

Taxation treatment of retirement income products in Australia focusing on variable annuity contracts

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Variable Annuities

Variable Annuities (VAs) were first introduced in the early 1950s and various 'GMxBs' have become available:

- Guaranteed Minimum Death Benefit introduced in 1980s.
- Guaranteed Minimum Living Benefits introduced in late 1990s.
 - **GMAB - Accumulation**,
 - GMIB - Income,
 - GMWB - Withdrawal,
 - (GLWB - Lifelong form of GMWB).

GMAB rider is the building block of all other GMxB rider and hence the focus of this research.

VA market over the world

- In the US, VA industry is huge: US\$2.08 trillion as of June 2021 (Insured Retirement Institute 2022).
- In Australia and Europe, the market is very thin:
 - In Australia, there are only a few notable players.¹
 - In Europe, the VAs' market was worth 188 billion in 2010 (EIOPA 2011). However, after the Global Financial Crisis, their popularity decreased and various life insurers stopped their VA offering.
- In Japan, the VA market grew from a market of less than \$1 billion in 2000 to over \$50 billion, subsequent to a period of financial deregulation in the late 90s (Zhang 2006).

¹e.g. AMP Financial Services, BT Financial Group and MLC. (Vassallo et al. 2016)

Surrender behavior

- Our GMAB and GMDB promise the return of the premium payment, or a higher stepped up value at the end of the accumulation period of the contract or death.
- Typically, the valuation frameworks study the effect of the underlying fund distribution (GBM, Levy, etc) on the fee.
- Recently, the surrender behavior is studied more closely in the literature (Bernard et al. 2014; Kang and Ziveyi 2018) as underpricing lapse risk has resulted in significant losses for insurers (Moody's Investor Service 2013).
- **Here:** the contract can be surrendered at any time prior to maturity, and the payments are liable for **taxes** (policyholder perspective) in the event of surrendering before the preservation age.
- We combine optimal stopping theory with uncertainty due to the event of death, in line with how such contracts are structured in practice.

Importance of Incorporating Tax

- One of the main attractive features of VAs is their tax-advantaged investing (Milevsky and Panyagometh 2001; Brown and Poterba 2006).
- Incorporating taxation in riders such as GMWB reconciles empirically observed fees with the theory (Moenig and Bauer 2015).
- The financial planning literature has long looked at ways of providing rules to follow so as to maximise post-tax returns (Sumutka et al. 2012; Horan and Robinson 2008).
- **In this study:** we examine the impact of taxation upon surrender before the preservation age when losses can offset gains, reflective of the Australian setting.

Two account setting

- The bulk of existing literature consider VA valuation whose guarantee fees are deducted from the investment account.
- In practice, some products are structured on a two account basis where the policyholder opens an investment account and a cash account as part of the VA contract.
- **Here:** we study this approach and consider management and transaction fees being deducted from the **investment account** indexed to the underlying fund.
- Guarantee fees funding the GMAB and GMDB riders are deducted from the **cash account**.

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The two accounts

- Let (S_ν) be the underlying asset which evolves according to the standard geometric Brownian motion (GBM) under the risk-neutral measure such that

$$dS_\nu = rS_\nu d\nu + \sigma S_\nu dW_\nu. \quad (1)$$

- The **investment component** of the VA (x_ν) can be expressed as $x_\nu = e^{-q_m \nu} S_\nu$ where q_m is the management fee rate.
- Applying Ito's Lemma to the process (x_ν) yields:

$$dx_\nu = (r - q_m)(x_\nu)d\nu + \sigma(x_\nu)dW_\nu. \quad (2)$$

- The **cash account** evolution is described as

$$c(\nu) = e^{r\nu} \int_\nu^{T_m} q_g G e^{(\delta-r)s} ds = q_g G e^{\delta\nu} \frac{e^{(\delta-r)(T_m-\nu)} - 1}{\delta - r}. \quad (3)$$

Toy example

- Management fees depend on the underlying, which is stochastic. Guarantee fees are deducted from a cash account, which is deterministic.
- This cash account decreases at a rate of $q_g G$ while also earning the risk free interest at rate r . At maturity, the cash account should be depleted ($c(T_m) = 0$).
- If the policyholder surrenders the contract prior to maturity, they receive the remaining amount in the cash account and can claim the reduction as a tax loss.

