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Decentralized Insurance
# DeIn and Insurance Eco-system

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Summary

Risk sharing

Risk transfer to the third party

Risk retained in the pool
Mutual Aid
Online Mutual Aid

- Zero entry cost;
- Ex-post payment;
- No pooled funds;
Equal Benefit Payment Model

Assume:

1. All participants are of homogeneous risk classification meaning the probability of loss is the same for all participants
2. All participants assume the same benefit amount
3. Losses are shared equally among participants
4. There is no frictional cost (no management fee)
Equal Benefit Payment Model

Notation:

- \( n \): number of participants
- \( b \): benefits per claim
- \( I_i \): indicator for the \( i \)-th participant’s claim status
  \[
  I_i = \begin{cases} 
  1 & \text{if } i \text{-th participant makes a claim} \\
  0 & \text{if } i \text{-th participant doesn’t make a claim}
  \end{cases}
  \]
- \( X_i \): claim from the \( i \)-th participant
  \[
  X_i = bI_i.
  \]
Equal Benefit Payment Model

There are several ways in which the total cost is allocated among members. Let \( Y \) be the cost of participation allocated to each member.

1. \textit{All to claimants}: All members pay for the costs associated with claimants.
   \[ Y = \frac{S}{n}. \]

2. \textit{Survivors to claimants}: Only survivors, those who do have legitimate claims, are asked to pay for the costs with claimants.
   \[ Y = \frac{S}{n - N}. \]

3. \textit{Capped cost}: To avoid unlimited liability for participants, the platforms often assure them by imposing a maximum cost to pay each period. Let \( d \) be the cap on cost.
   \[ Y = \frac{S}{n - N} \wedge d. \]
Peer-to-Peer Insurance
How Peer To Peer Insurance Works

P2P reverses the traditional insurance model. We treat the premiums you pay as if it’s your money, not ours. With P2P, everything becomes simple and transparent. We take a flat fee, pay claims super fast, and give back what’s left to causes you care about.

A transparent 20% fee to run everything

We pay claims super fast

If there’s money leftover, we give it back to causes
Homogeneous Models

Assumptions:

1. All participants are of homogeneous risk classification meaning the probability of loss is the same for all participants
2. All participants receive the same benefits
Homogeneous Models

Notation:

- $n$: number of participants
- $b$: benefits per claim
- $d$: premium amount
- $I_i$: indicator for the $i$th participant’s claim status
  
  $I_i = \begin{cases} 
  1 & \text{if } i\text{-th participant makes a claim} \\
  0 & \text{if } i\text{-th participant doesn’t make a claim} 
  \end{cases}$

- $X_i$: claim from the $i$-th participant

  $X_i = bI_i$. 
Homogeneous Model

$S$: total loss

$$S = \sum_{i=1}^{n} X_i.$$  

$D$: insufficient funds covered by reinsurance

$$[S - nd]_+^1$$

$D$: premium for excess-of-loss coverage

$$\Pi = \mathbb{E}[S - nd]_+$$

$S$: total surplus of the P2P Insurance fund, in general:

$$[nd - S]_+$$

$^1(x)_+, (x)_-$ are the positive part and the negative part of real number $x$, resp.
Homogeneous Model

○ Each member’s net cost of participation

\[
\frac{1}{n}\Pi + d - \frac{1}{n}(nd - S)_+.
\]

○ Let \( Y = S/n \) be each member’s share of the aggregate claim

○ Then the shared cost in a P2P insurance scheme is given by

\[
g^P(Y) = \mathbb{E}[(Y - d)_+] + d - (d - Y)_+ = \mathbb{E}[(Y - d)_+] + d \land Y.
\]
Takaful
回教保险（Takaful）

传统保险违背伊斯兰教义

参与者奉献互助金

被保险人从互助资金池中赔付

资金池余额返还

管理者承担蓄水池功能

Takaful - Wakala Model
RISK - Shared between participants
(Removes elements of uncertainty & Gambling)
Mudarabah Model

- Takaful provides a smoothing mechanism to reduce the fluctuation in the cost of participation over time.

- Had there been no smoothing mechanism, each participant is responsible for the cost \( Y = \frac{S}{n} \).

- Recall that each participant contributes \( b \) into the takaful fund. A proportion \( \rho_m \) of the surplus would be charged by the operator after the claim payment.

- A simplified form of the shared cost for each participant

\[
g^M(Y) = d - (1 - \rho_m)(d - Y)^+.
\]
CAT Risk Pooling
Risk Pooling Mechanism

CAT Risk Pooling

- **Coverage Limit**: The severity of an event at and above which the maximum payment is triggered.
- **Deductible**: The amount the country is required to pay before the reinsurance coverage kicks in.
- **Losses retained by the country**: The amount the country retains after the deductible.
- **Exhaustion Point**: The point at which the reinsurance coverage begins, typically 1/5 to 1/20 years.
- **Attachment Point**: The point at which the reinsurance coverage begins, typically 1/5 to 1/20 years.

Left panel is drawn from Bollman & Wang (2019)
Homogeneous Model

Consider the homogeneous case.

◦ Each participant is responsible for \( Y = \frac{S}{n} \).

◦ Divide the loss \( Y \) into three layers. The layers below \( d \) and above \( m \) are retained by the participant and the layer between \( d \) and \( m \) is ceded to the CAT risk pool.

◦ In a simplified model, the cost of participants is given by

\[
g^C(Y) = d \wedge Y + k + [Y - m]^+, \]

where \( k = \mathbb{E} \left[ (Y - d) I_{\{d < Y < m\}} \right] \),
Non-Olet Risk Pooling
Consider $n$ economic agents (individual, corporate, country, etc) to share risks with each other. Let $Y_i$ be the risk of agent $i$ prior to an exchange for $i = 1, 2, \cdots, n$. Therefore, the risk vector prior to the exchange is $(Y_1, \ldots, Y_n)$.

