INDEPENDENCE OR … COPULAS!

CAPTURING THE DEPENDENCE AMONG LARGE LOSSES USING EXTREME-VALUE COPULAS

THIAGO DUTRA DE ARAÚJO

- PhD candidate in Finance and Actuarial Science at University of São Paulo
- MSc in Statistics Education from University of São Paulo
- BSc in Actuarial Science and Mathematics from University of São Paulo

JOÃO VINÍCIUS DE FRANÇA CARVALHO

- Assistant Professor at University of São Paulo
- BSc in Actuarial Science from University of São Paulo
- MSc and PhD in Statistics from University of São Paulo

Thousands Evacuate in Worst Australian Floods in a Decade

Tens of thousands of people have been ordered to evacuate their homes and hundreds of thousands more have been told to prepare to flee as parts of Australia's southeast coast are inundated by the worst flooding in more than a decade.

By Associated Press | March 1, 2022, at 10:52 a.m.

Death toll in Brazil's Petropolis mudslides, floods hits 176; more than 110 missing

Published: 21 Feb 2022 - 04:01 pm | Last Updated: 21 Feb 2022 - 04:02 pm

Ukraine conflict: The civilian lives lost to Russia's war

COVID-19 loss of $44 bln is 3rd largest catastrophe cost to insurers

Reuters

January 4, 2022
1:41 PM GMT-3
Last Updated 2 months ago

OBJECTIVE/METHOD

Objective

Estimate the dependence structure among large losses from a single event that generated multiple claims in different lines of business using **extreme-value copulas** (EVC), applying to real-world insurance data and **compare** to other families of copulas.

Methodology

- Copula fitting → *cross-validation* Copula Information Criterion (CIC)
- Copula parameters estimation:
  I) Method-of-Moments Estimator (MME)
  II) Maximum Pseudo-Likelihood Estimator (MPLE)
  III) Method of Capéraà-Fougères-Genest (MCFG)

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- Copula fitting → *cross-validation* Copula Information Criterion (CIC)
- Higher the CIC → Most suitable is the model *(Grønneberg & Hjort, 2014)*
- Copula models analyzed:

Copula family	Elliptical copulas	Archimedean copulas	Extreme-value copulas (EVC)
Copula models	Gaussian	Clayton	Gumbel-Hougaard
	t-Student	Frank	Tawn
		Joe	Galambos
			Husler-Reiss

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- Copula parameter estimation ($\hat{\theta}$):

I) Method-of-Moments Estimator (MME) *(Oakes, 1982; Genest & Rivest, 1993)*

I.a) Calculate $\hat{\tau}$ (Kendall´s tau estimator [sample version]);

I.b) Calculate $\hat{\theta}$ by the inverse of the function $\tau(\theta)$.

II) Maximum Pseudo-Likelihood Estimator (MPLE) *(Kojadinovic & Yan, 2010)*

II.a) Create a sample of pseudo-observations $U_{i,n} = \left(\frac{r_{iX}}{n+1}, \frac{r_{iY}}{n+1}\right)$;

II.b) Calculate $\hat{\theta}$ using the maximum log-likelihood from $U_{i,n}$.

III) Method of Capéraà-Fougères-Genest (MCFG) *(only for EVC)*

III.a) Estimate *Pickands dependence function* $A(t)$;

III.b) Gudendorf & Segers (2012) version:

$$\widehat{A}'_{CFG}(t) = \exp\left[\frac{1}{n}\sum_{i=1}^{n}\left(\zeta_{i,n}(1) - \zeta_{i,n}(t)\right)\right], with\ \zeta_{i,n}(t) = \min_{j\in\{X,Y\}} \frac{-\log\left(U_{ij,n}\right)}{t}, 1 \le i \le n$$

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SIMULATED DATA (Dependence structure is known) → **REAL-WORLD DATA (Dependence structure is unknown)**

EFFECTIVENESS OF THE METHOD

Scenario	Details
Weak	10,000 pairs (X,Y) simulated from a Gaussian copula with $\hat{\tau}_{X,Y} = 0.1$
Moderate	10,000 pairs (X,Y) simulated from a Joe copula with $\hat{\tau}_{X,Y} = 0.4$
Strong	100 pairs (X,Y) simulated from a Husler-Reiss copula with $\hat{\tau}_{X,Y} = 0.7$

