Modeling the Reserving Cycle Using the Fourier Transform

IAA Colloquium 2022

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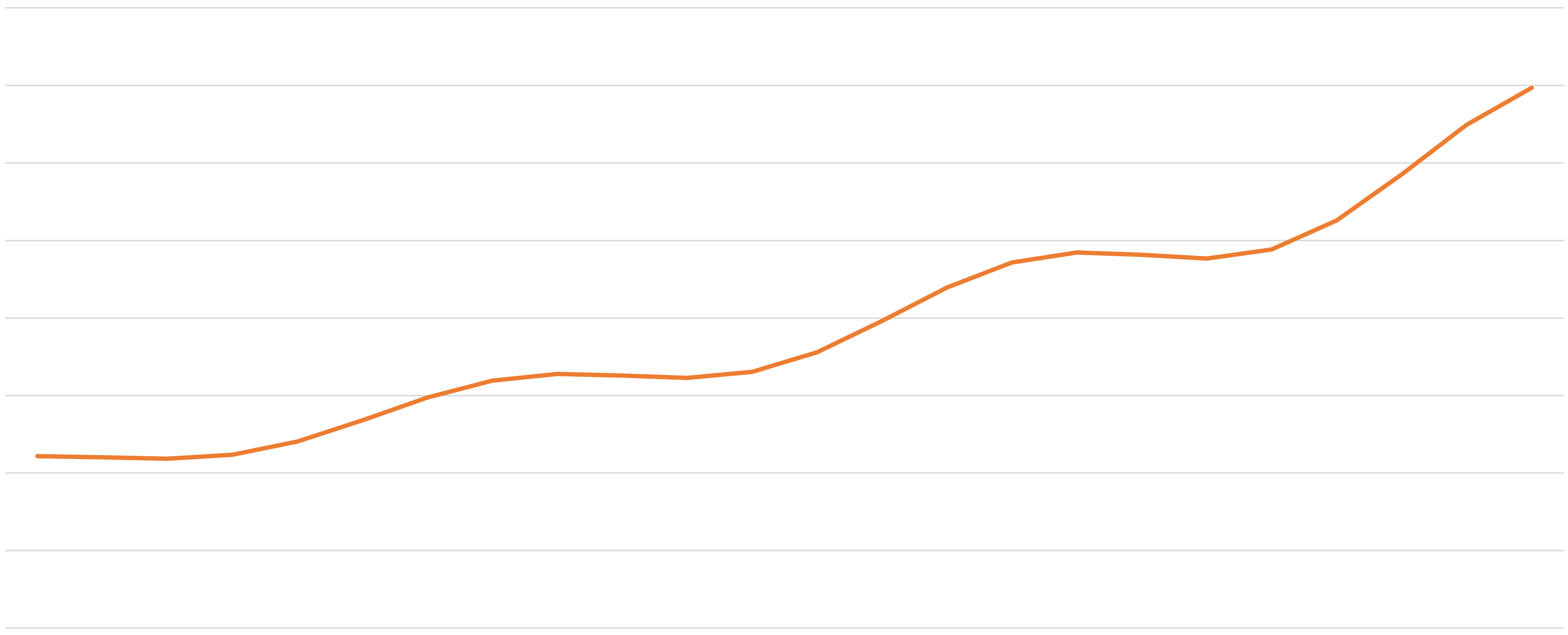
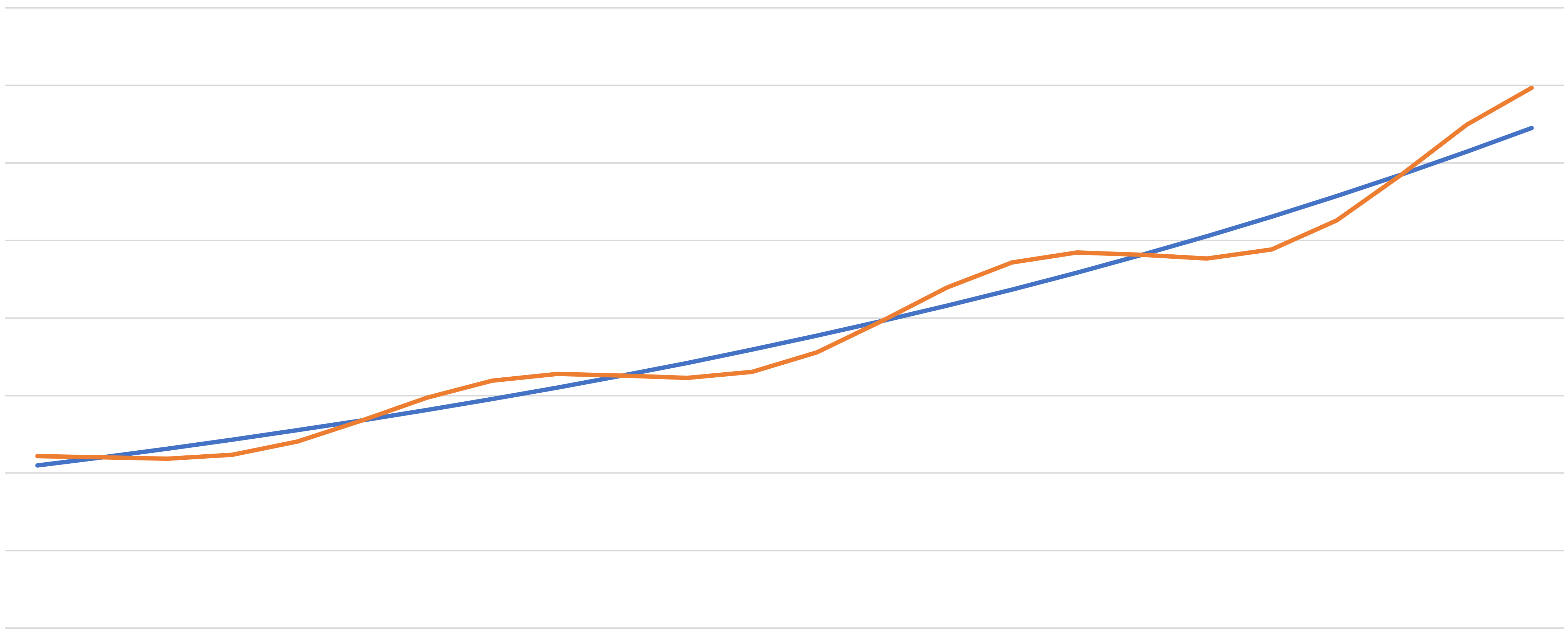
The Actuarial View of Inflation

