>instruments</td>
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<td>We get a full distribution</td>
<td>Model Risk</td>
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<td>of potential distributions</td>
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Bibliography