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# Modern Life-Care Tontines

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ASTIN AFIR-ERM Colloquium,
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joint work with:
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The New York Times

THE NEW OLD AGE

## Many Americans Will Need Long-Term Care. Most Won't be Able to Afford It.

A decade from now, most middle-income seniors will not be able to pay the rising costs of independent or assisted living.

Search jobs Sign in Search The Guardian International edition

## Residential care costs 'can soak up over 50% of property values'

**Study finds the cost of a typical residential care home stay around the UK to range from 18% to 56% of average house values**

Frankfurter Allgemeine
ZEITUNG FAZ.NET

VERBRAUCHERSCHÜTZER WARNEN

## „Steigende Pflegekosten sind soziale Zeitbombe"

AKTUALISIERT AM 21.08.2020 - 11:31

(The Guardian 2017, FAZ 2020, NYT 2018)

# Rise in long-term care expenditure

- Belgium: LTC spending (in terms of GDP) increased from **1.7% in 2000** to **2.3% in 2018** (source: Eurostat).

- United Nations projections: The number of **elderly people**, i.e. older than 65, is projected **to triple from 2020 to 2080** to reach 2.2 billion. The global **share of the elderly population** is expected to **rise from 9.4% in 2020 to 20.6% in 2080**.

Motivation

A fair, heterogeneous, modular mutual insurance scheme

Modern Life-Care Tontine

# Why pool mortality and morbidity risks?

- People moving into dependency need more money but have a reduced life expectancy!

  $\Longrightarrow$ **Natural hedge, diversification!**

- Individuals in bad health cannot receive long-term care insurance!

  $\Longrightarrow$ **Combined product gives access to insurance for a larger share of the population!**

- Cost reduction due to **reduced adverse selection**!

  $\Longrightarrow$ Combined product is **attractive for people in bad health...**

# Related literature

Mutual (life) insurance schemes gain popularity in academic literature:

- **(Natural) tontines**: Milevsky, Salisbury [2015, 2016], Chen, Hieber, Klein [2019], Chen, Hieber, Rach [2020], Chen, Qian, Yang [2021]. (many, many more . . .)

- **Pooled annuities, P2P insurance, (tontines)**: (Sabin [2010]), Qiao, Sherris [2013], Donnelly, Guillén, Nielsen [2013, 2014], Denuit [2019]. (many, many more. . .)

# Tontine products and mortality credits

Tontines were popular in the 17th / 18th century but gain popularity today as **modern tontines** / **pooled annuities** / **group self annuitization**:

- **Le Conservateur** (France).
- **The Tontine Trust**: `https://tontine.com/#About`.
- TIAA-CREF retirement fund (US).
- Lifetimeplus from Mercer (Australia).

Main idea of mortality credits: Survivors gain additional return based on (1) mortality risk and (2) amount invested.

(e.g. Donnelly, Guillén, Nielsen [2013, 2014], Denuit [2019])

# Modular mutual insurance scheme: Our contribution

- Based on Denuit [2019] (one-period scheme), we introduce a mutual insurance scheme that is:

  1. Able to pool heterogeneous mortality risks (by age, health).
  2. Discrete-time.
  3. Actuarially fair and fully funded in each period.
  4. Modular, flexible: Adding or removing policyholders does NOT change the AVERAGE payoff of pool members! (NEW property)

Modularity allows to easily add policyholders fairly! We share the risk, the average payoff is unaffected by pooling!

- Design a product sharing mortality AND morbidity (long-term care) risk.

# Related literature and "modularity/flexibility"

Usually, the average payoff depends on the other pool members:

- Donnelly, Guillén, Nielsen [2014]:

**Proposition 3.2.** *Conditional upon survival, the expected instantaneous actuarial gains for individual i in the kth group are*

$$\mathbb{E}\left(dG_t^{k,i}\middle|\mathcal{F}_{t-},\, N_t^{k,i}=0\right) = \lambda_t^k W_{t-}^k \left(1 - \frac{\lambda_t^k W_{t-}^k}{\sum_{m=1}^{M} W_{t-}^m \lambda_t^m L_{t-}^m}\right) dt, \quad (8)$$

*for all* $t \geq 0$ *and for each* $k = 1, \ldots, M$.

