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Generating unfavourable VaR scenarios with patchwork copulas

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Agenda

- Introduction
- Particular Examples
- A General Framework
- Case Study
- References

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Introduction

Motivation: Create unfavourable scenarios e.g. under Solvency II (i.e. with super-additive VaR, no diversification effects), in particular in the Non-Life segments or for reinsurance purposes (Nat-Cat portfolios)

- Find a "simple" copula construction under which the risk portfolio is in general *unfavourable*

- Give an explicit algorithm for corresponding Monte Carlo studies

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Introduction

Lemma 1. Let, for $d \geq 2$, $d \in \mathbb{N}$, $\mathbf{U} = (U_1, \cdots, U_d)$ and $\mathbf{V} = (V_1, \cdots, V_d)$ be d-dimensional random vectors over $[0,1]^d$ with continuous uniform margins (i.e., **U** and **V** represent d-dimensional copulas).

Let further I denote a binomially distributed random variable, independent of **U** and **V**, with $P(I=1) = p = 1-\beta \in (0,1)$. Then the random vector **W** with components

$$W_i := I \cdot p \cdot U_i + (1-I) \cdot [p + (1-p) \cdot V_i] \text{ for } 1 \leq i \leq d$$

also has continuous uniform margins, i.e. **W** represents a d-dimensional copula (patchwork copula, "ordinal sum").

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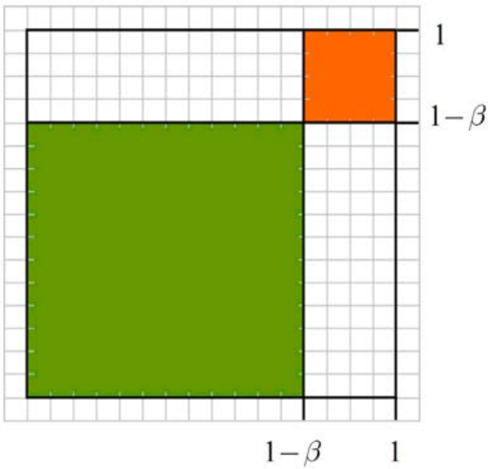
Introduction



Shape of the support of the underlying patchwork copula

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Particular Examples

General assumption: **U** and **V** have independent components

Example 1: Risks X_1 and X_2 with standard exponential distributions, $S = X_1 + X_2$, $f_S(x,\beta)$ corresponding density

$$f_S(x,\beta) = \begin{cases} \dfrac{xe^{-x}}{1-\beta}, & 0 \le x \le -\ln(\beta) \\ \dfrac{\left(-2\ln(\beta) - x\right)e^{-x}}{1-\beta}, & -\ln(\beta) \le x \le -2\ln(\beta) \\ \dfrac{\left(x + 2\ln(\beta)\right)e^{-x}}{\beta} & x \ge -2\ln(\beta) \end{cases}$$

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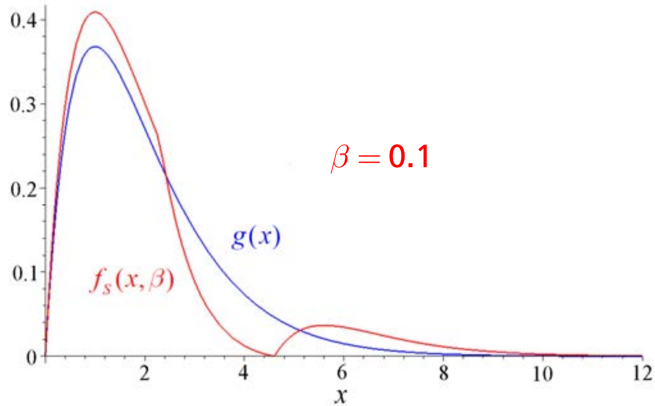
Particular Examples

Example 1: Risks X_1 and X_2 with standard exponential distributions, $S = X_1 + X_2$, $F_S(x,\beta)$ corresponding cumulative distribution function (cdf)

$$F_S(x,\beta) = \begin{cases} \dfrac{1-(1+x)e^{-x}}{1-\beta}, & 0 \le x \le -\ln(\beta) \\ \dfrac{1-2\beta+2e^{-x}\ln(\beta)+(1+x)e^{-x}}{1-\beta}, & -\ln(\beta) \le x \le -2\ln(\beta) \\ \dfrac{\beta-2e^{-x}\ln(\beta)-(1+x)e^{-x}}{\beta} & x \ge -2\ln(\beta). \end{cases}$$

