Capital requirements modeling for market and non-life premium risk in a dynamic insurance portfolio

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Introduction

Goal
The cash flows produced by the insurance business are invested and consequently they create a risk. As a result, not only do insurance companies face underwriting risk, but also market risk. In this regard, the aim of our paper is to describe a way of modeling the distributions of the annual rate of return and aggregate claim amount, in order to calculate the capital requirements for market and non-life premium risk and to illustrate their combined effect. We tackled a numerical analysis for a single-line and a multi-line insurance company in a multi-annual dynamic perspective and we calculated the Solvency Capital Requirements according to the Solvency II Standard Formula.

Data
We estimated the parameters of our numerical analysis, using current and available market data. For the financial side, we used the average of all the Euro Area government rates and European swaption-volatility quotes on September 30, 2020, and the Euro Stoxx 50 quotes between September 30, 2010, and September 30, 2020. For the technical side, we used Italian market data, provided by the National Association of Insurance Companies (ANIA) and Italian Insurance Supervisory Authority (IVASS).
The stochastic \textbf{risk reserve} (see [7]) at the end of the time $t$, in absence of reserve risk, reinsurance, taxes, dividends and premium reserve, is given by:

$$U_t = (1 + j_t)U_{t-1} + \left(B_t - X_t - E_t\right) + j_t L_{t-1}$$

where $j_t$ is the stochastic annual rate of return of the investments of the insurance company, $B_t$ is the gross premium amount, $X_t$ is the stochastic aggregate claim amount, $E_t$ is the expense amount and $L_{t-1}$ is the claims reserve at the end of the previous year. The gross premium amount, stochastic aggregate claim amount and expense amount are not capitalized, since they are accounted as realized at the end of the year.

The \textbf{gross premium amount} is given by:

$$B_t = \pi_t + \lambda \pi_t + c B_t$$

where $\pi_t$ is the risk premium amount, $\lambda$ is the safety loading coefficient and $c$ is the expense loading coefficient.
Risk reserve

We assume that the expense amount is equal to the expense loadings and that the claims reserve is equal to a constant percentage $\delta$ of the gross premium amount:

$$E_t = c B_t \quad \text{and} \quad L_t = \delta B_t$$

Since the insurance portfolio is dynamic (see [27]), we assume that the risk premium amount and the gross premium amount increase every year:

$$\pi_t = \pi_{t-1} (1 + i) (1 + g) \quad \text{and} \quad B_t = B_{t-1} (1 + i) (1 + g)$$

where $i$ is the claims inflation rate and $g$ is the real growth rate.

The stochastic annual net cash flows originated by the insurance business are given by:

$$F_t = B_t - E_t - \left( C_t^{CY} + C_t^{PY} \right) = B_t \left[ (1 - c) + \delta \left( 1 - \frac{1}{(1 + i)(1 + g)} \right) \right] - X_t$$

where $C_t^{CY}$ is the stochastic amount paid for the claims occurred in the current year and settled in the same year and $C_t^{PY}$ is the stochastic amount paid for the claims occurred in the previous years and settled in the current year.
We deal with three investments in non-dividend-paying stocks and five investments in risk-free zero-coupon bonds with time to maturity \( w = 1, 2, 3, 5, 10 \). Furthermore, we assume that the asset allocation is kept constant year by year.

The stochastic asset value of the portfolio at the end of time \( t \) is obtained from the combination of the stochastic values of the stock and bond portfolios:

\[
A_t = A^S_t + A^P_t \quad \text{with} \quad A_0 = U_0 + L_0
\]

The stochastic value of the stock portfolio is given by:

\[
A^S_t = \alpha \left( A_{t-1} \sum_{v=1}^{3} \beta_v \frac{S_v(t)}{S_v(t-1)} + F_t \right) = \alpha \left( A_{t-1} \frac{S_t}{S_{t-1}} + F_t \right)
\]