**Proof.** See Appendix. □

- Milevsky, Salisbury [2016]:

We use the notation $E_i$ to remind us that this is a conditional expectation, in which $N_i - 1 \sim \mathrm{Bin}(n_i - 1, {}_tp_{x_i})$, while the other $N_j \sim \mathrm{Bin}(n_j, {}_tp_{x_j})$. Call the above expression $w_i F_i(\pi_1, \ldots, \pi_K)$, so if $\pi = (\pi_1, \ldots, \pi_K)$, then

$$F_i(\pi) = \int_0^\infty e^{-rt}\, {}_tp_{x_i}\, w d(t)\, E_i\left[\frac{\pi_i}{\sum_j \pi_j w_j N_j(t)}\right] dt$$

**Problem**: Average payoffs are difficult to predict and depend on other (possibly future) pool members.

We add a (small) death benefit to work around this issue (see later slides)!

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# Some notation

- Pool members $\mathcal{L}_0 = \{1, 2, ..., n\}$. Time in periods $t = 0, 1, 2, \ldots$.
- Individual $j \in \mathcal{L}_0$ contributes single premium $c_j(0)$ at time 0.
- Deterministic, risk-free rate $\delta_t$, $t \geq 0$.
- Remaining lifetimes $T_j, j \in \mathcal{L}_0$, are assumed to be independent.
- Death probability: $q_{x_j}$. Maximal age $\omega \in \mathbb{N}$.
- Individual account value, fixed payoff $s_j(t)$:

$$c_j(t) = \begin{cases} e^{\int_{t-1}^{t} \delta_s \mathrm{d}s} c_j(t-1) - s_j(t)\,, & j \in \mathcal{L}_t \\ 0\,, & \text{otherwise} \end{cases} \tag{1}$$

# Mutual insurance: Insurer's view and actuarial fairness

For each $t = 0, 1, \ldots$, the premium equivalence holds: (pool view)

$$\underbrace{\sum_{j=1}^{n} c_j(t)}_{\text{total account values}} = \sum_{j=1}^{n} \underbrace{\sum_{s=t+1}^{\omega - x_j} e^{-\int_t^s \delta_u \mathrm{d}u} W_j(s)}_{\text{discounted future benefits individual } j} . \tag{2}$$

- Right hand side: **random** (big letter!)
- Left hand side: **deterministic**.

For each $t = 0, 1, \ldots$, the contract is fully-funded: (individual view)

$$\underbrace{c_j(t)}_{\text{retrospective reserve}} = \underbrace{\mathbb{E}_t \left[ \sum_{s=t+1}^{\omega - x_j} e^{-\int_t^s \delta_u \mathrm{d}u} W_j(s) \right]}_{\text{prospective reserve}} . \tag{3}$$

In case of death, the pool shares the remaining account value

$$X(t) := \sum_{j=1}^{n} \mathbb{1}_{j \in \mathcal{D}_t} \cdot e^{\int_{t-1}^{t} \delta_s \mathrm{d}s} c_j(t-1)\,.$$

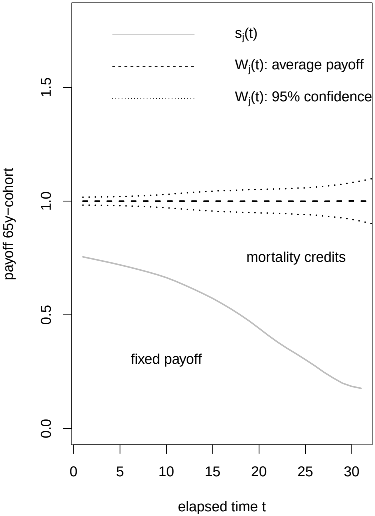
An individual $j \in \mathcal{L}_{t-1}$ receives a payoff of:

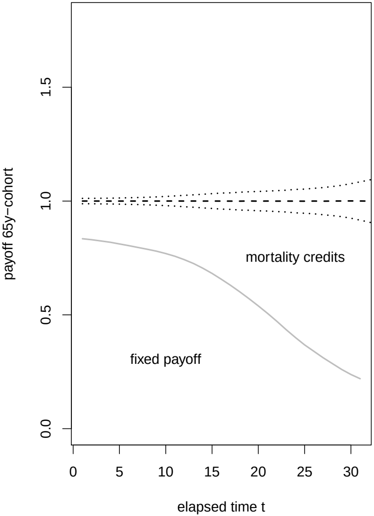
$$W_j(t) = \begin{cases} s_j(t) + \beta_j\big(X(t)\big), & \text{if } j \in \mathcal{L}_t \\ \beta_j\big(X(t)\big), & \text{if } j \in \mathcal{D}_t \end{cases} \tag{4}$$

decomposed of

- $s_j(t)$: individual, fixed withdrawal amount,
- $\beta_j\big(X(t)\big)$: collective part of the benefits, i.e. the mortality credits.