Particular Examples

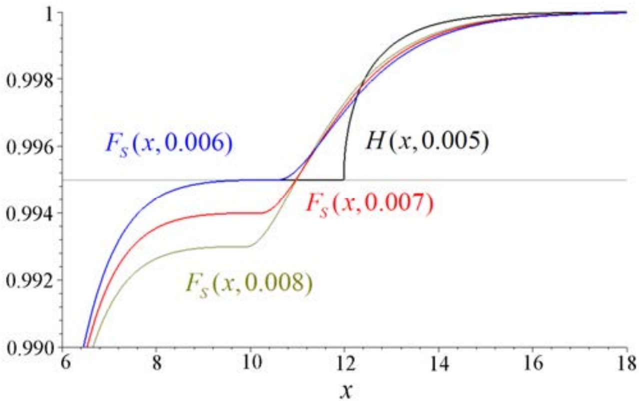
Example 1: Risks X_1 and X_2 with standard exponential distributions, $S = X_1 + X_2$, $f_S(x,\beta)$ corresponding density, g Gamma density for independent summands



Particular Examples

Example 1: Risks X_1 and X_2 with standard exponential distributions, $S = X_1 + X_2$, $F_S(x,\beta)$ corresponding cdf, $H_S(x,\beta)$ worst VaR cdf (V = lower Fréchet bound)

$\alpha = 0.005$;
Solvency II standard



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Particular Examples

Example 1: Risks X_1 and X_2 with standard exponential distributions, $S = X_1 + X_2$ (with patchwork copulas)

$\alpha = 0.005$	β				
	0.0050	0.0060	0.0068	0.0070	0.0080
$\text{VaR}_\alpha(S)$	10.5914	10.9630	10.9829	10.9821	10.9618
$\text{VaR}_\alpha(S)$ (independence)	7.4301	7.4301	7.4301	7.4301	7.4301
worst $\text{VaR}_\alpha(S)$	11.9829	11.9829	11.9829	11.9829	11.9829
$\text{VaR}_\alpha(X_1) + \text{VaR}_\alpha(X_2)$	10.5966	10.5966	10.5966	10.5966	10.5966

diversification   no diversification

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Particular Examples

Example 2: Risks X_1 and X_2 with standard Pareto distributions, $S = X_1 + X_2$, $F_S(x,\beta)$ corresponding cumulative distribution function (cdf)

$$F_S(x,\beta) = \begin{cases} \dfrac{x^2 + 2x - 2\ln(1+x)}{(2+x)^2(1-\beta)}, & 0 \le x \le \dfrac{1}{\beta} - 1 \\[2ex] \dfrac{(1-2\beta)x^2 + (4-6\beta)x - 4\beta + 4 + 2\ln\left(\beta x + 2\beta - 1\right)}{(2+x)^2(1-\beta)}, & \dfrac{1}{\beta} - 1 \le x \le 2\left(\dfrac{1}{\beta} - 1\right) \\[2ex] \dfrac{x^2 - 2x + \dfrac{2}{\beta}\ln\left(\beta x + 2\beta - 1\right)}{(2+x)^2} & x \ge 2\left(\dfrac{1}{\beta} - 1\right). \end{cases}$$

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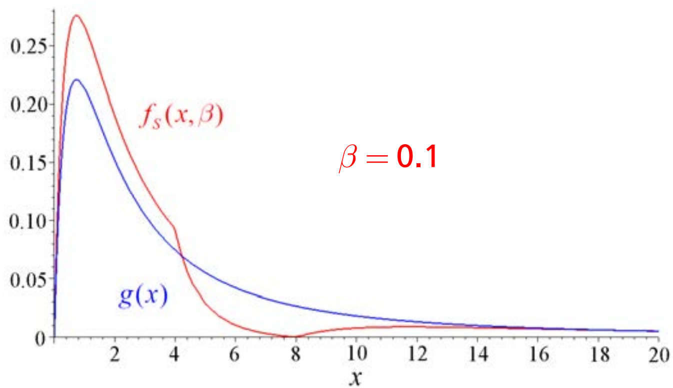
Particular Examples