CIC for copula models

Scenario	Gaussian	t-Student	Clayton	Frank	Joe	Gumbel	Tawn	Galambos	Husler-Reiss
Weak	118.537	118.535	70.839	109.322	9.217	75.841	51.404	75.254	70.826
Moderate	2048.75	2168.11	-32.16	1925.08	2953.65	2736.46	2726.26	2714.55	2673.84
Strong	69.2798	69.9732	26.681	61.8349	67.1129	74.7194	53.6635	75.1437	76.0469

EFFECTIVENESS OF THE METHOD

Estimating the copula parameter

Scenario	Method	$\hat{\theta}$	Standard deviation	Log-likelihood
Weak	MPLE	0.1539	000.010	119.5354
	MME	0.1547	636.400	119.5323
Moderate	MPLE	2.2358	0.031	2,954.5157
	MME	2.2249	*	2,954.3992
Strong	MPLE	3.2456	00,000.472	76.6722
	MME	3.0319	49,373.000	76.3523

EFFECTIVENESS OF THE METHOD

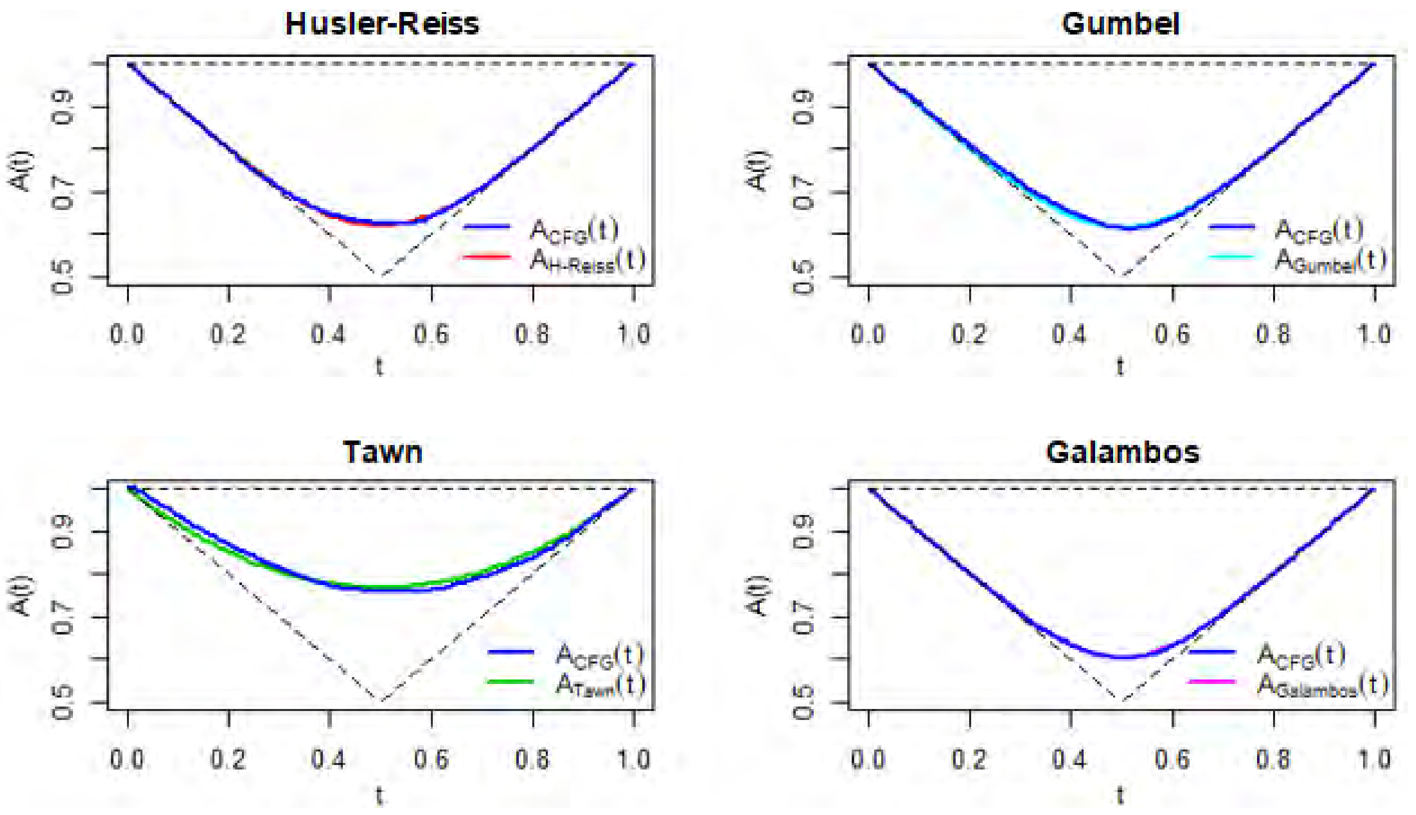
Estimating the copula parameter and upper tail coefficient