(Examples for $\beta_j$: linear (regression) rule / conditional mean risk sharing.)


s_j(t)
W_j(t): average payoff
W_j(t): 95% confidence
payoff 65y−cohort
1.5
1.0
0.5
0.0
mortality credits
fixed payoff
0
5
10
15
20
25
30
elapsed time t


payoff 65y−cohort
1.5
1.0
0.5
0.0
mortality credits
fixed payoff
0
5
10
15
20
25
30
elapsed time t

# Theorem (Backwards iteration)

*If an individual $j \in \mathcal{L}_t$ aims for an average payoff $b_j(t)$, the fixed payoff is given by:*

$$s_j(t) = \begin{cases} \frac{b_j(t)}{1+q_{\omega-1}}, & \text{for } t = \omega - x_j \\ \frac{b_j(t) - q_{x_j+t-1} \sum_{u=t+1}^{\omega-x_j} e^{-\int_t^u \delta_s \mathrm{d}s} s_j(u)}{1+q_{x_j+t-1}}, & \text{for } t = \omega - x_j - 1, \omega - x_j - 2, \ldots, 1 \end{cases} \tag{5}$$

*We derive the individual's account value as*

$$c_j(t) = \sum_{u=t+1}^{\omega-x_j} e^{-\int_t^u \delta_s \mathrm{d}s} s_j(u) \tag{6}$$

*and the initial single premium as $c_j(0)$.*

# Discussion

- The backwards iteration detects the split between **fixed payoff** $s_j(t)$ and **mortality credits** $\beta_j\big(X(t)\big)$ that leads to an average payoff of $b_j(t)$.
- The backwards iteration can be carried out **individually** for each $j \in \mathcal{L}_0$ (modularity / flexibility).
- This allows **different age cohorts** to share mortality risks in a fair way.

Motivation

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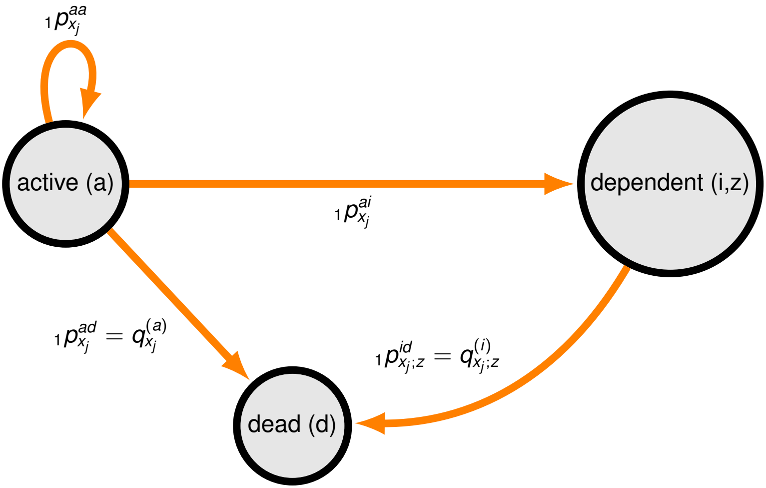
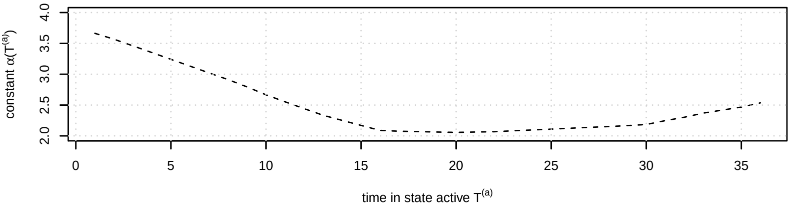
# Life-Care Tontine: semi-Markov model




$z$: time spent in dependency.