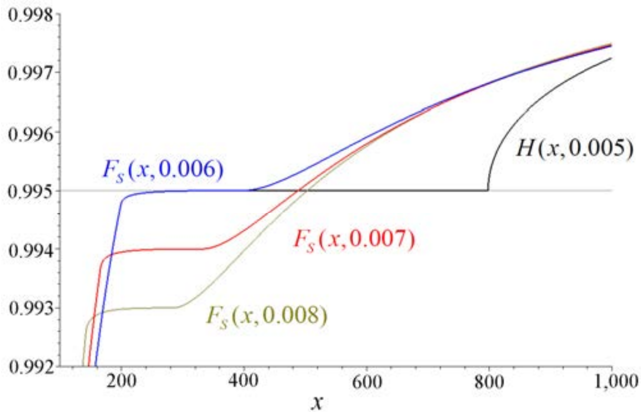
Example 1: Risks X_1 and X_2 with standard Pareto distributions, $S = X_1 + X_2$, $f_S(x,\beta)$ corresponding density, g density for independent summands



Particular Examples

Example 2: Risks X_1 and X_2 with standard Pareto distributions, $S = X_1 + X_2$, $F_S(x,\beta)$ corresponding cdf, $H_S(x,\beta)$ worst VaR cdf (V = lower Fréchet bound)

$\alpha = 0.005$; Solvency II standard



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Particular Examples

Example 2: Risks X_1 and X_2 with standard Pareto distributions, $S = X_1 + X_2$ (with patchwork copulas)

$\alpha = 0.005$	β				
	0.0050	0.0070	0.0089	0.0100	0.0110
$\text{VaR}_\alpha(S)$	397.3168	503.2848	509.3804	508.6489	507.0076
$\text{VaR}_\alpha(S)$ (independence)	403.9161	403.9161	403.9161	403.9161	403.9161
worst $\text{VaR}_\alpha(S)$	798.0000	798.0000	798.0000	798.0000	798.0000
$\text{VaR}_\alpha(X_1) + \text{VaR}_\alpha(X_2)$	398.0000	398.0000	398.0000	398.0000	398.0000

diversification   no diversification

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A General Framework

Lemma 2. For $d \in \mathbb{N}, d > 1$ let $\mathbf{I}_d$ denote the d-dimensional unit matrix, $\mathbf{e}_d = (1,\cdots,1)$ the d-dimensional row vector consisting of just ones, and $\mathbf{E}_d = \mathbf{e}_d^{tr}\mathbf{e}_d$ the $d \times d$ matrix with all entries equal to unity. Then

$$\Sigma_d = (1-r)\mathbf{I}_d + r\mathbf{E}_d = \begin{pmatrix} 1 & r & r & \cdots & r \\ r & 1 & r & \cdots & r \\ r & r & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & 1 & r \\ r & r & \cdots & r & 1 \end{pmatrix}$$

is a correlation matrix iff $-\dfrac{1}{d-1} \le r \le 1.$

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A General Framework

In the general case, the latent roots λ_i of Σ_d are given by

$$\lambda_1 = 1 + (d-1)r \text{ and } \lambda_i = 1 - r,\ i = 2, \cdots, d.$$

An orthonormal basis $T_1, \cdots, T_d$ of corresponding latent vectors is given by

$T_1 = \frac{1}{\sqrt{d}} \mathbf{e}^{tr}$ and $T_j = \left(t_{1j}, \cdots, t_{dj}\right)^{tr}$ for $2 \le j \le d$ where

$$t_{ij} = \begin{cases} -\frac{1}{\sqrt{j(j-1)}}, & 1 \le i < j \\ \sqrt{\frac{j-1}{j}}, & j = i \\ 0, & i > j. \end{cases}$$

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A General Framework

Hence Σ_d possesses the spectral decomposition $\Sigma_d = \mathbf{A}\mathbf{A}^{tr}$ with $\mathbf{A} = \mathbf{T}\sqrt{\Delta}$ where $\mathbf{T} = [T_1, \cdots, T_d]$ and $\Delta = \operatorname{diag}(\lambda_1, \cdots, \lambda_d)$.