The goal is to find a risk exchange $(X_1, \cdots, X_n)$ so that

$$\sum_{i=1}^{n} Y_i = \sum_{i=1}^{n} X_i.$$  

(Loss conservation)
Most non-olet risk pooling schemes are based on Pareto optimality. A risk pooling scheme \((Y_1^*, \ldots, Y_n^*)\) is Pareto-optimal if there is no other risk pooling scheme \((Y_1, \ldots, Y_n)\) that satisfies the conservation condition and

- \(Y_i^* \succeq Y_i\) for all \(i = 1, \ldots, n\);
- \(Y_i^* \succ Y_i\) for at least one \(i = 1, \ldots, n\).

Pareto-optimal risk pooling schemes may not be unique or even be better than the original risks. Therefore, we often work with the Pareto-optimal risk pooling scheme \((Y_1, \ldots, Y_n)\) that satisfies two additional properties.

- (Individual rationality) \(Y_i \succeq X_i\) for all \(i = 1, \ldots, n\);
- (Actuarial fairness) \(p[Y_i] = p[X_i]\) for some premium principle \(p\) and all \(i = 1, \ldots, n\).
Non-Olet

It is often the case that Pareto optimal risk exchange is always based on a division of the aggregate risk $S$.

This type of risk exchanges has been referred to in the literature as “non-olet” risk exchange. All losses are first merged, and then divided up. The term “non olet” (“l’argent n’a pas d’odeur”) is used in reference to a Vespasian anecdote.

Suppose that each agent uses an exponential utility function,

$$ u_i(x) = \frac{1}{\alpha_i} (1 - e^{-\alpha_i x}), \quad i = 1, \ldots, n, $$

where $\alpha_i$ is the constant risk aversion of agent $i$.

The fair Pareto optimal risk exchange is given by

$$ Y_i = \mathbb{E}[X_i] + \frac{\alpha}{\alpha_i} (S - \mathbb{E}[S]), $$

where $1/\alpha = \sum_{i=1}^{n} 1/\alpha_i$.

Borch (1964), Denuit and Robert (2021), etc.
P2P Networks
P2P Networks

Ideal decentralized systems would involve no central fund or facilitator, hence the theory behind P2P networks. Cash flows directly between participants.
The P2P risk sharing framework enables each claim to be paid directly from one participant to another without the process of aggregation. In other words, 

\[ \sum_{i=1}^{n} X_i = \sum_{i=1}^{n} Y_i, \quad Y_i = h_i(X_1, \ldots, X_n) \text{ for } i = 1, \ldots, n, \]

for some deterministic functions \( h_i, i = 1, \ldots, n \).

Example: quota share risk exchange \( Y = AX \), where \( Y = (Y_1, \ldots, Y_n), X = (X_1, \ldots, X_n), A = (\alpha_{ij})_{i,j=1,\ldots,n} \).

Charpentier et al. (2021)
The allocation coefficient $A$ is determined by the optimization problem

$$\min_A \sum_{i=1}^n \text{Var}(Y_i) = \min_A \text{tr}(A\Sigma A^\top)$$

due to the introduction of a fairness constraint

$$\hat{A} = A\mu = \mu, \quad e^\top A = e^\top$$

where $\mathbb{E}(X) = \mu$ and $\text{Cov}(X) = \Sigma$ are mean and positive definite covariance matrix of all agents’ pre-exchange losses.

The optimal allocation matrix is

$$\hat{A} = \frac{1}{n}ee^\top + k \left( I - \frac{1}{n}ee^\top \right) \mu \mu^\top \Sigma^{-1} \quad \text{where} \quad k^{-1} = \mu^\top \Sigma^{-1} \mu.$$
Unified Framework
Unified Framework

- Mutualization (risk diversification)
  Share losses among peers

\[ \sum X_i = \sum Y_i \]

- Risk transfer to third parties

\[ Y_i = g(Y_i) + \text{(ceded risk)}. \quad \text{(retained risk)} \]

- Examples:
  - Traditional ins: \( g^T(Y) = \mathbb{E}[Y] \);
  - P2P ins: \( g^P(Y) = \mathbb{E}[(Y - d)_+] + d \wedge Y \);
  - Mutual aid: \( g(Y) = Y \);
  - Takaful (mudarabah): \( g^M(Y) = d - (1 - \rho_m)(d - Y)_+ \).
  - Cat risk pooling: \( g^C(Y) = d \wedge Y + k + [Y - m]_+ \).
Unified Framework

- Mutualization (risk diversification)
- Risk transfer to third parties
Unified Framework

- Compare existing risk exchange schemes;
- Design new risk exchange schemes.
  - Hybrid risk sharing: $g^H = g^M \circ g^W$.
  - P2P insurance, takaful, cat for heterogeneous risks.
  - Particular graphic structure.
  - Insureds versus investors
Ordering

- Convex order
  \[ X \leq_{cx} Y \text{ if } \mathbb{E}[\phi(X)] \leq \mathbb{E}[\phi(Y)] \text{ for all convex functions } \phi. \]

- A risk sharing scheme \( X \) is considered to be less risky than another scheme \( Y \) if \( X \leq_{cx} Y \).
We can show that these decentralized insurance schemes can all be put in convex order from the smallest to largest,

$$\mathbb{E}[X_i] \leq_{cx} g^M(Y_i) \leq_{cx} g^P(Y_i) \leq_{cx} g^C(Y_i) \leq_{cx} Y_i \leq_{cx} X_i.$$


