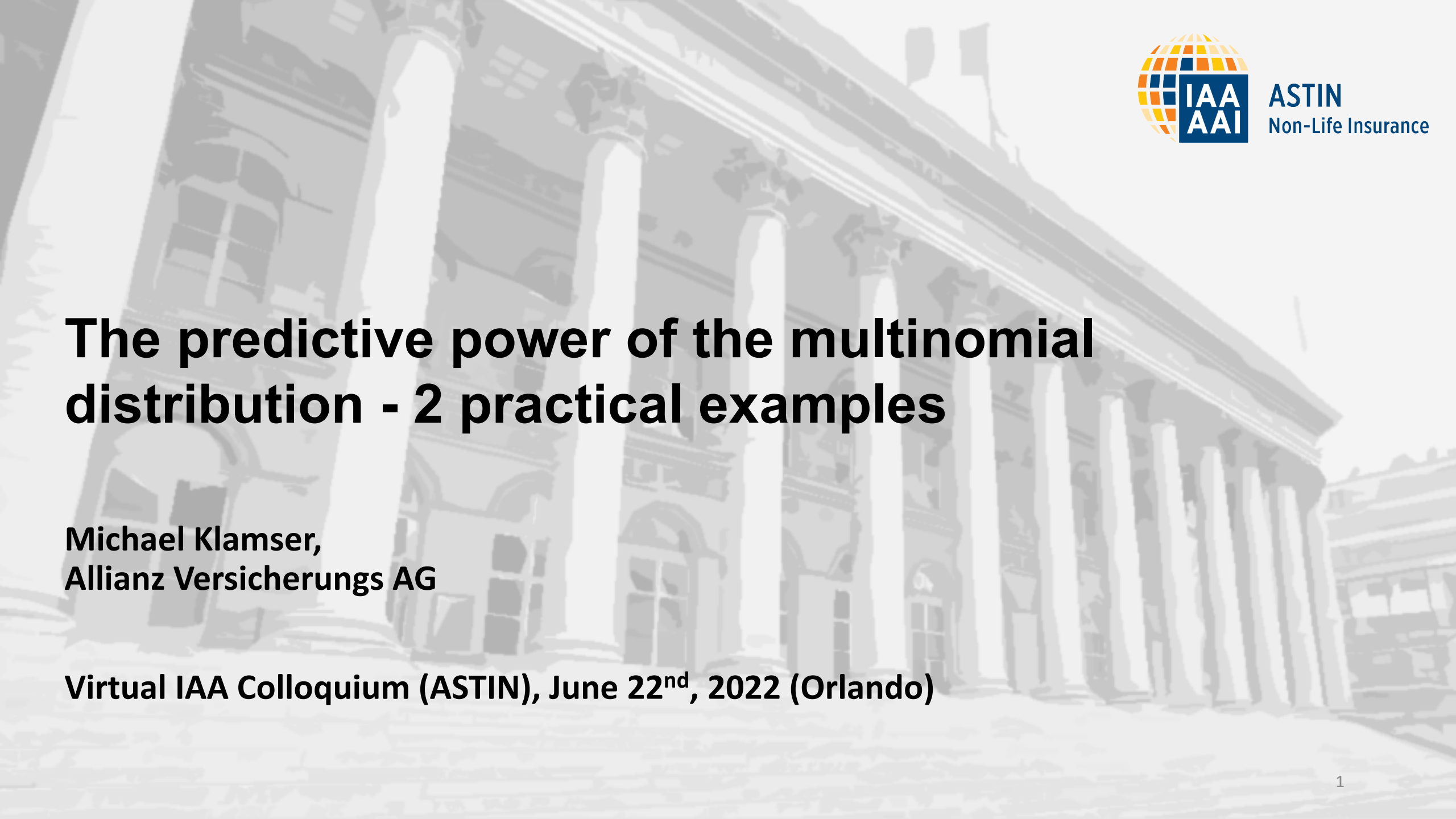

The predictive power of the multinomial distribution - 2 practical examples

Michael Klamser,
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Virtual IAA Colloquium (ASTIN), June 22nd, 2022 (Orlando)