Note: Σ_d is a particular symmetric Toeplitz matrix for which the latent roots λ_i^* and normalized latent vectors T_i^* can alternatively be expressed via trigonometric functions, i.e.

$$\lambda_j^* = 1 + r\sum_{i=1}^{d-1}\cos\left(\frac{2\pi ij}{d}\right) = \begin{cases} 1-r, & j = 1,\cdots,d-1 \\ 1+(d-1)r, & j = d \end{cases}$$

and

$$t_{ij}^* = \frac{\cos\left(\frac{2\pi ij}{d}\right) + \sin\left(\frac{2\pi ij}{d}\right)}{\sqrt{d}}, \quad 1 \le i, j \le d.$$

A General Framework

We call a Gaussian copula derived from the correlation matrix

$$\Sigma_d = \frac{d}{d-1}\mathbf{I}_d - \frac{1}{d-1}\mathbf{E}_d \text{ for } r = -\frac{1}{d-1}$$

a *minimal correlation Gaussian copula*.

Note that the corresponding multivariate normal distribution is degenerated since Σ_d is singular, i.e. a random vector $\mathbf{X}$ with zero mean and correlation matrix Σ_d has the representation $\mathbf{X} = A\mathbf{Y}$ where $\mathbf{Y}$ has a standard multivariate normal distribution with mean zero and variance-covariance matrix $\mathbf{I}_d$. For $d = 2$, the minimal correlation Gaussian copula is identical to the lower Fréchet bound or countermonotonicity copula.

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Case Study

The following study shows the effects of such an approach for the 19-dimensional data set discussed in Pfeifer et al. (2019).

It contains insurance losses from a non-life portfolio of natural perils in $d = 19$ adjacent areas in central Europe over a time period of 20 years.

The losses are given in million monetary units (MMU).

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Case Study

Year	Area 1	Area 2	Area 3	Area 4	Area 5	Area 6	Area 7	Area 8	Area 9	Area 10
1	23.664	154.664	40.569	14.504	10.468	7.464	22.202	17.682	12.395	18.551
2	1.080	59.545	3.297	1.344	1.859	0.477	6.107	7.196	1.436	3.720
3	21.731	31.049	55.973	5.816	14.869	20.771	3.580	14.509	17.175	87.307
4	28.99	31.052	30.328	4.709	0.717	3.530	6.032	6.512	0.682	3.115
5	53.616	62.027	57.639	1.804	2.073	4.361	46.018	22.612	1.581	11.179
6	29.95	41.722	12.964	1.127	1.063	4.873	6.571	11.966	15.676	24.263
7	3.474	14.429	10.869	0.945	2.198	1.484	4.547	2.556	0.456	1.137
8	10.020	31.283	21.116	1.663	2.153	0.932	25.163	3.222	1.581	5.477
9	5.816	14.804	128.072	0.523	0.324	0.477	3.049	7.791	4.079	7.002
10	170.725	576.767	108.361	41.599	20.253	35.412	126.698	71.079	21.762	64.582
11	21.423	50.595	4.360	0.327	1.566	64.621	5.650	1.258	0.626	3.556
12	6.380	28.316	3.740	0.442	0.736	0.470	3.406	7.859	0.894	3.591
13	124.665	33.359	14.712	0.321	0.975	2.005	3.981	4.769	2.006	1.973
14	20.165	49.948	17.658	0.595	0.548	29.350	6.782	4.873	2.921	6.394
15	78.106	41.681	13.753	0.585	0.259	0.765	7.013	9.426	2.180	3.769
16	11.067	444.712	365.351	99.366	8.856	28.654	10.589	13.621	9.589	19.485
17	6.704	81.895	14.266	0.972	0.519	0.644	8.057	18.071	5.515	13.163
18	15.550	277.643	26.564	0.788	0.225	1.230	26.800	64.538	2.637	80.711
19	10.099	18.815	9.352	2.051	1.089	6.102	2.678	4.064	2.373	2.057
20	8.492	138.708	46.708	3.680	1.132	1.698	165.600	7.926	2.972	5.237

Loss data (I)