Real Life Inflation Varies



Trend and Cycle



Concept

We can represent variable inflation as wave around an exponential function.

A complex exponential captures both the exponential growth and the wave.

US Workers Compensation
Paid Link Ratios

	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120
AY									
1997									1.011
1998								1.018	1.010
1999							1.026	1.013	1.022
2000						1.037	1.019	1.021	1.036
2001					1.048	1.025	1.033	1.034	1.002
2002				1.082	1.044	1.043	1.042	1.005	1.052
2003			1.141	1.084	1.058	1.043	1.012	1.061	1.019
2004		1.261	1.138	1.084	1.061	1.014	1.075	1.026	1.022
2005	1.931	1.274	1.136	1.092	1.026	1.087	1.029	1.027	1.016
2006	1.981	1.297	1.161	1.053	1.101	1.038	1.036	1.020	1.017
2007	2.056	1.307	1.113	1.135	1.058	1.044	1.029	1.022	
2008	2.072	1.267	1.198	1.088	1.058	1.040	1.026		
2009	1.971	1.344	1.143	1.089	1.058	1.033			
2010	2.122	1.299	1.142	1.089	1.046				
2011	2.030	1.297	1.148	1.078					
2012	2.049	1.289	1.134						
2013	2.068	1.284							
2014	2.041								