About the speaker

Name

- **Michael Klamser (Senior Actuary)**
- **1986-1994:** Studies in Econometrics (TU Karlsruhe)
- **1994:** Entering Allianz Insurance Company (Actuarial Department)
- **1994-2000:** Actuarial Department (Motor business – retail and commercial)
- **2000-today:** Commercial Motor Department
- **Since 1999:** Actuary at the German Association of Actuaries (DAV)

Allianz Group (Non-Life) - 2019

- **Turnover:** 59,2 bln. €,
- **Operating profit:** 5,0 bln €
- **Loss ratio:** 68,0 % (German fleet market/before run-off: 92,0 %)
- **Combined Ratio:** 95,5 % (German fleet market/after run-off: 102,0 %)

Disclaimer:

All the figures/KPIs in the following slides which are connected
with the Allianz fleet portfolio, do not correspond with the figures in reality.

Still, the deductions done in the presentation respectively
during the session are the same as the ones based on the real figures.

Glossary:

TP: (actuarially correct) technical premium

CP: commercial premium (before any adjustments)

AP: actual respectively offered premium

LR: loss-ratio (**not** lapse-ratio!!)

MRP: Manual Renewal-Probability

Overview

1. The flatrate model cred/Bonus-Malus
2. The flatrate model cred/new calc.
3. (cred/Bonus-Malus)
   Prediction of the Loss-Ratio through Multinomial Approach
4. (cred/new calc.)
   Prediction of the market adjustment through Multinomial Approach

Overview

1. **The flatrate model cred/Bonus-Malus**
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1. The flatrate model cred/Bonus-Malus

Basics:

- Introduced in 2013.
- Each new fleet has to be calculated on the basis of credibility.
- An essential model to increase the profitability of the overall flatrate portfolio.
- Includes an **optional** premium adjustment-clause
  (➔ to compensate for the loss in GWP due to automatic renewal).
- Enables a new calculation of the fleet if certain criteria are met.
- As of end of 2020: approx. 1.000 fleets with an AP of 70 Mio. EUR.

1. The flatrate model cred/Bonus-Malus

Rules for automatic renewal (dependent of 8 LR-classes):

- LR<45 % ➔ -15 % discount,
- LR in (45%,55%) ➔ -10 % discount,
- ….
- LR in (85%,95%) ➔ +15 % loading,
- LR > 95 % ➔ new calculation on the basis of credibility.

Overview

1. The flatrate model cred/Bonus-Malus
2. **The flatrate model cred/new calc.**
3. (cred/Bonus-Malus)
   Prediction of the Loss-Ratio through Multinomial Approach
4. (cred/new calc.)
   Prediction of the market adjustment through Multinomial Approach

2. The flatrate model cred/new calc.

Basics:

- Introduced in 2010.
- Paradigm change in the calculation of large fleets
  (➔ from burning-cost to credibility approach).
- Each year every fleet has to be calculated anew on the basis of credibility.
- Yearly adjustment only possible through overall tariff level
  (can be differentiated for each fleet by AP/TP and LR-combination).
- As of end of 2020: approx. 900 fleets with an AP of 130 Mio. EUR.

Overview

1. The flatrate model cred/Bonus-Malus
2. The flatrate model cred/new calc.
3. **(cred/Bonus-Malus)**
   Prediction of the Loss-Ratio through Multinomial Approach
4. (cred/new calc.)
   Prediction of the market adjustment through Multinomial Approach

3. (cred/Bonus-Malus)
Prediction of the Loss-Ratio through Multinomial Approach

Predicament:

An eventual overall premium-adjustment in addition to the automatic renewal has to be decided no later than in August (due to technical restraints).

➔ Prediction of the loss-ratio as of 31st of December on the basis of 31st of July is of paramount importance so as to get a hint of the topline in the following year.

Possible solution (see also **SAS/STAT – PROC GENMOD, examples**) :

Application of the Generalized Linear Model with

- the **multinomial distribution** and
- the **cumulative logit function**.