The stochastic value of the bond portfolio is given by:

\[
A^P_t = (1 - \alpha) \left( A_{t-1} \sum_{w \in \{1,2,3,5,10\}} \gamma_w \frac{P(t, t-1 + w)}{P(t-1, t-1 + w)} + F_t \right) = (1 - \alpha) \left( A_{t-1} \frac{P_t}{P_{t-1}} + F_t \right)
\]
As a result, the stochastic **asset value of the portfolio** is found to be:

\[
A_t = \alpha \left( A_{t-1} \frac{S_t}{S_{t-1}} + F_t \right) + (1 - \alpha) \left( A_{t-1} \frac{P_t}{P_{t-1}} + F_t \right)
\]

where \( \alpha \) and \( 1 - \alpha \) are the percentages invested in the stock and bond portfolios respectively. Moreover, \( \beta_v \) is the percentage invested in the \( v \)-th stock, so that \( S_v \) is the stochastic average stock price, and \( \gamma_w \) is the percentage invested in the bond with time to maturity \( w \), so that \( P_w \) is the stochastic average bond price.

In conclusion, the stochastic **annual rate of return** of the investments of the insurance company is given by:

\[
j_t = \frac{(A_t - F_t) - A_{t-1}}{A_{t-1}} = \alpha \frac{S_t}{S_{t-1}} + (1 - \alpha) \frac{P_t}{P_{t-1}} - 1
\]
We assume that the market is frictionless, meaning that all securities are perfectly divisible and that no short-sale restrictions, transaction costs, or taxes are present. The security trading is continuous and there are no riskless arbitrage opportunities.

The real-world process for the $v$-th non-dividend-paying stock price (see [17]) follows a geometric Brownian motion that satisfies the following stochastic differential equation:

$$
\text{d}S_v(t) = \mu_v S_v(t) \text{d}t + \sigma_v S_v(t) \text{d}W^\mathbb{P}_v(t)
$$

where $\mu_v$ is the drift coefficient, $\sigma_v > 0$ is the diffusion coefficient and $W^\mathbb{P}_v(t)$ is the standard Brownian motion under the real measure $\mathbb{P}$.

The short-rate follows the Two-Additive-Factor Gaussian model (i.e. $\text{G2}++$ model) that is given by:

$$
r(t) = x(t) + y(t) + \varphi(t)
$$

where $x(t)$ and $y(t)$ are the stochastic state variables and $\varphi(t)$ is a deterministic function of time that allows the model to fit perfectly the term structure observed in the market.
The **real-world processes** for the state variables (see [4]) satisfy the following stochastic differential equations:

\[
\begin{align*}
    \frac{dx(t)}{dt} &= a \left( d_x(t) - x(t) \right) dt + \sigma \, dW^P_x(t) \quad \text{with} \quad x(0) = 0 \\
    \frac{dy(t)}{dt} &= b \left( d_y(t) - y(t) \right) dt + \eta \, dW^P_y(t) \quad \text{with} \quad y(0) = 0
\end{align*}
\]

The corresponding **risk-neutral processes** for the state variables (see [4]) satisfy the following stochastic differential equations:

\[
\begin{align*}
    \frac{dx(t)}{dt} &= -a \, x(t) \, dt + \sigma \, dW^Q_x(t) \quad \text{with} \quad x(0) = 0 \\
    \frac{dy(t)}{dt} &= -b \, y(t) \, dt + \eta \, dW^Q_y(t) \quad \text{with} \quad y(0) = 0
\end{align*}
\]

where \(a > 0\) and \(b > 0\) are the constant speeds of mean reversion, \(d_x(t)\) and \(d_y(t)\) are the deterministic mean reversion levels, \(\sigma > 0\) and \(\eta > 0\) are the constant diffusion coefficients, \(W^P_x(t)\) and \(W^P_y(t)\) are the standard Brownian motions under the real measure \(\mathbb{P}\) with instantaneous correlation \(-1 \leq \rho \leq 1\) and \(W^Q_x(t)\) and \(W^Q_y(t)\) are the standard Brownian motions under the risk-neutral measure \(\mathbb{Q}\).
The stochastic **aggregate claim amount** (see [7]) at the end of the time \( t \) is given by the following collective risk model:

\[
X_t = \sum_{k=1}^{K_t} Z_{k,t}
\]

where \( K_t \) is the stochastic number of claims and \( Z_{k,t} \) is the stochastic amount for the \( k \)-th claim. We assume that:

1. the random variables \( Z_{k,t} \) are independent;
2. the random variables \( Z_{k,t} \) are identically distributed;
3. the random variables \( Z_{k,t} \) and \( K_t \) are independent.

The first and second assumption are satisfied, in particular, for limited time periods. The third assumption is usually satisfied, but it could be refuted in some situation. In case of storm, for example, the number of claims and single claim amount increase significantly at the same time.
Number of claims

There are several distributions associated with the number of claims, such as the Poisson and Negative Binomial. We point out that the aggregate claim amount is found to be zero, in case that the number of claims is zero as well.

We assume that the number of claims is distributed as a Mixed Poisson, i.e. a Poisson with stochastic parameter \( n_t > 0 \), such that:

\[
    n_t = n_t \cdot q
\]

In order to describe the number of claims, considering the short-term fluctuations only, we assume that \( q \) is distributed as a Gamma with equal parameters \((h, h)\) and with mean, variance and skewness given by:

\[
    \mathbb{E}(q) = 1 \quad \text{and} \quad \mathbb{V}(q) = \frac{1}{h} \quad \text{and} \quad \mathbb{S}(q) = \frac{2}{\sqrt{h}} = 2 \cdot \sqrt{\mathbb{V}(q)}
\]

As a result, the Mixed Poisson distribution is found to be a **Negative Binomial** with parameters \((h, p_t)\) and with \( 0 < p_t = \frac{h}{h + n_t} < 1 \).
There are several distributions associated with the single claim amount, such as the Lognormal, Gamma, Weibull, Inverse Normal and Pareto. The first four distributions fit better the attritional claims, i.e. the most frequent and least expensive claims. The last distribution fits better the large claims, i.e. the least frequent and most expensive claims or catastrophic claims.

We assume that the single claim amount is distributed as a Lognormal distribution with parameters \((m_t, s)\) and it is defined by the following probability density function:

\[
f_{Z_t}(z) = \frac{1}{z \sqrt{2\pi s}} \exp \left[-\frac{1}{2} \left(\frac{\ln(z) - m_t}{s}\right)^2\right]
\]

with \(z > 0\) and \(-\infty < m_t < +\infty\) and \(s > 0\)

A popular alternative is to use distributions where the attritional and large claims are described by different random variables.
Multi-line insurance company

The capital requirements are here calculated using a Monte Carlo approach based on 100,000 simulations and using a risk measure that takes into account the expected return produced by the financial investment of the resources created by the insurance business. The minimum Risk-Based Capital over the time horizon \((0,t)\) within the confidence level \(1 - \varepsilon\) is given by:

\[
RBC(0,t) = U_0 - \frac{U_\varepsilon(t)}{\prod_{k=1}^{t} 1 + \mathbb{E}\{j_k\}}
\]

where \(U_\varepsilon(t)\) is the \(\varepsilon\)-th order quantile of the risk reserve.

We now assume that the insurance company is multi-line, because not only does it work in the Motor Third-Party Liability (MTPL) line of business, but also in the Motor Other Damages (MOD) and General Third-Party Liability (GTPL) lines of business. We thus use the Gaussian copula or Gumbel copula to inject some dependence structure in the aggregate claim amount. We now present the parameters of our numerical analysis.
Multi-line insurance company

Relevant parameters of our numerical analysis

<table>
<thead>
<tr>
<th>$U_0 / B_0$</th>
<th>$F_0$</th>
<th>$i$</th>
<th>$g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25%</td>
<td>0</td>
<td>1.5%</td>
<td>2%</td>
</tr>
</tbody>
</table>

