Taxation treatment of retirement income products in Australia focusing on variable annuity contracts

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Variable Annuities (VAs) were first introduced in the early 1950s and various ‘GMxBs’ have become available:

- Guaranteed Minimum Death Benefit introduced in 1980s.
- Guaranteed Minimum Living Benefits introduced in late 1990s.
  - GMAB - Accumulation,
  - GMIB - Income,
  - GMWB - Withdrawal,
  - (GLWB - Lifelong form of GMWB).

GMAB rider is the building block of all other GMxB rider and hence the focus of this research.
VA market over the world

- In the US, VA industry is huge: US$2.08 trillion as of June 2021 (Insured Retirement Institute 2022).
- In Australia and Europe, the market is very thin:
  - In Australia, there are only a few notable players.¹
  - In Europe, the VAs’ market was worth 188 billion in 2010 (EIOPA 2011). However, after the Global Financial Crisis, their popularity decreased and various life insurers stopped their VA offering.
- In Japan, the VA market grew from a market of less than $1 billion in 2000 to over $50 billion, subsequent to a period of financial deregulation in the late 90s (Zhang 2006).

¹e.g. AMP Financial Services, BT Financial Group and MLC. (Vassallo et al. 2016)
Surrender behavior

- Our GMAB and GMDB promise the return of the premium payment, or a higher stepped up value at the end of the accumulation period of the contract or death.

- Typically, the valuation frameworks study the effect of the underlying fund distribution (GBM, Levy, etc) on the fee.

- Recently, the surrender behavior is studied more closely in the literature (Bernard et al. 2014; Kang and Ziveyi 2018) as underpricing lapse risk has resulted in significant losses for insurers (Moody’s Investor Service 2013).

- **Here:** the contract can be surrendered at any time prior to maturity, and the payments are liable for taxes (policyholder perspective) in the event of surrendering before the preservation age.

- We combine optimal stopping theory with uncertainty due to the event of death, in line with how such contracts are structured in practice.
Importance of Incorporating Tax

- One of the main attractive features of VAs is their tax-advantaged investing (Milevsky and Panyagometh 2001; Brown and Poterba 2006).
- Incorporating taxation in riders such as GMWB reconciles empirically observed fees with the theory (Moenig and Bauer 2015).
- The financial planning literature has long looked at ways of providing rules to follow so as to maximise post-tax returns (Sumutka et al. 2012; Horan and Robinson 2008).
- **In this study:** we examine the impact of taxation upon surrender before the preservation age when losses can offset gains, reflective of the Australian setting.
Two account setting

• The bulk of existing literature consider VA valuation whose guarantee fees are deducted from the investment account.

• In practice, some products are structured on a two account basis where the policyholder opens an investment account and a cash account as part of the VA contract.

• Here: we study this approach and consider management and transaction fees being deducted from the investment account indexed to the underlying fund.

• Guarantee fees funding the GMAB and GMDB riders are deducted from the cash account.
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The two accounts

- Let \((S_\nu)\) be the underlying asset which evolves according to the standard geometric Brownian motion (GBM) under the risk-neutral measure such that

\[
dS_\nu = rS_\nu \, d\nu + \sigma S_\nu \, dW_\nu. \tag{1}
\]

- The **investment component** of the VA \((x_\nu)\) can be expressed as

\[
x_\nu = e^{-q_m\nu} S_\nu \text{ where } q_m \text{ is the management fee rate.}
\]

- Applying Ito’s Lemma to the process \((x_\nu)\) yields:

\[
dx_\nu = (r - q_m)(x_\nu) \, d\nu + \sigma (x_\nu) \, dW_\nu. \tag{2}
\]

- The **cash account** evolution is described as

\[
c(\nu) = e^{rv} \int_\nu^{T_m} q_g Ge^{(\delta - r)s} \, ds = q_g Ge^{\delta \nu} \frac{e^{(\delta - r)(T_m - \nu)} - 1}{\delta - r}. \tag{3}
\]
Toy example

- Management fees depend on the underlying, which is stochastic. Guarantee fees are deducted from a cash account, which is deterministic.
- This cash account decreases at a rate of $q_g G$ while also earning the risk free interest at rate $r$. At maturity, the cash account should be depleted ($c(T_m) = 0$).
- If the policyholder surrenders the contract prior to maturity, they receive the remaining amount in the cash account and can claim the reduction as a tax loss.