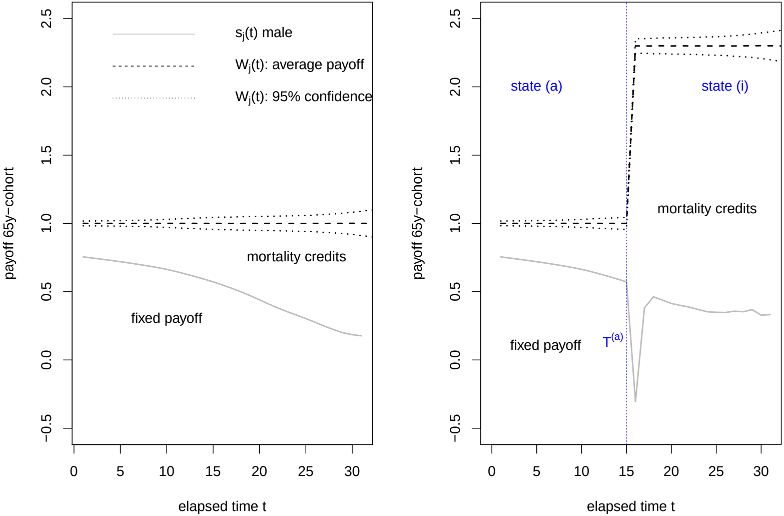
# Modern Life-Care Tontine

“Natural increase”: French mortality data shows dependent people receive an $a(T^{(a)})$ times higher payoff when moving in dependency at time $T^{(a)}$:

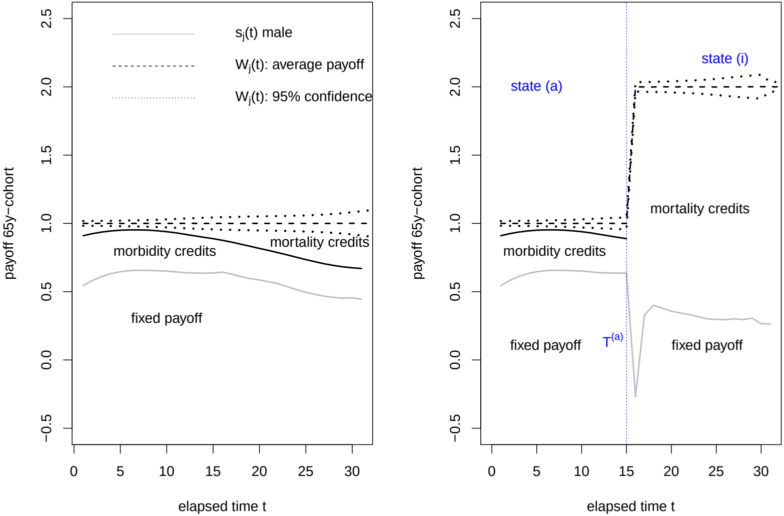



Mortality credits of a dependent person depend on the death probability $q^{(i)}_{x_j+t-1} > q^{(a)}_{x_j+t-1}$.

The product shares mortality and morbidity risk within a pool. We may further adapt the share $a(T^{(a)})$ and distribute “morbidity credits”.



s_j(t) male
W_j(t): average payoff
W_j(t): 95% confidence
mortality credits
fixed payoff
payoff 65y−cohort
elapsed time t
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
0
5
10
15
20
25
30
state (a)
state (i)
mortality credits
fixed payoff
T(a)
payoff 65y−cohort
elapsed time t


payoff 65y−cohort
$s_j$(t) male
$W_j$(t): average payoff
$W_j$(t): 95% confidence
morbidity credits
mortality credits
fixed payoff
elapsed time t
state (a)
state (i)
mortality credits
morbidity credits
fixed payoff
$T^{(a)}$
fixed payoff
elapsed time t

# Discussion and conclusion

- It is beneficial to **pool mortality and long-term care (morbidity) risks**.
- We propose a fair, modular / flexible mututal insurance scheme ($b_j(t)$ for each individual $j$, we share the risk, the average payment is unaffected by pooling!).
- We show how this scheme can be adapted to a **life-care tontine** introducing the concept of **morbidity credits**.
- The scheme allows to pool different age cohorts.
- It is **fully-funded at all times**, allowing individuals to later join the scheme!

# Thank you!

Denuit, M. (2019). Size-biased transform and conditional mean risk sharing, with application to P2P insurance and tontines. *ASTIN Bulletin*, 49(3), 591-617.