Other relevant parameters of our numerical analysis

<table>
<thead>
<tr>
<th>$LoB$</th>
<th>$B_0$</th>
<th>$\lambda$</th>
<th>$c$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MTPL</td>
<td>50 million</td>
<td>0.87%</td>
<td>21.24%</td>
<td>156.10%</td>
</tr>
<tr>
<td>MOD</td>
<td>25 million</td>
<td>13.81%</td>
<td>30.30%</td>
<td>22.88%</td>
</tr>
<tr>
<td>GTPL</td>
<td>25 million</td>
<td>6.61%</td>
<td>32.30%</td>
<td>416.52%</td>
</tr>
<tr>
<td>Total</td>
<td>100 million</td>
<td>4.99%</td>
<td>26.27%</td>
<td>187.90%</td>
</tr>
</tbody>
</table>

Relevant indicators of the distributions of the single claim amount and number of claims

<table>
<thead>
<tr>
<th>$LoB$</th>
<th>$\mathbb{E}(Z_0)$</th>
<th>$\mathbb{CV}(Z)$</th>
<th>$n_0$</th>
<th>$\mathbb{Var}(q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MTPL</td>
<td>4,000</td>
<td>7</td>
<td>9,760</td>
<td>0.0067</td>
</tr>
<tr>
<td>MOD</td>
<td>2,500</td>
<td>2</td>
<td>6,124</td>
<td>0.0025</td>
</tr>
<tr>
<td>GTPL</td>
<td>10,000</td>
<td>12</td>
<td>1,587</td>
<td>0.0218</td>
</tr>
</tbody>
</table>
Multi-line insurance company

Parameters of the Lognormal and Negative Binomial distributions

<table>
<thead>
<tr>
<th>$LoB$</th>
<th>$m_0$</th>
<th>$s$</th>
<th>$h$</th>
<th>$p_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MTPL</td>
<td>6.3380</td>
<td>1.9779</td>
<td>148.47</td>
<td>0.0150</td>
</tr>
<tr>
<td>MOD</td>
<td>7.0193</td>
<td>1.2686</td>
<td>399.40</td>
<td>0.0612</td>
</tr>
<tr>
<td>GTPL</td>
<td>6.7220</td>
<td>2.2309</td>
<td>45.78</td>
<td>0.0280</td>
</tr>
</tbody>
</table>

Correlation coefficients of the Gaussian copula for the aggregate claim amount

<table>
<thead>
<tr>
<th>MTPL and MOD</th>
<th>MTPL and GTPL</th>
<th>MOD and GTPL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5000</td>
<td>0.5000</td>
<td>0.2500</td>
</tr>
</tbody>
</table>

Parameters of the Gumbel copula for the aggregate claim amount

<table>
<thead>
<tr>
<th>Year</th>
<th>MTPL and MOD</th>
<th>MTPL aggregated to MOD and GTPL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5000</td>
<td>1.4893</td>
</tr>
<tr>
<td>2</td>
<td>1.5000</td>
<td>1.4893</td>
</tr>
<tr>
<td>3</td>
<td>1.5000</td>
<td>1.4892</td>
</tr>
</tbody>
</table>
Multi-line insurance company

Asset allocation of our numerical analysis

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$\gamma_3$</th>
<th>$\gamma_5$</th>
<th>$\gamma_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15%</td>
<td>60%</td>
<td>30%</td>
<td>10%</td>
<td>40%</td>
<td>25%</td>
<td>15%</td>
<td>10%</td>
<td>10%</td>
</tr>
</tbody>
</table>

Real-world parameters of the geometric Brownian motions

<table>
<thead>
<tr>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
<th>$\mu_3$</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$\sigma_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0269</td>
<td>0.0359</td>
<td>0.0448</td>
<td>0.1547</td>
<td>0.2063</td>
<td>0.2579</td>
</tr>
</tbody>
</table>