3. (cred/Bonus-Malus)
Prediction of the Loss-Ratio through Multinomial Approach

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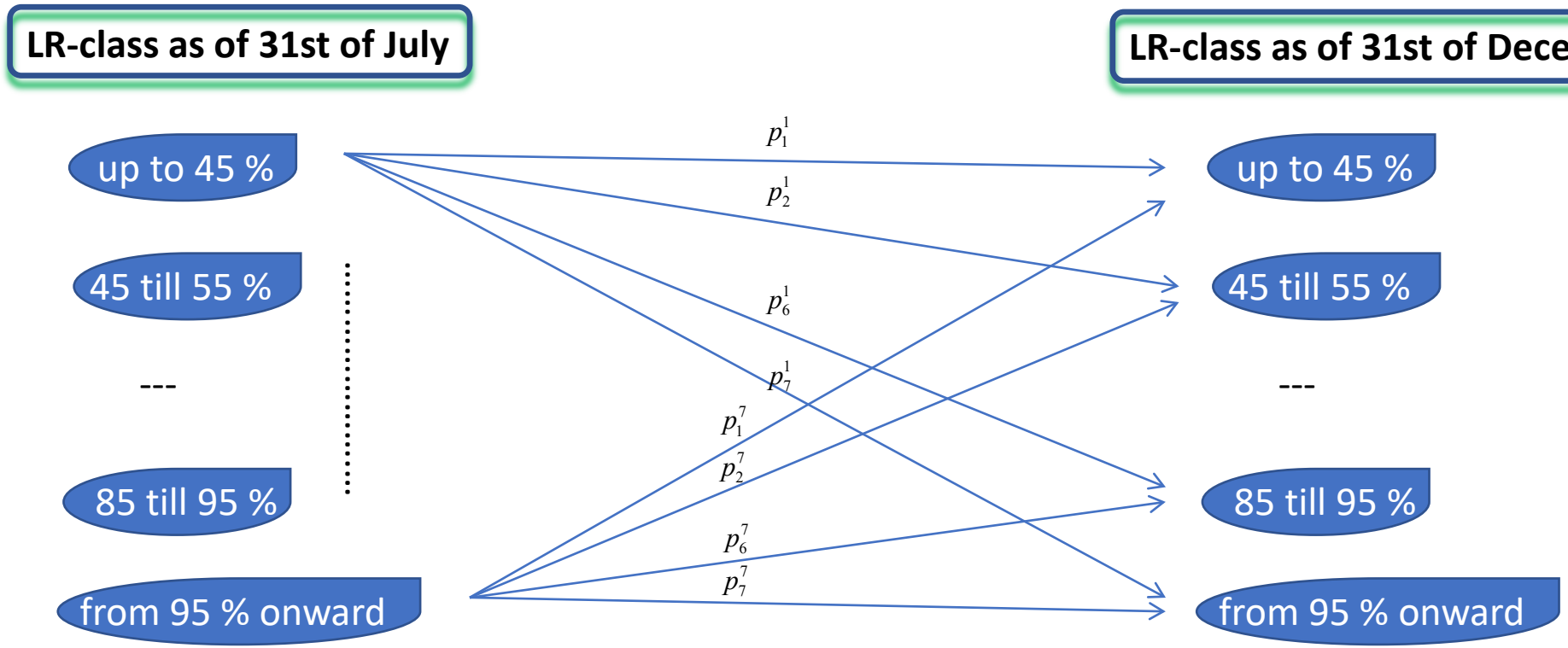
Can be any (statistically sensible) grouping

Should be the same as for automatic renewal



Modelling takes place with 2019-figures!!

3. (cred/Bonus-Malus)
Prediction of the Loss-Ratio through Multinomial Approach

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- **In a nutshell (premise: manually renewed fleets are not taken into account!):**

(1) Creating an **ordinal-scaled predictor** "**LR as of 31st of July**" -
grouped into classes „up to 45 %", „45 to 55 %",….,till "higher than 95 %"
(**Attention:** Further grouping should be envisaged in the modelling process!).

(2) Defining the **ordinal scaled response** "**LR as of 31st of December**"
on the basis of the „rules for automatic renewal" (see chapter 1 ➔ 7 LR-classes).

(3) For each of the k LR-classes as of 31st of July (k=1 to 7), be p_i^k the **transfer-probability**
that the fleet falls into the i-th LR-class as of 31st of December (i=1 to 7).
Then the cumulative logit function for the i-th LR-class is $g(p_1^k, p_2^k, ..., p_i^k) = \log(\sum_{j=1}^{i} p_j^k / (1 - \sum_{j=1}^{i} p_j^k))$

(4) Finally, through a simple recursion all the estimates for the p_i can be determinded –
and this in dependence of the respective linear predictor η.