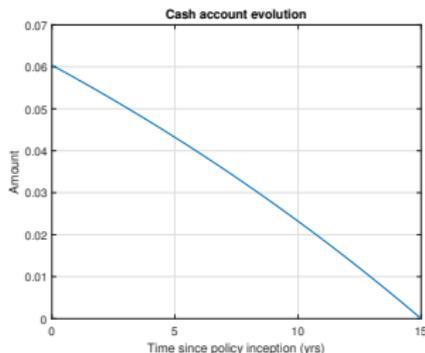


Figure: Evolution of the cash account. All parameters are as in the illustrative Table 1.

Item	Held until maturity	Early surrender at $t=7.5$
Fund value at commencement (a)	100	100
Fund value at termination (b)	120	110
Surrender penalty (c)	0	5*
Cash account payout (d)	0	1.20^2
Capital gain on inv acc $[(b) + (d) - (c) - (a) = (e)]$	20	6.20
Capital loss on cash acc $[(d) - c(0) = (f)]$	4.32	3.12
Total capital gain $[(e) - (f) = (g)]$	15.68	3.09
Tax paid $[0.30*(g) = (h)]$	4.70	0.93
Inv account payout $[(b) - (c) - (h) = (i)]$	115.30	107.07

Table: Illustrative example with cashflows associated with the cash account. We consider a VA contract with maturing in 10 years, $\delta = 0$, guarantee fee rate 0.005, $r = 3\%$, $c(0) = 4.32$ corresponding to the initial investment in the cash account.

*An arbitrary surrender penalty is used for illustrative purposes.

²This corresponds to $c(7.5)$.

The hybrid VA product

- T_d time at death, T_m maturity, α age at inception

$$\begin{aligned}
 V_\alpha(x, T_m) &= \operatorname{ess\,sup}_{\nu \leq \epsilon^* \leq T_m} \mathbb{E}^{\mathbb{Q}} \left[\overbrace{\mathbb{1}_{\{T_d > T_m\}}}^{\text{Survival}} \overbrace{\left(e^{-\int_\nu^{\epsilon^*} r(s) ds} g(\epsilon^*, x_{\epsilon^*}) \right)}^{\text{PV GMAB payout}} \right. \\
 &\quad \left. + \overbrace{\mathbb{1}_{\{T_d < T_m\}}}^{\text{Death before maturity}} \overbrace{\left(e^{-\int_\nu^{T_d} r(s) ds} h(\nu, x_\nu) \right)}^{\text{PV of GMDB payout}} \middle| \mathcal{F}_\nu \right] \\
 &= T_m \rho_\alpha \overbrace{\left[\operatorname{ess\,sup}_{\nu \leq \epsilon^* \leq T_m} \mathbb{E}^{\mathbb{Q}} \left[e^{-\int_\nu^{\epsilon^*} r(s) ds} g(\epsilon^*, x_{\epsilon^*}) \right] \right]}^{\text{Only living benefits to optimise}} \\
 &\quad + \mathbb{E}^{\mathbb{Q}} \left[\mathbb{1}_{\{T_d < T_m\}} e^{-\int_\nu^{T_d} r(s) ds} h(\nu, x_\nu) \right] \middle| \mathcal{F}_\nu \\
 &= T_m \rho_\alpha u^m(x, \nu, T_m) + \int_\nu^{T_m} \left\{ \mathbb{E}^{\mathbb{Q}} [\mu_\alpha(k) \cdot k \rho_\alpha | \mathcal{F}_\nu] \mathbb{E}^{\mathbb{Q}} \left[e^{-\int_\nu^k r(s) ds} h(\nu, x_\nu) \middle| \mathcal{F}_\nu \right] \right\} dk \\
 &\equiv V_\alpha^{LB}(x, T_m) + V_\alpha^{DB}(x, T_m),
 \end{aligned}$$