Donnelly, C., Guillén, M., and Nielsen, J. P. (2014). Bringing cost transparency to the life annuity market. *Insurance: Mathematics and Economics*, 56, 14-27.

Milevsky, M. A., and Salisbury, T. S. (2015). Optimal retirement income tontines. *Insurance: Mathematics and Economics*, 64, 91-105.

Chen, A., Hieber, P., and Klein, J. K. (2019). Tonuity: A novel individual-oriented retirement plan. *ASTIN Bulletin*, 49(1), 5-30.

# Definition (Fair distribution rule: mortality credits)

A fair distribution rule $\beta_j\big(X(t)\big)$ satisfies:

- **Self-sufficiency property**: $\sum_{j \in \mathcal{L}_{t-1}} \beta_j\big(X(t)\big) = X(t)$.
- **Positivity property**: $\beta_j\big(X(t)\big) \geq 0$.
- **Fairness property**:

$$\mathbb{E}_{t-1}\left[\beta_j\big(X(t)\big)\right] = \underbrace{\mathbb{E}_{t-1}\left[\mathbb{1}_{j \in \mathcal{D}_t}\right]}_{\text{probability to die in } (t-1,\, t]} \cdot \underbrace{e^{\int_{t-1}^{t} \delta_s \mathrm{d}s} c_j(t-1)}_{\text{amount at risk at time } t}, \quad (7)$$

where $\mathbb{E}_t := \mathbb{E}[\,\cdot\,|\,\mathcal{F}_t]$ is an expectation conditional on the information $\mathcal{F}_t := \sigma(\mathcal{L}_t)$.

# Example (Linear risk sharing rule)

At time $t$, each individual $j \in \mathcal{L}_{t-1}$ receives the mortality credit (respectively death benefit):

$$\beta_j(X(t)) = \frac{q_{x_j+t-1} \cdot c_j(t-1)}{\sum_{j \in \mathcal{L}_{t-1}} q_{x_j+t-1} \cdot c_j(t-1)} \cdot X(t) \,. \tag{8}$$

(see, e.g., Donnelly, Guillén, Nielsen [2013, 2014], Schumacher [2018]

# Example (Linear regression rule)

At time $t$, each individual $j \in \mathcal{L}_{t-1}$ receives the mortality credit (respectively death benefit):

$$\beta_j(X(t)) = \mathbb{E}_{t-1}[\,X_j(t)\,] + \frac{\mathrm{Cov}_{t-1}[X_j(t), X(t)]}{\mathrm{Var}_{t-1}[X(t)]}\big(X(t) - \mathbb{E}_{t-1}[\,X(t)\,]\big) \,. \tag{9}$$

## Example (Conditional mean risk sharing rule)

At time $t$, each individual $j \in \mathcal{L}_{t-1}$ receives the mortality credit (respectively death benefit):

$$\beta_j\big(X(t)\big) = \mathbb{E}_{t-1}[\,X_j(t)\,|\,X(t)\,]\,. \tag{10}$$

(see, e.g., Denuit and Dhaene [2012], Denuit [2019])

Individual $j \in \mathcal{L}_t$'s time-$t$ account value is given by:

$$c_j(t) = \sum_{u=t+1}^{\omega - x_j} e^{-\int_t^u \delta_s \mathrm{d}s} s_j(u) \,. \tag{11}$$

How do we choose $s_j(u)$, $u = 1, 2, \dots, \omega - x_j$?

For example, choose the average payoff to be constant, equal to $b_j > 0$:

$$\begin{aligned} \mathbb{E}_{t-1}[W_j(t) \,|\, j \in \mathcal{L}_t] &= \mathbb{E}_{t-1}\left[\mathbb{1}_{j \in \mathcal{L}_t} \cdot s_j(t) + \mathbb{1}_{j \in \mathcal{L}_{t-1}} \cdot \beta_j\big(X(t)\big) \,|\, j \in \mathcal{L}_t\right] \\ &= s_j(t) + \mathbb{E}_{t-1}\left[\beta_j\big(X(t)\big)\right] \\ &= s_j(t) + q_{x_j+t-1} e^{\int_{t-1}^t \delta_s \mathrm{d}s} c_j(t-1) \stackrel{!}{=} b_j \,. \end{aligned} \tag{12}$$

*((12) is a system of equations backwards in time!)*