Real-world parameters of the G2++ model

<table>
<thead>
<tr>
<th>$a$</th>
<th>$b$</th>
<th>$\sigma$</th>
<th>$\eta$</th>
<th>$\rho$</th>
<th>$d_x$</th>
<th>$d_y$</th>
<th>$l_x$</th>
<th>$l_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2810</td>
<td>0.0554</td>
<td>0.0154</td>
<td>0.0121</td>
<td>-0.9877</td>
<td>-0.0008</td>
<td>0.0178</td>
<td>-0.0296</td>
<td>0.0182</td>
</tr>
</tbody>
</table>

Instantaneous correlations between standard Brownian motions

<table>
<thead>
<tr>
<th>$v$-th stock and $w$-th stock</th>
<th>$v$-th stock and variable $x$</th>
<th>$v$-th stock and variable $y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7500</td>
<td>-0.0011</td>
<td>-0.0011</td>
</tr>
</tbody>
</table>
Market risk

We now isolate the effect of the market risk. In this regard, we drop the underwriting result by the risk reserve, but we still consider the interest on the investment of its average (i.e. the safety loadings). We also assume that the interest rate risk only affects the bond investments and the equity risk only affects the stock investments.

As a result, the risk reserve is found to be:

\[
U_t = (1 + j_t) U_{t-1} + j_t \delta B_{t-1} + j_t \sum_{k=1}^{t-1} \lambda \pi_k
\]

According to the Standard Formula, the Solvency Capital Requirement (SCR) for equity risk (without SA), calculated on a single stock investment, is found to be:

\[
SCR_{S_v} = 39\% A^S_0 \quad \text{with} \quad v = 1, 2, 3
\]

Hence, according to the Standard Formula, the SCR for equity risk is found to be:

\[
SCR_{equity} = \sum_{v=1}^{3} SCR_{S_v} = 39\% A^S_0
\]
Market risk

According to the Standard Formula, the **SCR for interest rate risk**, calculated on a single bond investment, is found to be:

\[
SCR_{Pw} = A_{0}^{Pw} - A_{0}^{Pw} \left[ 1 + R^{euro}_{a}(0,w) \right]^{w} \left[ 1 + R^{eiopea \text{up}}_{a}(0,w) + R^{euro}_{a}(0,w) - R^{eiopea}_{a}(0,w) \right]^{-w}
\]

with \( w = 1,2,3,5,10 \)

where \( R^{euro}_{a}(0,w) \), \( R^{eiopea}_{a}(0,w) \) and \( R^{eiopea \text{up}}_{a}(0,w) \) are respectively the average of all the spot Euro Area government rates (annually compounded) with maturity \( w \) and the spot Eiopa risk-free rates (annually compounded) without VA and with maturity \( w \), either without any shock or with positive interest rate shock.

Hence, according to the Standard Formula, the SCR for interest rate risk is found to be:

\[
SCR_{\text{interest rate}} = \sum_{\{w=1,2,3,5,10\}} SCR_{Pw}
\]

According to the Standard Formula, the **SCR for market risk** is found to be:

\[
SCR_{\text{market}} = \sqrt{SCR^{2}_{\text{equity}} + SCR^{2}_{\text{interest rate}}}
\]
Results

Capital requirements over the initial gross premium amount and percentage diversification benefits according to the Standard Formula (SF) and our Model (M) over a period of one, two and three years (values in %)