- It is only the GMAB rider component which is influenced by policyholder behaviour and hence can be formulated as an optimal surrender problem which we express here as

$$V_{\alpha}^{LB}(x, T_m) = T_m p_{\alpha} u^m(x, \nu, T_m), \quad (4)$$

where

$$u^m(x, \nu, T_m) = \operatorname{ess\,sup}_{\nu \leq \epsilon^* \leq T_m} \mathbb{E}^{\mathbb{Q}} \left[e^{-\int_{\nu}^{\epsilon^*} r(s) ds} g(\epsilon^*, x_{\epsilon^*}) \right] | \mathcal{F}_{\nu}, \quad (5)$$

with

$$g(\nu, x_{\nu}) = c(\nu) + \begin{cases} e^{-\kappa(T_m - \nu)} x_{\nu}, & \nu < T_m \\ \max(x_{\nu}, G), & \nu = T_m. \end{cases}$$

- Likewise, the GMDB rider can be expressed as

$$\begin{aligned}
 V_{\alpha}^{DB}(x, T_m) &= \int_{\nu}^{T_m} \left\{ \mathbb{E}^{\mathbb{Q}} [\mu_{\alpha}(k) \cdot {}_k p_{\alpha} | \mathcal{F}_{\nu}] \mathbb{E}^{\mathbb{Q}} \left[e^{-\int_{\nu}^k r(s) ds} h(\nu, x_{\nu}) | \mathcal{F}_{\nu} \right] \right\} dk, \\
 &= \int_{\nu}^{T_m} \left\{ \mathbb{E}^{\mathbb{Q}} [\mu_{\alpha}(k) \cdot {}_k p_{\alpha} | \mathcal{F}_{\nu}] u^D(x, \nu, k) \right\} dk,
 \end{aligned} \tag{6}$$

where

$$u^D(x, \nu, T_m) = \mathbb{E}^{\mathbb{Q}} \left[e^{-\int_{\nu}^{T_m} r(s) ds} h(\nu, x_{\nu}) | \mathcal{F}_{\nu} \right]. \tag{7}$$

with

$$h(\nu, x_{\nu}) = c(\nu) + \max(x_{\nu}, G). \tag{8}$$

- The policyholder receives the remaining value of the cash account upon surrender and death.

The governing PDE

- The fund evolves as a Geometric Brownian Motion process.
- If t represents the contract's time to maturity, u will satisfy the PDE:

$$\frac{1}{2}\sigma^2x^2u_{xx}^m + (r - q_m) \cdot xu_x - ru - u_t = q_g Ge^{\delta(T_m-t)}, \quad (9)$$

where $0 < t < T_m$ and $0 < x < s(t)$, with $s(t)$ being the optimal surrender boundary for a rational policyholder.

- The boundary conditions capture the taxes paid upon surrender or maturity if age lies below the preservation age. Boundary conditions
- We use the Method of Lines to solve Equation (9) (Meyer and Van der Hoek 1997).

Mortality model

We assume that the policyholder's mortality evolves in a stochastic fashion and is governed by the Renshaw and Haberman (2003) model:

$$\ln(m_{x,t}) = a_x + b_x k_t + b_x^2 \gamma_{t-x} + \epsilon_{x,t} \quad (10)$$

where $m_{x,t}$ is the central mortality estimate, b_x and b_x^2 are age-dependent parameters, k_t is a time dependent parameter, γ_{t-x} is a cohort parameter and $\epsilon_{x,t}$ is a Poisson error term.

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Financial base case parameters

Parameter	Value	Parameter	Value
x_0	1	T	5,10,15
σ	0.20	κ	0.001
r	0.03	τ	0.225
δ	0.015		

Table: Base case parameters for numerical illustrations

- Weekly time discretisation.
- Maximum possible fund value and total fees set to $4 \cdot G$.