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Case Study

Year	Area 11	Area 12	Area 13	Area 14	Area 15	Area 16	Area 17	Area 18	Area 19
1	1.842	4.100	46.135	14.698	44.441	7.981	35.833	10.689	7.299
2	0.429	1.026	7.469	7.058	4.512	0.762	14.474	9.337	0.740
3	0.209	2.344	22.651	4.117	26.586	3.920	13.804	2.683	3.026
4	0.521	0.696	31.126	1.878	29.423	6.394	18.064	1.201	0.894
5	2.715	1.327	40.156	4.655	104.691	28.579	17.832	1.618	3.402
6	4.832	0.701	16.712	11.852	29.234	7.098	17.866	5.206	5.664
7	0.268	0.580	11.851	2.057	11.605	0.282	16.925	2.082	1.008
8	0.741	0.369	3.814	1.869	8.126	1.032	14.985	1.390	1.703
9	0.524	6.554	5.459	3.007	8.528	1.920	5.638	2.149	2.908
10	9.882	6.401	106.197	44.912	191.809	90.559	154.492	36.626	36.276
11	1.052	8.277	22.564	8.961	19.817	16.437	25.990	2.364	6.434
12	0.136	0.364	28.000	7.574	3.213	1.749	12.735	1.744	0.558
13	1.990	15.176	57.235	23.686	110.035	17.373	7.276	2.494	0.525
14	0.630	0.762	25.897	3.439	8.161	3.327	24.733	2.807	1.618
15	0.770	15.024	36.068	1.613	6.127	8.103	12.596	4.894	0.822
16	0.287	0.464	24.211	38.616	51.889	1.316	173.080	3.557	11.627
17	0.590	2.745	16.124	2.398	20.997	2.515	5.161	2.840	3.002
18	0.245	0.217	12.416	4.972	59.417	3.762	24.603	7.404	19.107
19	0.415	0.351	10.707	2.468	10.673	1.743	27.266	1.368	0.644
20	0.566	0.708	22.646	6.652	14.437	63.692	113.231	7.218	2.548

Loss data (II)

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Case Study

A statistical analysis of the data shows a good fit to lognormal $\mathcal{LN}(\mu,\sigma)$-distributions for the losses per Area k, $k=1,\cdots,19$. The parameters μ_k and σ_k for Area k were hence estimated from the log data by calculating means and standard deviations.

Parameter	Area 1	Area 2	Area 3	Area 4	Area 5	Area 6	Area 7	Area 8	Area 9	Area 10
μ_k	2.8063	4.0717	3.1407	0.6375	0.3984	1.2227	2.3210	2.2123	1.0783	2.1055
σ_k	1.2161	1.0521	1.2110	1.5685	1.2998	1.5987	1.1980	0.9882	1.1445	1.2531

Parameter	Area 11	Area 12	Area 13	Area 14	Area 15	Area 16	Area 17	Area 18	Area 19
μ_k	–0.3231	0.3815	3.0198	1.7488	3.0409	1.5501	3.0700	1.2444	0.9378
σ_k	1.0881	1.3353	0.8027	1.0033	1.1221	1.4765	0.9622	0.8577	1.2141