(5) For each LR-group (as of 31st of July), the weighted LR-group (as of 31st of December) is calculated
(the weights consist of the transfer-probabilities).

(6) If need be, a correction factor is added to the weighted LR-group (e. g. 5 %-pts.) so as to heighten the predictivity.

3. (cred/Bonus-Malus)
Prediction of the Loss-Ratio through Multinomial Approach

In a nutshell (tbc) – clarification for point 5 + 6:

correction factor: 5 %-pts

LR as of 31.7. (grouped)	LR-interval (31.7.)	LR as of 31.12. (weighted) [1)]	incl. corrction factor	LR-interval (31.12.) prognosed
35%	0-45%	20,9%	25,9%	0-45%
45%	45-55%	43,5%	48,5%	45-55%
55%	55-65%	49,6%	54,6%	45-55%
65%	65-75%	55,4%	60,4%	55-65%
75%	75-85%	61,0%	66,0%	65-75%
85%	85-95%	66,2%	71,2%	65-75%
95%	95 % and higher	71,0%	76,0%	75-85%

1) weighted with transfer-probabilities

Beware!
Whilst examining the predictivity next year, the same correction factor must be used as in the modelling.

Determination of the correction factor:

- **Heuristic approach:** get into the middle of the interval (as of 31.7.)
- **Optimization of the validation results:**

  see later „Validation of the model“.
- **Backtesting** before applying the final model to the most actual period.

3. (cred/Bonus-Malus)
Prediction of the Loss-Ratio through Multinomial Approach

General modelling results:

Difference between observed/estimated transfer-probability		F- and Chi-Square-statistics			
Std-Deviance	Std. Error of Mean	FValue	ProbF	ChiSq	ProbChiSq
0,1392	0,0096	291,74	<.0001	291,74	<.0001

Predictor „LR as of 31st of July (grouped)“ highly significant

with regard to the Response

„LR as of 31st of December (grouped according to rules for automatic renewal)“

3. (cred/Bonus-Malus)
Prediction of the Loss-Ratio through Multinomial Approach

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Parameter Estimates Statistics:

Parameter	lower conf.limit	Estimate	upper conf.limit	Std.error	ProbChiSq
Intercept1	0,43	0,75	1,07	0,16	<.0001
Intercept2	1,21	1,56	1,91	0,18	<.0001
Intercept3	1,96	2,35	2,75	0,20	<.0001
Intercept4	2,61	3,04	3,47	0,22	<.0001
Intercept5	3,15	3,61	4,08	0,24	<.0001
Intercept6	3,61	4,10	4,59	0,25	<.0001
LR (31/7) – grouped	-5,12	-4,53	-3,94	0,30	<.0001

The intercepts behave very organic and there is in any case practically

no overlapping of the conf. limits with the former/latter parameter estimate.

3. (cred/Bonus-Malus)
Prediction of the Loss-Ratio through Multinomial Approach

Validation of the model (20 % sample):

max	q99	q95	q90	q75	q50	q25	q10	q5	q1	min
18,4%	18,4%	6,7%	4,7%	2,1%	0,5%	-4,6%	-9,1%	-13,4%	-29,0%	-29,0%

The median of the difference between the estimated transition-probability (modelled via the 80 %-test sample) and the observed one in the validation sample is pretty close to zero.

But the tendency is clearly towards a bigger observed value than estimated ones (➔ skewness towards left).

3. (cred/Bonus-Malus)
Prediction of the Loss-Ratio through Multinomial Approach

Final transition probabilities (2 examples):

LR-class (as of 31st of July)	LR-class (as of 31st of Dec.)	transition prob. JUL==>DEC	lower conf.limit	transition prob. (cumulative)	upper conf.limit
0.45%	0-45%	70,1%	65,1%	70,1%	75,1%
0.45%	45-55%	14,4%	79,5%	84,5%	89,5%
0.45%	55-65%	9,9%	89,4%	94,4%	98,0%
0.45%	65-75%	2,7%	92,1%	97,1%	98,9%
0.45%	75-85%	1,3%	93,4%	98,4%	99,5%
0.45%	85-95%	0,8%	94,2%	99,2%	99,4%
0.45%	higher than 95%	0,8%	---	---	---

85-95%	0-45%	7,1%	2,1%	7,1%	8,7%
85-95%	45-55%	7,4%	9,5%	14,5%	16,6%
85-95%	55-65%	13,7%	23,2%	28,2%	32,8%
85-95%	65-75%	17,1%	40,3%	45,3%	51,2%
85-95%	75-85%	15,9%	56,2%	61,2%	67,0%
85-95%	85-95%	19,5%	75,7%	80,7%	84,8%
85-95%	higher than 95%	19,3%	---	---	---

Confidence limits show the high reliability of the estimators for the transition probability.