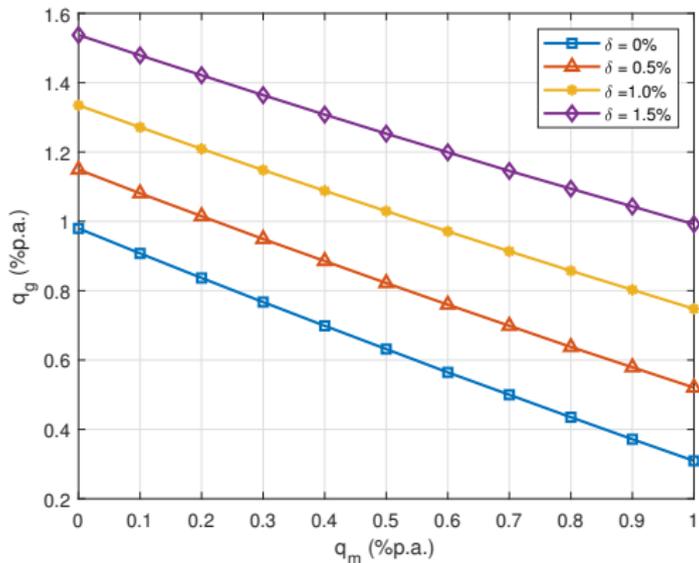
- Intuitively, $q_m \nearrow$, the policyholder's fair guarantee fee $q_g \searrow$. Indeed, they are paying more for the product in the form of administrative fees deducted from the investment account.
- However, if management fees q_m high \rightarrow investment account \searrow and guarantee becomes more likely. d
- Thus, it is of interest to examine the fair fee charging structures and the trade-off between the guarantee and the investment management fees.

qm\delta	0.0%	0.5%	1.0%	1.5%
0.0%	0.98%	1.15%	1.33%	1.54%
0.1%	0.91%	1.08%	1.27%	1.48%
0.2%	0.84%	1.02%	1.21%	1.42%
0.3%	0.77%	0.95%	1.15%	1.36%
0.4%	0.70%	0.89%	1.09%	1.31%
0.5%	0.63%	0.82%	1.03%	1.25%
0.6%	0.56%	0.76%	0.97%	1.20%
0.7%	0.50%	0.70%	0.91%	1.15%
0.8%	0.44%	0.64%	0.86%	1.09%
0.9%	0.37%	0.58%	0.80%	1.04%
1.0%	0.31%	0.52%	0.75%	0.99%

- Return-on-premium ($\delta = 0\%$) with no management fees have a comparable guarantee fee to $\delta = 1.5\%$ with 1% management fees.
- Guarantee fees decrease with management fees.
- Guarantee fees increase with roll-up.