Distributional parameters for fitted lognormal loss distributions

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Case Study

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	A19
A1	1	0.46	0.03	0.16	0.47	0.20	0.35	0.49	0.41	0.24	0.78	0.64	0.91	0.63	0.85	0.66	0.30	0.67	0.56
A2	0.46	1	0.64	0.78	0.67	0.36	0.51	0.76	0.57	0.51	0.58	-0.04	0.59	0.84	0.68	0.58	0.87	0.77	0.90
A3	0.03	0.64	1	0.93	0.41	0.26	0.11	0.16	0.33	0.16	0.08	-0.09	0.13	0.64	0.25	0.10	0.74	0.14	0.35
A4	0.16	0.78	0.93	1	0.54	0.36	0.16	0.25	0.43	0.19	0.22	-0.10	0.30	0.79	0.36	0.19	0.84	0.32	0.49
A5	0.47	0.67	0.41	0.54	1	0.41	0.35	0.51	0.84	0.63	0.59	0.02	0.64	0.67	0.59	0.50	0.58	0.71	0.67
A6	0.20	0.36	0.26	0.36	0.41	1	0.07	0.11	0.28	0.19	0.28	0.14	0.31	0.42	0.24	0.27	0.39	0.27	0.40
A7	0.35	0.51	0.11	0.16	0.35	0.07	1	0.44	0.27	0.19	0.48	-0.07	0.46	0.35	0.45	0.91	0.64	0.61	0.49
A8	0.49	0.76	0.16	0.25	0.51	0.11	0.44	1	0.50	0.75	0.61	-0.03	0.54	0.47	0.71	0.53	0.40	0.75	0.90
A9	0.41	0.57	0.33	0.43	0.84	0.28	0.27	0.50	1	0.66	0.68	-0.01	0.52	0.60	0.50	0.41	0.46	0.65	0.63
A10	0.24	0.51	0.16	0.19	0.63	0.19	0.19	0.75	0.66	1	0.33	-0.12	0.27	0.28	0.43	0.24	0.23	0.45	0.65
A11	0.78	0.58	0.08	0.22	0.59	0.28	0.48	0.61	0.68	0.33	1	0.19	0.79	0.65	0.80	0.73	0.43	0.84	0.74
A12	0.64	-0.04	-0.09	-0.10	0.02	0.14	-0.07	-0.03	-0.01	-0.12	0.19	1	0.44	0.21	0.28	0.17	-0.12	0.13	0.03
A13	0.91	0.59	0.13	0.30	0.64	0.31	0.46	0.54	0.52	0.27	0.79	0.44	1	0.71	0.86	0.74	0.47	0.76	0.65
A14	0.63	0.84	0.64	0.79	0.67	0.42	0.35	0.47	0.60	0.28	0.65	0.21	0.71	1	0.74	0.54	0.79	0.68	0.72
A15	0.85	0.68	0.25	0.36	0.59	0.24	0.45	0.71	0.50	0.43	0.80	0.28	0.86	0.74	1	0.69	0.47	0.71	0.75
A16	0.66	0.58	0.10	0.19	0.50	0.27	0.91	0.53	0.41	0.24	0.73	0.17	0.74	0.54	0.69	1	0.63	0.77	0.64
A17	0.30	0.87	0.74	0.84	0.58	0.39	0.64	0.40	0.46	0.23	0.43	-0.12	0.47	0.79	0.47	0.63	1	0.59	0.64
A18	0.67	0.77	0.14	0.32	0.71	0.27	0.61	0.75	0.65	0.45	0.84	0.13	0.76	0.68	0.71	0.77	0.59	1	0.86
A19	0.56	0.90	0.35	0.49	0.67	0.40	0.49	0.90	0.63	0.65	0.74	0.03	0.65	0.72	0.75	0.64	0.64	0.86	1

Correlations between losses

yellow = high, green = low

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Case Study

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	A19
A1	1	0.27	0.30	0.16	0.17	0.45	0.28	0.32	0.32	0.29	0.67	0.51	0.76	0.34	0.67	0.74	0.18	0.21	0.29
A2	0.27	1	0.48	0.66	0.39	0.37	0.71	0.69	0.52	0.64	0.30	-0.02	0.45	0.66	0.58	0.45	0.73	0.74	0.78
A3	0.30	0.48	1	0.70	0.40	0.31	0.42	0.51	0.58	0.53	0.18	0.07	0.21	0.32	0.54	0.26	0.47	0.21	0.57
A4	0.16	0.66	0.70	1	0.77	0.47	0.46	0.47	0.59	0.49	0.18	-0.13	0.33	0.50	0.47	0.18	0.76	0.43	0.54
A5	0.17	0.39	0.40	0.77	1	0.59	0.30	0.20	0.49	0.39	0.28	0.08	0.35	0.56	0.44	0.16	0.55	0.36	0.41
A6	0.45	0.37	0.31	0.47	0.59	1	0.14	0.01	0.36	0.34	0.33	0.12	0.48	0.46	0.48	0.37	0.59	0.17	0.50
A7	0.28	0.71	0.42	0.46	0.30	0.14	1	0.52	0.27	0.40	0.45	-0.07	0.31	0.31	0.46	0.62	0.63	0.58	0.57
A8	0.32	0.69	0.51	0.47	0.20	0.01	0.52	1	0.64	0.81	0.27	-0.02	0.38	0.35	0.56	0.35	0.28	0.62	0.63
A9	0.32	0.52	0.58	0.59	0.49	0.36	0.27	0.64	1	0.78	0.40	0.19	0.27	0.50	0.44	0.30	0.33	0.57	0.61
A10	0.29	0.64	0.53	0.49	0.39	0.34	0.40	0.81	0.78	1	0.21	-0.02	0.21	0.37	0.52	0.30	0.31	0.53	0.81
A11	0.67	0.30	0.18	0.18	0.28	0.33	0.45	0.27	0.40	0.21	1	0.47	0.49	0.45	0.60	0.67	0.20	0.45	0.39
A12	0.51	-0.02	0.07	-0.13	0.08	0.12	-0.07	-0.02	0.19	-0.02	0.47	1	0.44	0.21	0.24	0.46	-0.23	0.25	0.05
A13	0.76	0.45	0.21	0.33	0.35	0.48	0.31	0.38	0.27	0.21	0.49	0.44	1	0.55	0.60	0.71	0.37	0.39	0.24
A14	0.34	0.66	0.32	0.50	0.56	0.46	0.31	0.35	0.50	0.37	0.45	0.21	0.55	1	0.59	0.43	0.57	0.58	0.53
A15	0.67	0.58	0.54	0.47	0.44	0.48	0.46	0.56	0.44	0.52	0.60	0.24	0.60	0.59	1	0.59	0.36	0.35	0.63
A16	0.74	0.45	0.26	0.18	0.16	0.37	0.62	0.35	0.30	0.30	0.67	0.46	0.71	0.43	0.59	1	0.38	0.43	0.39
A17	0.18	0.73	0.47	0.76	0.55	0.59	0.63	0.28	0.33	0.31	0.20	-0.23	0.37	0.57	0.36	0.38	1	0.52	0.56
A18	0.21	0.74	0.21	0.43	0.36	0.17	0.58	0.62	0.57	0.53	0.45	0.25	0.39	0.58	0.35	0.43	0.52	1	0.60
A19	0.29	0.78	0.57	0.54	0.41	0.50	0.57	0.63	0.61	0.81	0.39	0.05	0.24	0.53	0.63	0.39	0.56	0.60	1