3. (cred/Bonus-Malus)
Prediction of the Loss-Ratio through Multinomial Approach

Check of predictivity – AP 2021 prognosed (through model in 2019) vs. observed:

Prerequisites:

- Fleet was automatically renewed
  (i. e. manually renewed fleets were not taken into account).

- Technical premium is available
  (➔ un-typical large fleets – e. g. with only a few vehicles left - are excluded).

3. (cred/Bonus-Malus)
Prediction of the Loss-Ratio through Multinomial Approach

Check of predictivity – transition probabilities in 2020:

LR-class (as of 31st of July)	LR-class (as of 31st of December)	# fleets	transition prob. JUL==>DEC (observed)	transition prob. JUL==>DEC (prognosed)

55_ - below 65 %	below 45 %	22	27,5%	26,6%
55_ - below 65 %	45_ - below 55 %	22	27,5%	19,0%
55_ - below 65 %	55_ - below 65 %	14	14,6%	26,5%
55_ - below 65 %	65_ - below 75 %	17	19,4%	18,3%

85_ - below 95 %	45_ - below 55 %	9	10,1%	7,5%
85_ - below 95 %	55_ - below 65 %	12	19,2%	17,8%
85_ - below 95 %	65_ - below 75 %	16	31,2%	19,6%
85_ - below 95 %	75_ - below 85 %	12	19,2%	15,7%
85_ - below 95 %	85_ - below 95 %	10	13,2%	16,7%
85_ - below 95 %	from 95 % onwards	7	4,1%	14,2%

The partial lockdown towards the end of 2020 in Germany due to Covid clearly to be seen in the 1st block (observed transition probability tends to be lower than the expected one).

3. (cred/Bonus-Malus)
Prediction of the Loss-Ratio through Multinomial Approach

Check of predictivity – AP 2021 prognosed (through model in 2019) vs. observed:

LR-class (as of 31st of July)	# fleets	AP 2020	AP 2021 (observed)	AP 2021 (prognosed)	relative discrepancy (progn. vs obs.)	absolute discrepancy (progn. vs obs.)
below 45 %	297	17.219.036	15.110.641	14.553.020	-3,7%	-557.621
45_- below 55 %	80	5.323.524	4.999.047	4.763.949	-4,7%	-235.098
55_- below 65 %	62	4.176.167	4.035.603	3.944.817	-2,2%	-90.787
65_- below 75 %	39	2.466.358	2.385.370	2.329.727	-2,3%	-55.643
75_- below 85 %	32	2.871.756	2.929.079	2.855.439	-2,5%	-73.640
85_- below 95 %	33	3.219.262	3.377.151	3.200.971	-5,2%	-176.181
from 95 % onwards	38	2.414.876	2.541.611	2.521.212	-0,8%	-20.399
	581	**37.690.977**	**35.378.503**	**34.169.134**	**-3,4%**	**-1.209.369**

Predictivity pretty satisfactory with a discrepancy of „only“ 1,2 Mio. € between observed and prognosed (despite the afore metioned partial lockdown due to Covid).

Overview

1. The flatrate model cred/Bonus-Malus
2. The flatrate model cred/new calc.
3. (cred/Bonus-Malus)
   Prediction of the Loss-Ratio through Multinomial Approach
4. **(cred/new calc.)**
   Prediction of the market adjustment through Multinomial Approach

4. (cred/new calc.)
Prediction of the market adjustment through multinom. appr.

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Predicament:

Since each fleet has to be calculated anew on the basis of the CP,

the need arises to predict the market adjustments for the prognosis of the topline.

- ➔ Prediction of the **market adjustment (AP/CP-1 in %)** as of 1st of January next year on the basis of the AP/CP as of 31st of July .
- ➔ Eventual price optimization while fixing the premium level for each fleet.