Scenario	Method	$\hat{\theta}$	Standard deviation	Log-likelihood	Upper tail coefficient
Weak	MPLE	0.1539	000.010	119.5354	0
	MME	0.1547	636.400	119.5323	
Moderate	MPLE	2.2358	0.031	2,954.5157	0.636
	MME	2.2249	*	2,954.3992	
Strong	MPLE	3.2456	00,000.472	76.6722	0.758
	MME	3.0319	49,373.000	76.3523	

EFFECTIVENESS OF THE METHOD

Comparative graphs between $\widehat{A}_{CFG}$ and $A(t)$

EVC	Dataset
Husler-Reiss	150 pairs simulated from a Husler-Reiss copula with $\hat{\tau}_{X,Y} = 0.7$
Gumbel	150 pairs simulated from a Gumbel copula with $\hat{\tau}_{X,Y} = 0.7$
Tawn	150 pairs simulated from a Tawn copula with $\hat{\tau}_{X,Y} = 0.4$
Galambos	150 pairs simulated from a Galambos copula with $\hat{\tau}_{X,Y} = 0.7$



REAL-WORLD MICRODATA

- The real-world microdata used contain all the policies of an insurance company whose claims incurred by a single event generated payments (in Brazilian Real BRL currency) in (at least) two different Brazilian insurance lines of business (LoB) in which the company operates, in the period from Jan/2007 to May/2012;

LoB Code	LoB
0531	Automobile – Hull
0553	Automobile – Motor third-party liability
0351	General third-party liability
0378	Professional third-party liability
0118	Business comprehensive
0520	Automobile – Passenger personal accidents

Pair of lines (X,Y)	(0531,0553)	(0351,0378)	(0118,0351)	(0520,0531)
Number of observations	13676	25	17	16
Mean of X + Y	13,297.03	50,958.47	108,390.74	48,384.23
Median of X + Y	7,440.30	41,968.50	15,231.74	38,738.98
St. deviation of X + Y	25,987.45	49,888.17	293,507.80	35,489.41
Maximum of X + Y	1,131,667.96	165,004.24	1,230,452.83	128,636.08

RESULTS

Scenario	Details
1	All data of the pair (0531,0553)
2	90%-quantile in each of lines 0531 and 0553
3	95%-quantile in each of lines 0531 and 0553
4	99.5%-quantile in each of lines 0531 and 0553
5	90%-quantile in sum of lines 0531 and 0553
6	95%-quantile in sum of lines 0531 and 0553
7	99.5%-quantile in sum of lines 0531 and 0553
8	All data of the pair (0351,0378)
9	All data of the pair (0118,0351)
10	All data of the pair (0520,0531)

RESULTS

Scenario	Details	Copula family
1	All data of the pair (0531,0553)	**EVC**
2	90%-quantile in each of lines 0531 and 0553	**EVC**
3	95%-quantile in each of lines 0531 and 0553	Elliptical
4	99.5%-quantile in each of lines 0531 and 0553	**EVC**
5	90%-quantile in sum of lines 0531 and 0553	Elliptical
6	95%-quantile in sum of lines 0531 and 0553	Archimedean
7	99.5%-quantile in sum of lines 0531 and 0553	Elliptical
8	All data of the pair (0351,0378)	**EVC**
9	All data of the pair (0118,0351)	Archimedean
10	All data of the pair (0520,0531)	Archimedean