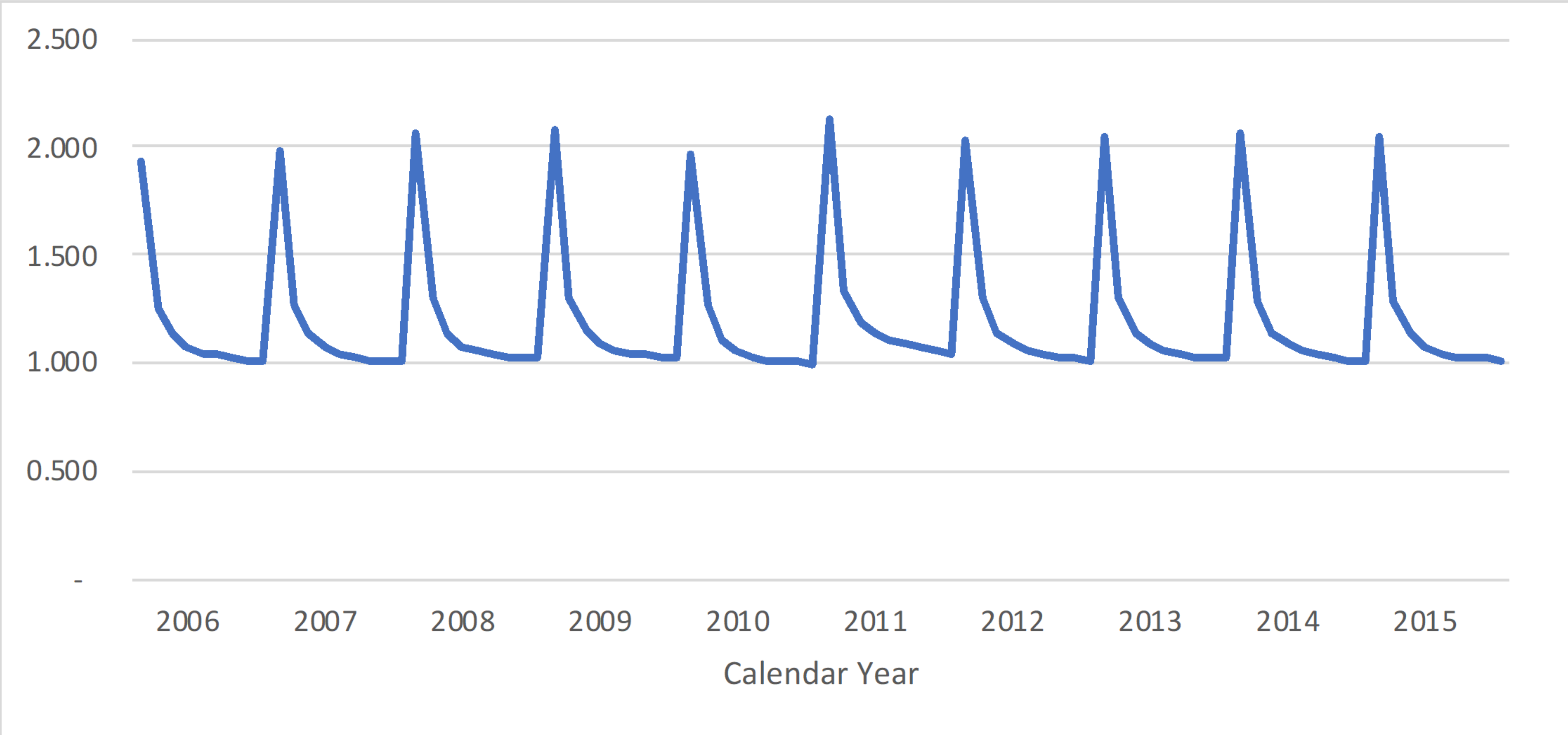
How to Model the Reserving Cycle

Sorting data by CY and age expresses incremental data as a repeated cycle.

Applying the DFT to the sequence, i.e. interpolate with a complex wave

The low frequency noise comprises the reserving cycle.

Link Ratios sorted by CY and Age



Why use Transforms?

Some in finance consider the LaPlace Transform to represent stochastic present value.

The LaPlace Transform agrees with our present value formula for deterministic cash flows and interest rate.

They agree in present value for stochastic interest rates and/or interest rates.

The LaPlace Transform is useful for solving convolution and differential equations.

Present Value or Transform

$$\int e^{-dt} f(t)$$

Real d gives present value of f

Imaginary d gives the Fourier Transform aka characteristic function of f

Complex d gives the LaPlace Transform, which is the characteristic function of discounted f

How Can This Help You?

Transforms give an explanation for the Link-Ratio method.

The link-ratio distribution at any age is a transform of the loss emergence pattern or payment pattern distribution.

A Typical Reserving Assumption

$$Cumulative(n+1) = Cumulative(n) * Incremental(n+1)$$

Transformed Reserving Assumption

$$T[Cumulative(n+1)] = T[Cumulative(n)]T[Incremental(n+1)]$$

Taking transforms converts convolution to multiplication

Rearrange Terms

$$\frac{T[Cumulative(n+1)]}{T[Cumulative(n)]} = T[incremental(n+1)]$$