Initial problem to be solved:

Grouping of both KPIs can‘t be derived automatically as in the former chapter

(➔ rules for automatic renewal are fixed for cred/Bonus-Malus).

4. (cred/new calc.)
Prediction of the market adjustment through multinom. appr.

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<u>determination of the relevant predictors:</u>

A possible discount-budget doesn‘t only depend on the AP/CP (➔ Authorities).

<u>explanation conc. authorities:</u>

If certain criteria are met, the underwriter gets for each fleet a discount-budget

example: for a fleet size from 100 vehicles on ➔ maximum discount of 15 %,

for a fleet size below 100 vehicles on ➔ maximum discount of 3 %.

<u>The following variants are examined:</u>

V1: 1 predictor (AP/CP grouped),

V2: 2 predictors (AP/CP grouped + flag for authorities).

4. (cred/new calc.)
Prediction of the market adjustment through multinom. appr.

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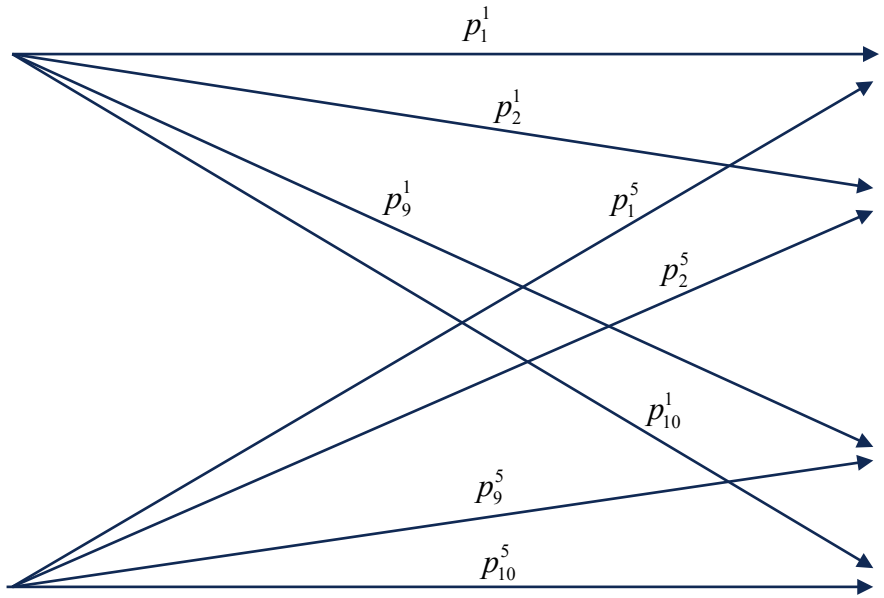
AP/CP (grouped) as of 31.7.2020 ●	
k=1	up to 70%
k=2	70-80%
k=3	80-90%
k=4	90-100%
k=5	from 100%

● 5 classes



AP/CP (grouped) as of 1.1.2021 ●		representative as of 1.1.2021
i=1	0,30 up to 0,65	0,50
i=2	0,65 to -0,70	0,65
	in 5%-steps	...
i=9	1,00 to 1,05	1,00
i=10	1,05 to 3,00	1,20

● 10 classes

4. (cred/new calc.)
Prediction of the market adjustment through multinom. appr.

The weighted adjustment-group (as of 31st of December) for each AP/CP-group (as of 31st of July) – incl. corr.factor:

AP/CP-interval (31.7.)	AP/CP as of 1.1. (weighted) 1)
30-70%	61,7%
70-80%	72,6%
80-90%	82,6%
90-100%	91,5%
from 100%	95,2%

1) representatives of AP/CP-interval (as of 1.1.) weighted with transfer-probabilities

Beware!
Whilst examining the predictivity next year, the same representatives are to be used as in the modelling (see p. 26).

Calculation of the AP (prognosed):

AP/CP as of 1.1. (weighted) multiplied by CP as of 31.7.

4. (cred/new calc.)
Prediction of the market adjustment through multinom. appr.

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General modelling results:

Source	deviance		Fisher-Statistics		significance level	
	variant 1	Variant 2	variant 1	Variant 2	variant 1	Variant 2
Intercepts	540,4	616,6	---		---	
AP/CP (grouped)	95,8	160,2	204,2	312,1	<.0001	<.0001
flag for authorities	---	141,8	---	12,5	---	0,0006

Predictor „AP/CP as of 31.7.2020 (grouped)“ together with the flag for authorities highly significant with regard to the response „adjustment group as of 1.1.2021“

4. (cred/new calc.)
Prediction of the market adjustment through multinom. appr.