Figure: Evolution of the cash account. All parameters are as in the illustrative Table 1.
<table>
<thead>
<tr>
<th>Item</th>
<th>Held until maturity</th>
<th>Early surrender at t=7.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fund value at commencement (a)</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Fund value at termination (b)</td>
<td>120</td>
<td>110</td>
</tr>
<tr>
<td>Surrender penalty (c)</td>
<td>0</td>
<td>5*</td>
</tr>
<tr>
<td>Cash account payout (d)</td>
<td>0</td>
<td>1.20^2</td>
</tr>
<tr>
<td>Capital gain on inv acc [(b) + (d) - (c) - (a) = (e)]</td>
<td>20</td>
<td>6.20</td>
</tr>
<tr>
<td>Capital loss on cash acc [(d) - c(0) = (f)]</td>
<td>4.32</td>
<td>3.12</td>
</tr>
<tr>
<td>Total capital gain [(e) - (f) = (g)]</td>
<td>15.68</td>
<td>3.09</td>
</tr>
<tr>
<td>Tax paid [0.30*(g) = (h)]</td>
<td>4.70</td>
<td>0.93</td>
</tr>
<tr>
<td>Inv account payout [(b) - (c) - (h) = (i)]</td>
<td>115.30</td>
<td>107.07</td>
</tr>
</tbody>
</table>

**Table:** Illustrative example with cashflows associated with the cash account. We consider a VA contract with maturing in 10 years, \( \delta = 0 \), guarantee fee rate 0.005, \( r = 3\% \), \( c(0) = 4.32 \) corresponding to the initial investment in the cash account.

*An arbitrary surrender penalty is used for illustrative purposes.

\(^2\)This corresponds to \( c(7.5) \).
The hybrid VA product

- \( T_d \) time at death, \( T_m \) maturity, \( \alpha \) age at inception

\[
V_{\alpha}(x, T_m) = \underbrace{\operatorname{ess sup}_{\nu \leq \epsilon^* \leq T_m} \mathbb{E}^Q \left[ \mathbb{1}_{\{T_d > T_m\}} \left( e^{-\int_{\nu}^{\epsilon^*} r(s) \, ds} g(\epsilon^*, x_{\epsilon^*}) \right) \right]}_{\text{Survival}} + \underbrace{\mathbb{E}^Q \left[ \mathbb{1}_{\{T_d < T_m\}} \left( e^{-\int_{\nu}^{T_d} r(s) \, ds} h(\nu, x_{\nu}) \right) \right]}_{\text{PV of GMAB payout}}
\]

\[
= T_m \, p_{\alpha} \underbrace{\operatorname{ess sup}_{\nu \leq \epsilon^* \leq T_m} \mathbb{E}^Q \left[ e^{-\int_{\nu}^{\epsilon^*} r(s) \, ds} g(\epsilon^*, x_{\epsilon^*}) \right]}_{\text{Only living benefits to optimise}} + \mathbb{E}^Q \left[ \mathbb{1}_{\{T_d < T_m\}} e^{-\int_{\nu}^{T_d} r(s) \, ds} h(\nu, x_{\nu}) \right] \mathcal{F}_\nu
\]

\[
= T_m \, p_{\alpha} u^m(x, \nu, T_m) + \int_{\nu}^{T_m} \left\{ \mathbb{E}^Q \left[ \mu(\nu) \cdot k \right] \mathcal{F}_\nu \mathbb{E}^Q \left[ e^{-\int_{\nu}^{k} r(s) \, ds} h(\nu, x_{\nu}) \right] \mathcal{F}_\nu \right\} \, dk
\]

\[
= V_{LB}^{\alpha}(x, T_m) + V_{DB}^{\alpha}(x, T_m),
\]
It is only the GMAB rider component which is influenced by policyholder behaviour and hence can be formulated as an optimal surrender problem which we express here as

\[ V_{\alpha}^{LB}(x, T_m) = T_m \ p_{\alpha} u^{m}_{\alpha}(x, \nu, T_m), \]  

(4)

where

\[ u^{m}(x, \nu, T_m) = \text{ess sup}_{\nu \leq \epsilon^{*} \leq T_m} \ E^{\mathbb{Q}} \left[ e^{-\int_{\nu}^{\epsilon^{*}} r(s) ds} g(\epsilon^{*}, x_{\epsilon^{*}}) \right] |F_{\nu}, \]  

(5)

with

\[ g(\nu, x_{\nu}) = c(\nu) + \begin{cases} 
  e^{-\kappa(T_m-\nu)}x_{\nu}, & \nu < T_m \\
  \max(x_{\nu}, G), & \nu = T_m.
\end{cases} \]
Likewise, the GMDB rider can be expressed as

\[
V^{DB}_\alpha(x, T_m) = \int_{\nu}^{T_m} \left\{ E^Q \left[ \mu_\alpha(k) \cdot k p_\alpha | F_\nu \right] E^Q \left[ e^{-\int_{\nu}^{k} r(s) ds} h(\nu, x_\nu) | F_\nu \right] \right\} dk,
\]

\[
= \int_{\nu}^{T_m} \left\{ E^Q \left[ \mu_\alpha(k) \cdot k p_\alpha | F_\nu \right] u^D(x, \nu, k) \right\} dk,
\]

where

\[
u^D(x, \nu, T_m) = E^Q \left[ e^{-\int_{\nu}^{T_m} r(s) ds} h(\nu, x_\nu) | F_\nu \right].
\]

with

\[
h(\nu, x_\nu) = c(\nu) + \max(x_\nu, G).
\]

The policyholder receives the remaining value of the cash account upon surrender and death.
The governing PDE

- The fund evolves as a Geometric Brownian Motion process.
- If $t$ represents the contract’s time to maturity, $u$ will satisfy the PDE:

$$
\frac{1}{2} \sigma^2 x^2 u_{xx}^m + (r - q_m) \cdot x u_x - ru - u_t = q_g Ge^{\delta(T_m-t)},
$$

(9)

where $0 < t < T_m$ and $0 < x < s(t)$, with $s(t)$ being the optimal surrender boundary for a rational policyholder.