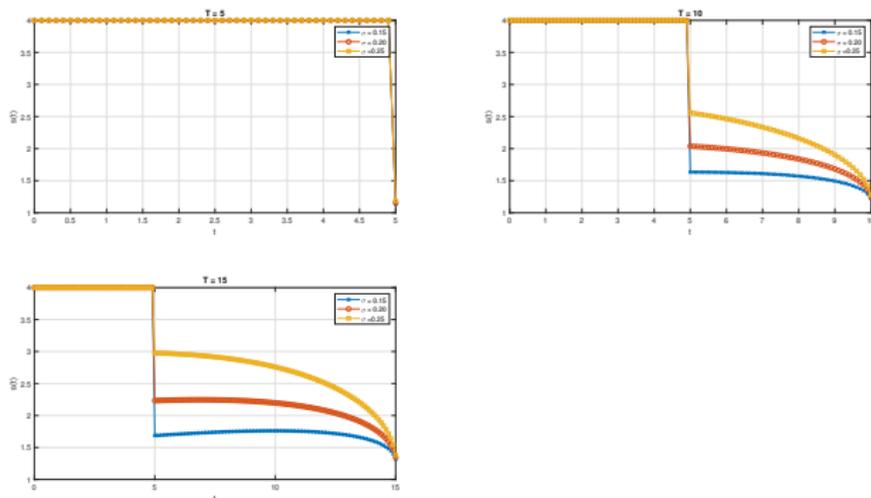
Varying roll up rates

Figure: Fair fee curves for various guarantee growth rates



Early surrender boundaries³

Figure: Volatility

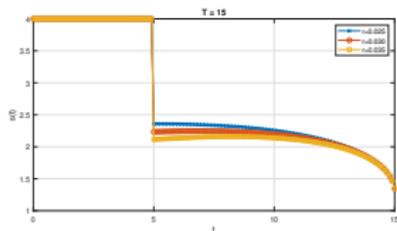
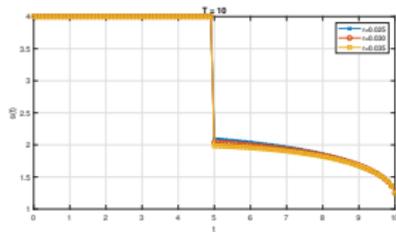


- Boundaries increases with volatility.
- *Contrary to classical results?*

³Recall $4 * G$ is the maximum in the numerical scheme.

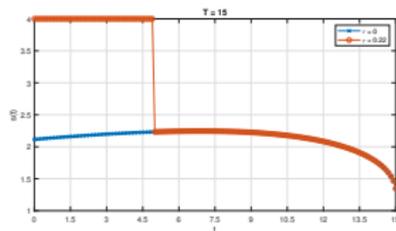
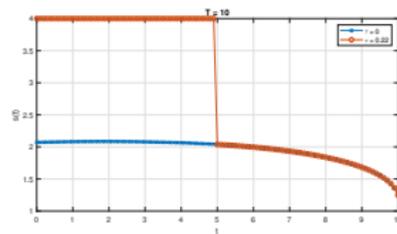
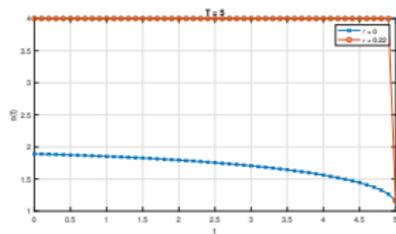
Early surrender boundaries (C'td)

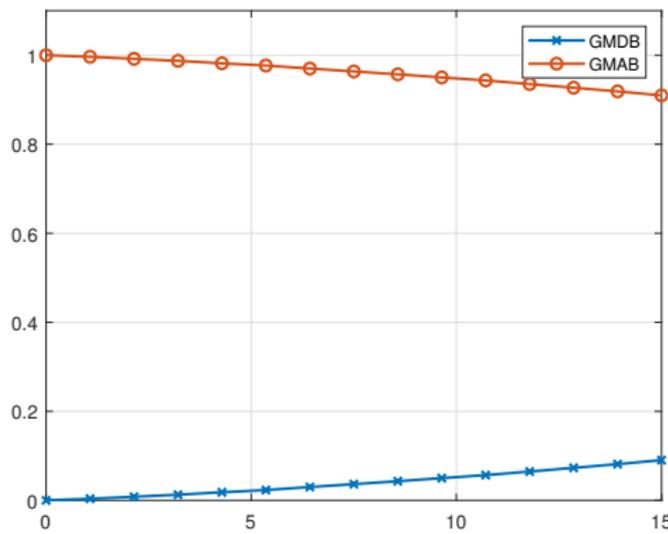
Figure: Risk free rate



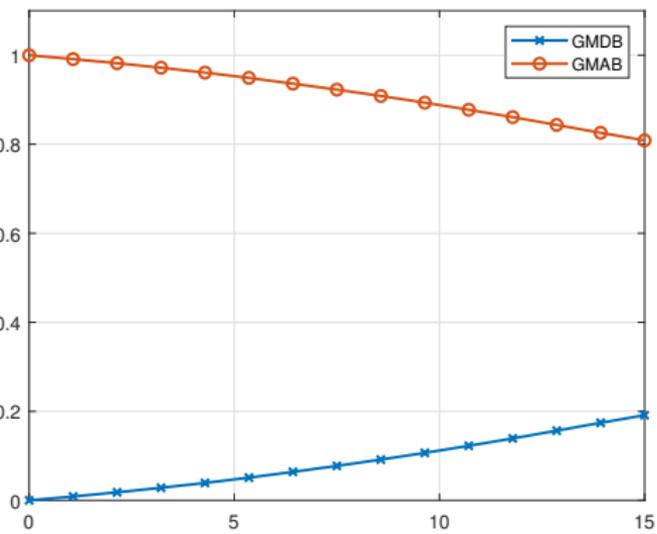
Early surrender boundaries (C'td)