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Parameter Estimates Statistics:

	estimate		standard error		significance level (Chi-Square)	
Source	variant 1	variant 2	variant 1	variant 2	variant 1	variant 2
Intercept1	2,84	1,46	0,42	0,34	<.0001	<.0001
Intercept2	3,82	2,42	0,42	0,34	<.0001	<.0001
Intercept3	4,90	3,49	0,45	0,36	<.0001	<.0001
Intercept4	5,96	4,55	0,49	0,40	<.0001	<.0001
Intercept5	7,08	5,69	0,54	0,43	<.0001	<.0001
Intercept6	7,78	6,42	0,57	0,46	<.0001	<.0001
Intercept7	8,68	7,35	0,61	0,49	<.0001	<.0001
Intercept8	9,77	8,46	0,66	0,53	<.0001	<.0001
Intercept9	10,52	9,24	0,69	0,56	<.0001	<.0001
APCP (grouped)	-2,12	-2,10	0,13	0,10	<.0001	<.0001
flag for authorities	---	1,20	---	0,19	---	0,0004

- Organic behaviour of the intercepts (similar to the model for cred/Bonus-malus).
- The standard error in a satisfying range (variant 2 seems to be slightly superior).
- All intercepts (including the predictors) are highly significant

  ➔ granularity of the adjustment groups still acceptable.

4. (cred/new calc.)
Prediction of the market adjustment through multinom. appr.

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Validation of the model (20 % sample):

	Max	99%-Q.	95%-Q.	90%-Q.	75%-Q.	50%-Q.	25%-Q.	10%-Q.	5%-Q.	1%-Q.	Min
variant 1	0,10	0,10	0,10	0,04	0,02	-0,01	-0,04	-0,14	-0,17	-0,24	-0,24
variant 2	0,13	0,13	0,09	0,04	0,02	-0,05	-0,16	-0,41	-0,46	-0,48	-0,48

	standard deviance	standard error of the mean
variant 1	0,167	0,018
variant 2	0,245	0,029

Variant 1 gets significantly better validation results than variant 2 –

both with regard to the skewness of the quantile distribution (➔ skewness towards left)

and the standard error.

But: Higher granularity of variant 2.

4. (cred/new calc.)
Prediction of the market adjustment through multinom. appr.

	transfer-p
	20-30%
	10-20%
	bis 10%

Final transition probabilities (for 2 AP/CP-groups):

	AP/CP (grouped) as of 1.1.2021										
AP/CP 31.7.2020 (grouped)	**AP 7/2020**	**(group 1) 0,30 to 0,65**	**(group 2) 0,65 to 0,70**	**(group 3) 0,70 to 0,75**	**(group 4) 0,75 to 0,80**	**(group 5) 0,80 to 0,85**	**(group 6) 0,85 to 0,90**	**(group 7) 0,90 to 0,95**	**(group 8) 0,95 to 1,00**	**(group 9) 1,00 to 1,05**	**(group 10) 1,05 to 3,00**
...						---					
0,70-0,80	31.494.366	26,2%	22,4%	25,0%	15,4%	7,1%	1,9%	1,2%	0,5%	0,1%	0,1%
...						---					
0,90-1,00	23.914.357	0,3%	0,5%	1,5%	4,1%	10,8%	12,4%	21,2%	24,6%	11,3%	13,3%
...						---					

possible results:

- The Technical Area succeeds to keep the majority of the profitable fleets in the profitable segment, but
- Obvious difficulty to reduce the losses for highly unprofitable fleets (possible reasons:
  - „political fleets“ respectively very large fleets,
  - overall profitable customer (only the fleet is highly unprofitable).

4. (cred/new calc.)
Prediction of the market adjustment through multinom. appr.

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Check of predictivity – AP 2022 prognosed (through model in 2020) vs. observed:

Prerequisites:

- Fleet not flagged for portfolio cleaning (due to problems to fit an adequate model).

- Technical premium is available
  (➔ un-typical „large fleets“ – e. g. with only a few vehicles left - are excluded).