<table>
<thead>
<tr>
<th></th>
<th>Year 1 (SF)</th>
<th>Year 1 (M)</th>
<th>Year 2 (M)</th>
<th>Year 3 (M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock 1</td>
<td>7.47</td>
<td>6.01</td>
<td>6.24</td>
<td>6.44</td>
</tr>
<tr>
<td>Stock 2</td>
<td>3.74</td>
<td>3.77</td>
<td>3.86</td>
<td>3.94</td>
</tr>
<tr>
<td>Stock 3</td>
<td>1.25</td>
<td>1.48</td>
<td>1.50</td>
<td>1.52</td>
</tr>
<tr>
<td>Total stocks</td>
<td>12.45</td>
<td>11.25</td>
<td>11.60</td>
<td>11.90</td>
</tr>
<tr>
<td>Bond with maturity 1</td>
<td>0.72</td>
<td>0.35</td>
<td>4.42</td>
<td>6.36</td>
</tr>
<tr>
<td>Bond with maturity 2</td>
<td>0.90</td>
<td>0.46</td>
<td>2.60</td>
<td>3.74</td>
</tr>
<tr>
<td>Bond with maturity 3</td>
<td>0.80</td>
<td>0.43</td>
<td>1.50</td>
<td>2.15</td>
</tr>
<tr>
<td>Bond with maturity 5</td>
<td>0.88</td>
<td>0.67</td>
<td>1.08</td>
<td>1.45</td>
</tr>
<tr>
<td>Bond with maturity 10</td>
<td>1.72</td>
<td>2.00</td>
<td>2.06</td>
<td>2.28</td>
</tr>
<tr>
<td>Total bonds</td>
<td>5.02</td>
<td>3.91</td>
<td>11.65</td>
<td>15.98</td>
</tr>
<tr>
<td>MARKET RISK</td>
<td>13.43</td>
<td>12.28</td>
<td>17.32</td>
<td>21.43</td>
</tr>
<tr>
<td>Market percentage diversification benefit</td>
<td>23.15</td>
<td>19.01</td>
<td>25.51</td>
<td>23.16</td>
</tr>
</tbody>
</table>
We now isolate the effect of the non-life premium risk. In this regard, we drop the investment result by the risk reserve, assuming that the annual rate of return is null.

As a result, the risk reserve is found to be:

\[ U_t = U_{t-1} + [(1 + \lambda) \pi_t - X_t] \]

where the stochastic aggregate claim amount is obtained using the **Gaussian** or **Gumbel copula** to join the aggregate claim amounts of the lines of business.

Simulated aggregate claim amounts after one year (MTPL in red, MOD in black, GTPL in blue and x-axis values in millions)
Market and non-life premium risk

According to the Standard Formula, the **SCR for non-life premium and reserve risk** (that in our case is equal to the non-life premium risk) is found to be:

\[ SCR_{nl\text{ prem res}} = 3 \sigma_{nl} V_{nl} \]

where \( \sigma_{nl} \) and \( V_{nl} \) are the standard deviation, in relative terms, and the volume measure for non-life premium and reserve risk (respectively 8.5% and 103.53 million in our case).

We now finally take into account both the market risk and non-life premium risk, considering the original risk reserve equation.

According to the Standard Formula, the **SCR for market and non-life underwriting risk** (that in our case is equal to the non-life premium risk) is found to be:

\[ SCR = \sqrt{SCR_{market}^2 + \frac{1}{2} SCR_{market} SCR_{non-life} + SCR_{non-life}^2} \]

where \( SCR_{non-life} \) is the SCR for non-life underwriting risk.
Results using Gaussian copula

Capital requirements over the initial gross premium amount and percentage diversification benefits according to the Standard Formula (SF) and our Model (M) (using the Gaussian copula) over a period of one, two and three years (values in %)

<table>
<thead>
<tr>
<th></th>
<th>Year 1 (SF)</th>
<th>Year 1 (M)</th>
<th>Year 2 (M)</th>
<th>Year 3 (M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MTPL</td>
<td>15.53</td>
<td>13.00</td>
<td>18.27</td>
<td>21.87</td>
</tr>
<tr>
<td>MOD</td>
<td>6.21</td>
<td>0.27</td>
<td>-0.95</td>
<td>-2.40</td>
</tr>
<tr>
<td>GTPL</td>
<td>10.87</td>
<td>21.07</td>
<td>28.71</td>
<td>33.87</td>
</tr>
<tr>
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<td>34.35</td>
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<td>19.39</td>
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<td>23.71</td>
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<tr>
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<td>35.16</td>
<td>39.61</td>
<td>44.80</td>
<td>46.60</td>
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</table>
Results using Gumbel copula