Correlations between log losses

yellow = high, green = low

Case Study

The following graphs for the tail cdf's correspond to a rank based Bernstein copula **U** with a minimal correlation Gaussian copula **V**:

$p = 1$ $[F_1(x)]$ (independence case); $p = 0.99$ $[F_2(x)]$; $p = 0.994$ $[F_3(x)]$

and a rank based Bernstein copula **U** with

$p = 0.994$

but different copulas **V**:
upper Fréchet bound $[F_4(x)]$; independence copula $[F_5(x)]$.

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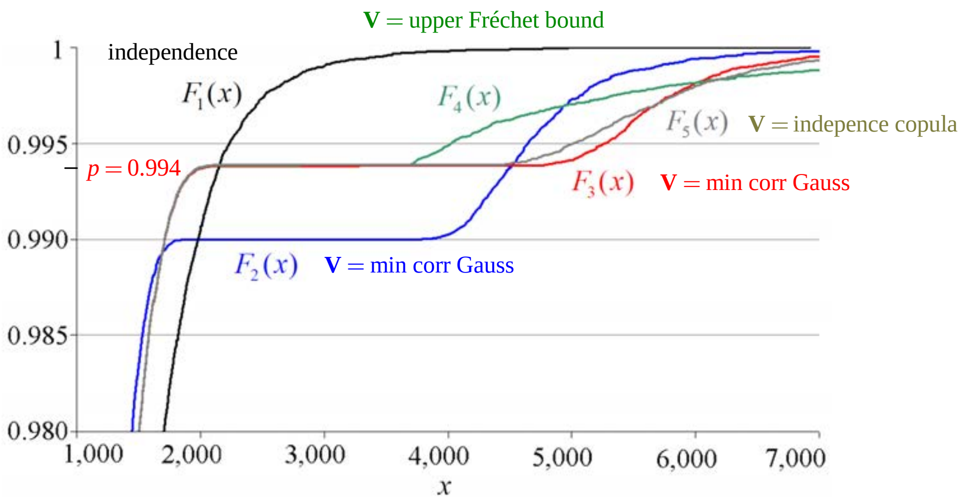
Case Study



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Case Study

p	0.990	0.994	0.994	0.994	1
V	min corr Gauss	min corr Gauss	upper Fréchet	independence	---
$\mathrm{VaR}_\alpha(S)$	4,647 MMU	5,272 MMU	3,976 MMU	5,018 MMU	2,229 MMU
$\sum_{k=1}^{19}\mathrm{VaR}_\alpha(X_k)$	3,976 MMU	3,976 MMU	3,976 MMU	3,976 MMU	3,976 MMU

$$\alpha = 0.005$$

As can clearly be seen, the patchwork construction with the minimal correlation Gaussian copula representing V with no tail dependence gives the largest VaR estimate here and is typically larger than for the construction with the upper Fréchet bound which has a positive tail dependence. Note that using the Bernstein copula alone would lead to a truly diversified portfolio while all other copula models do not.

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