- The boundary conditions capture the taxes paid upon surrender or maturity if age lies below the preservation age.

- We use the Method of Lines to solve Equation (9) (Meyer and Van der Hoek 1997).
Mortality model

We assume that the policyholder’s mortality evolves in a stochastic fashion and is governed by the Renshaw and Haberman (2003) model:

\[
\ln(m_{x,t}) = a_x + b_x k_t + b_x^2 \gamma_{t-x} + \epsilon_{x,t}
\]  

(10)

where \( m_{x,t} \) is the central mortality estimate, \( b_x \) and \( b_x^2 \) are age-dependent parameters, \( k_t \) is a time dependent parameter, \( \gamma_{t-x} \) is a cohort parameter and \( \epsilon_{x,t} \) is a Poisson error term.
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### Financial base case parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0$</td>
<td>1</td>
<td>$T$</td>
<td>5,10,15</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.20</td>
<td>$\kappa$</td>
<td>0.001</td>
</tr>
<tr>
<td>$r$</td>
<td>0.03</td>
<td>$\tau$</td>
<td>0.225</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.015</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table:** Base case parameters for numerical illustrations

- Weekly time discretisation.
- Maximum possible fund value and total fees set to $4 \cdot G$. 
Intuitively, $q_m \uparrow$, the policyholder’s fair guarantee fee $q_g \downarrow$. Indeed, they are paying more for the product in the form of administrative fees deducted from the investment account.

However, if management fees $q_m$ high $\rightarrow$ investment account $\downarrow$ and guarantee becomes more likely. 

Thus, it is of interest to examine the fair fee charging structures and the trade-off between the guarantee and the investment management fees.
Return-on-premium ($\delta = 0\%$) with no management fees have a comparable guarantee fee to $\delta = 1.5\%$ with 1% management fees.

- Guarantee fees decrease with management fees.
- Guarantee fees increase with roll-up.
Varying roll up rates

Figure: Fair fee curves for various guarantee growth rates
Early surrender boundaries\textsuperscript{3}

**Figure: Volatility**

- Boundaries increases with volatility.
- *Contrary to classical results?*

\textsuperscript{3}Recall $4 \times G$ is the maximum in the numerical scheme.
Early surrender boundaries (C’td)

**Figure:** Risk free rate
Early surrender boundaries (C’td)

Figure: Taxation

Numerical Results
Early surrender boundaries for changing market variables

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VA with tax
Numerical Results

Comparison of hybrid variable annuity with standalone riders

![Figure: GMAB and GMDB components](image)

(a) Entry age 55

(b) Entry age 65.

Figure: GMAB and GMDB components
Figure: GMAB and GMDB components $T = 5$
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- Presented a hybrid framework for VAs embedded with GMAB and GMDB riders as is often the case in practice.
- A novel two account structure mimicking some product offerings in the Australian market.
- Incorporating taxation in the event of contract exercised before the preservation age in Australia.
- Sensitivity analyses have been performed for varying parameter settings, providing insights on the interplay of management fees and guarantee fees.
References


Thank you for your attention

Questions?
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Boundary conditions

\[ u^m(x, 0) = \max (x, Ge^\delta T_m) - \tau [\alpha + T_m - t \leq 60] \left[ \max (x, Ge^\delta T_m) + c(T_m) - x_0 - c(0) \right]_+, \] (11)

\[ u^m(s(t), t) = \gamma_t e^{\kappa(T_m-t)} s(t) - \tau [\alpha + T_m - t \leq 60] \left[ (s(t)\gamma_t e^{\kappa(T_m-t)} + c(T_m - t) - x_0 - c(0)) \right]_+, \] (12)

\[ u^m(0, t) = (Ge^\delta T_m - \tau [\alpha + T_m - t \leq 60]) [Ge^\delta T_m - x_0 - c(0)]_+ e^{-rt}, \] (13)

\[ u_x^m(s(t), t) = e^{\kappa(T_m-t)} \gamma_t - \tau [\alpha + T_m - t \leq 60] \gamma_t e^{\kappa(T_m-t)} \left[ s(t)\gamma_t e^{\kappa(T_m-t)} + c(T_m - t) - x_0 - c(0) > 0 \right]_+, \] (14)

on the domain \((x, t) \in [0, X] \times [0, T_m]\). Back to Main story