Capital requirements over the initial gross premium amount and percentage diversification benefits according to the Standard Formula (SF) and our Model (M) (using the Gumbel copula) over a period of one, two and three years (values in %)

<table>
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<th></th>
<th>Year 1 (SF)</th>
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</table>
Conclusions

Results

In our paper, together with other results, we showed that:

- the capital requirements, according to our Model, are usually smaller than in the case of the Standard Formula, because the Standard Formula was calibrated on riskier distributions than in our Model and the correlation assumptions are more favourable in the case of our Model rather than in the Standard Formula;
- the capital requirements for non-life premium risk, according to our Model, are bigger in the case of the Gumbel copula rather than in the case of the Gaussian copula, because the upper tail dependence of the Gumbel copula is higher.

Improvements

Possible improvements can include the introduction of:

- an interest rate sensitive claims reserve;
- other sources of risk (e.g. spread risk, default risk and non-life reserve risk);
- a more complex and realistic asset allocation;
- some reinsurance forms.
Bibliography


[23] OECD. (2020). *Short or long-term interest rates and short or long-term interest rates forecast (indicators)*.


Thank you for your attention
Appendix: Short-rate

The deterministic function of time that allows the model to fit perfectly the term structure observed in the market is given by:

\[
\varphi(t) = f^M(0,t) + \frac{\sigma^2}{2a} \left( 1 - e^{-at} \right)^2 + \frac{\eta^2}{2b} \left( 1 - e^{-bt} \right)^2 + \rho \frac{\sigma \eta}{ab} \left( 1 - e^{-at} \right) \left( 1 - e^{-bt} \right)
\]

where \(f^M(0,t)\) is the instantaneous forward rate at initial time for a maturity \(t\) implied by the term structure observed in the market.

The price at time \(t\) of a zero-coupon bond with maturity in \(T > t\) is thus found to be:

\[
P(t, T) = \exp \left\{ - \int_t^T \varphi(u) \, du - \frac{1 - e^{-a(T-t)}}{a} x(t) - \frac{1 - e^{-b(T-t)}}{b} y(t) + \frac{1}{2} V(t, T) \right\}
\]

where the integral admits an explicit solution that depends on the prices at initial time of the zero-coupon bonds implied by the term structure observed in the market and \(V(t, T)\) is the variance of the integral between \(t\) and \(T\) of the sum of the state variables.
The relation between the expected zero-coupon interest rate (continuously compounded) at time $t$ for a term of $T - t$ under the real measure $\mathbb{P}$ and risk-neutral measure $\mathbb{Q}$ is found to be:

$$
\mathbb{E}_0^\mathbb{P}\{R(t, T)\} = \mathbb{E}_0^\mathbb{Q}\{R(t, T)\} + \frac{1}{T - t} \left[ \frac{1 - e^{-a(T-t)}}{a} R_{Px}(t) + \frac{1 - e^{-b(T-t)}}{b} R_{Py}(t) \right]
$$

where $R_{Px}(t)$ and $R_{Py}(t)$ are the actual risk premiums of the short rate between time 0 and $t$ for the state variables:

$$
R_{Px}(t) = \int_0^t e^{-a(t-u)} a d_x(u) \, du \quad \text{and} \quad R_{Py}(t) = \int_0^t e^{-b(t-u)} b d_y(u) \, du
$$

We assume that the mean reversion levels are given by step functions:

$$
d_x(t) = I_{t\leq\tau} d_x + I_{t>\tau} l_x \quad \text{and} \quad d_y(t) = I_{t\leq\tau} d_y + I_{t>\tau} l_y
$$

where $d_x$, $l_x$, $d_y$ and $l_y$ are real valued constants, $\tau$ is a constant time parameter (that is 15 months in our case) and $I$ is the indicator function.