4. (cred/new calc.)
Prediction of the market adjustment through multinom. appr.

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Check of predictivity – AP 2022 prognosed (through model in 2020) vs. observed:

a.) Adjustment factor for AP (as of 31.7.2021) - observed vs prognosed (quantil analysis)

		maximum	95%-Q.	90%-Q.	75%-Q.	median	25%-Q.	10%-Q.	5%-Q.	minimum
observed		2,61	1,10	1,03	0,94	0,89	0,83	0,76	0,70	0,22
variant 1	prognosed	1,05	0,95	0,94	0,91	0,89	0,85	0,82	0,79	0,78
variant 2	prognosed	1,24	1,12	1,09	1,06	1,04	1,00	0,97	0,94	0,91

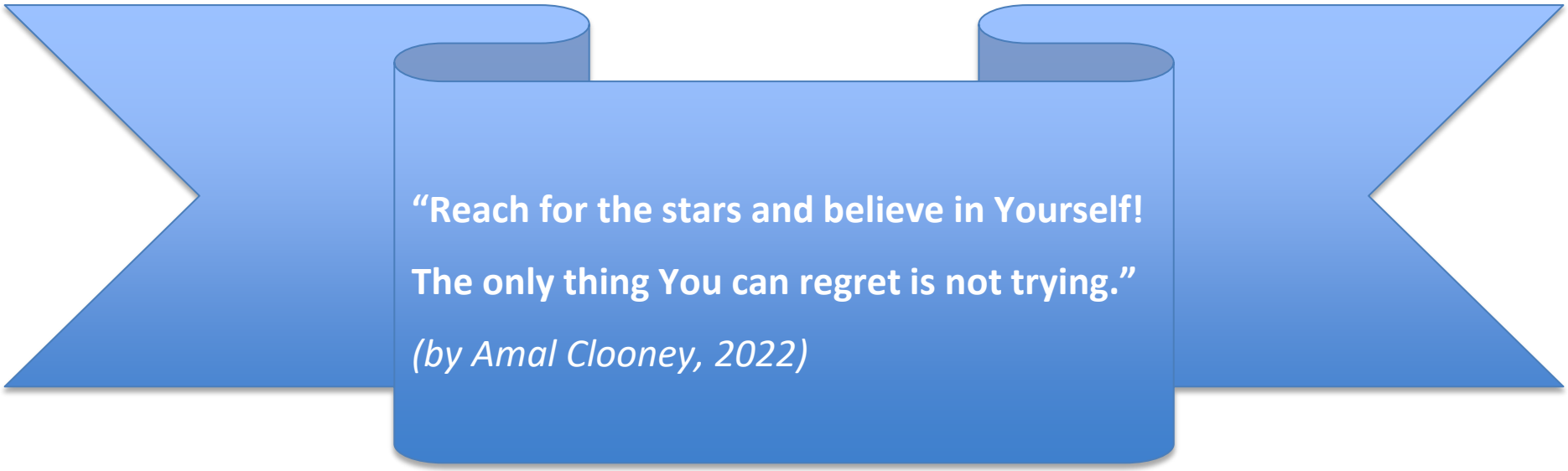
✓ Variant 1 allows for better prognosis than variant 2 (though the latter one being more granular)

variant 1:

AP/CP-group (as of July 2021)	number of fleets	AP 2021 (JUL)	AP 2022 (JAN) observed	AP 2022 (JAN) prognosed	AP2022 progn. vs observed (in %)	AP2022 progn. vs observed (in EUR)
30-70%	93	18.893.207	19.061.678	18.755.180	-1,6%	-306.498
70-80%	121	12.032.039	11.708.568	12.093.895	3,3%	385.327
80-90%	223	41.782.325	41.626.014	41.924.646	0,7%	298.632
90-100%	156	29.366.966	28.108.916	28.694.992	2,1%	586.076
from 100%	126	22.341.472	21.558.399	21.478.370	-0,4%	-80.029
total	**719**	**124.416.009**	**122.063.575**	**122.947.083**	**0,7%**	**883.508**

With variant 1, the discrepancy between prognosed and observed AP is just

+0,7 % respectively 0,9 mio. EUR.



Though Amal Clooney addressed herself specifically to women,

the coping with challenges („to go for it“) is intrisically human.

Thank you for your attention

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