Figure: Taxation



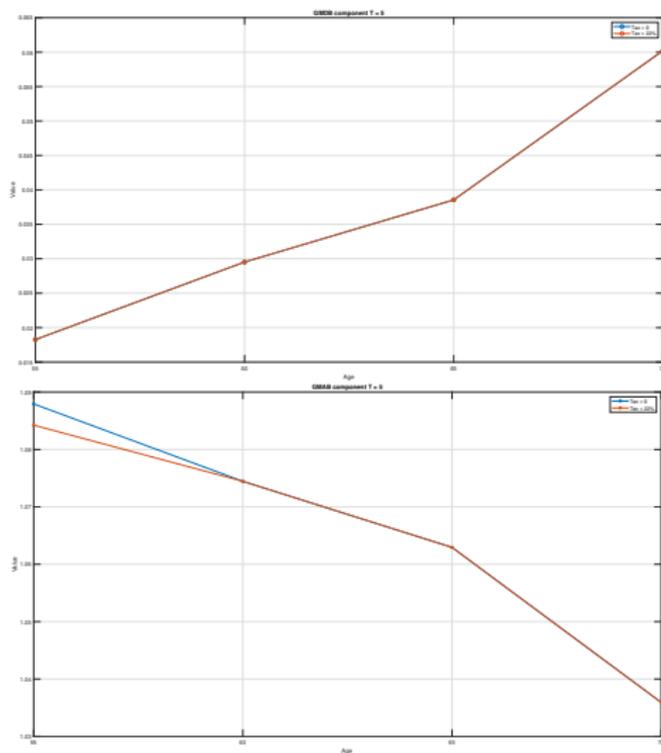


(a) Entry age 55



(b) Entry age 65.

Figure: GMAB and GMDB components

Figure: GMAB and GMDB components $T = 5$ 

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Conclusion

- Presented a hybrid framework for VAs embedded with GMAB and GMDB riders as is often the case in practice.
- A novel two account structure mimicking some product offerings in the Australian market.
- Incorporating taxation in the event of contract exercised before the preservation age in Australia.
- Sensitivity analyses have been performed for varying parameter settings, providing insights on the interplay of management fees and guarantee fees.

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Thanks

Thank you for your attention
Questions?

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Boundary conditions

$$u^m(x, 0) = \max(x, Ge^{\delta T_m}) - \tau \mathbb{1}_{\{\alpha + T_m - t \leq 60\}} [\max(x, Ge^{\delta T_m}) + c(T_m) - x_0 - c(0)]_+, \quad (11)$$

$$u^m(s(t), t) = \gamma_t e^{\kappa(T_m - t)} s(t) - \tau \mathbb{1}_{\{\alpha + T_m - t \leq 60\}} [(s(t)\gamma_t e^{\kappa(T_m - t)} + c(T_m - t) - x_0 - c(0))_+, \quad (12)$$

$$u^m(0, t) = (Ge^{\delta T_m} - \tau \mathbb{1}_{\{\alpha + T_m - t \leq 60\}} [Ge^{\delta T_m} - x_0 - c(0)]_+) e^{-rt}, \quad (13)$$

$$u_x^m(s(t), t) = e^{\kappa(T_m - t)} \gamma_t - \tau \mathbb{1}_{\{\alpha + T_m - t \leq 60\}} \gamma_t e^{\kappa(T_m - t)} \mathbb{1}_{\{s(t)\gamma_t e^{\kappa(T_m - t)} + c(T_m - t) - x_0 - c(0) > 0\}}, \quad (14)$$

on the domain $(x, t) \in [0, X] \times [0, T_m]$. [Back to Main story](#)