RESULTS

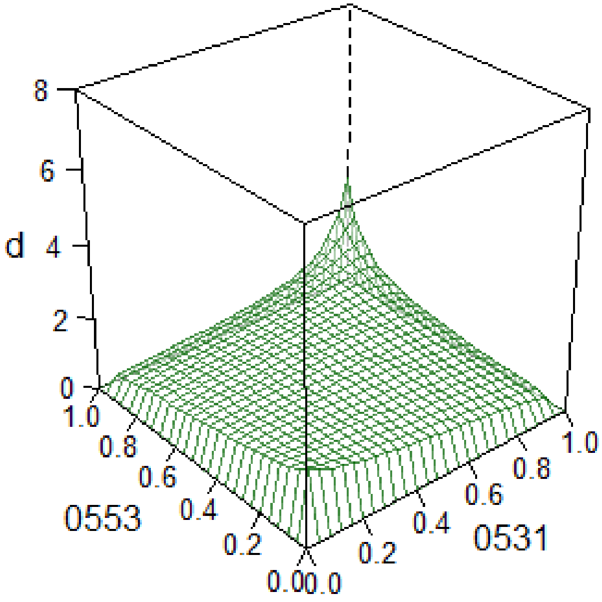
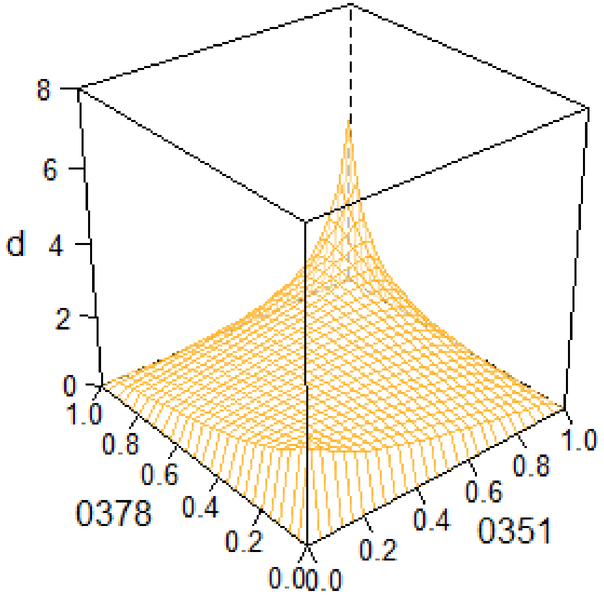
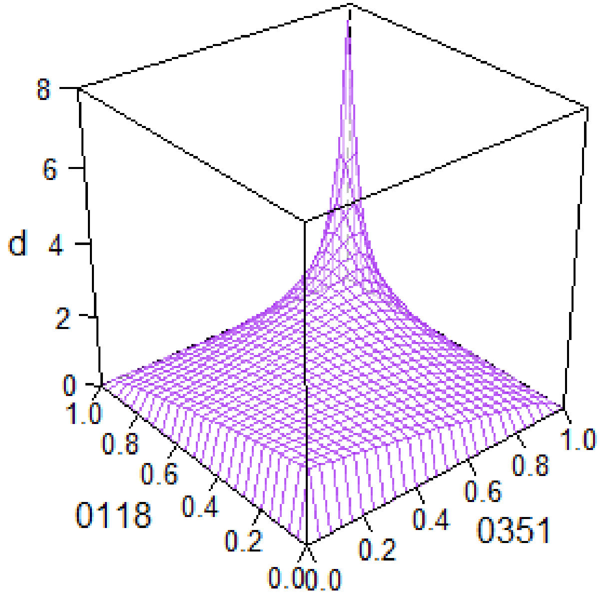
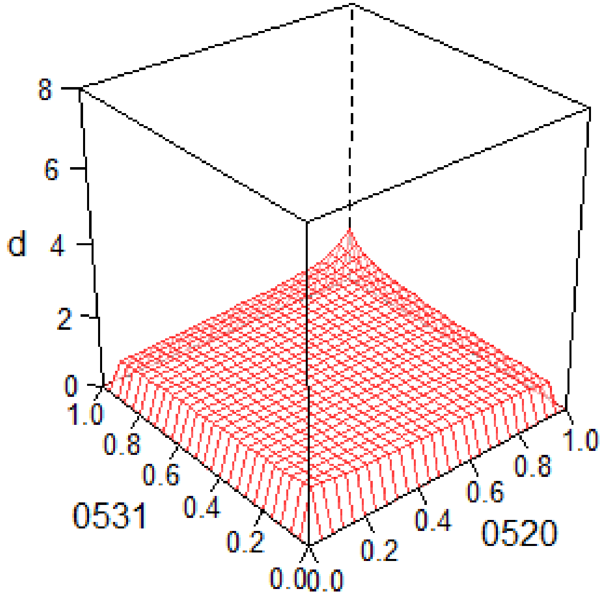
Scenario	Details	Copula family	Copula
1	All data of the pair (0531,0553)	**EVC**	**Gumbel**
2	90%-quantile in each of lines 0531 and 0553	**EVC**	**Husler-Reiss**
3	95%-quantile in each of lines 0531 and 0553	Elliptical	Gaussian
4	99.5%-quantile in each of lines 0531 and 0553	**EVC**	**Tawn**
5	90%-quantile in sum of lines 0531 and 0553	Elliptical	t-Student
6	95%-quantile in sum of lines 0531 and 0553	Archimedean	Frank
7	99.5%-quantile in sum of lines 0531 and 0553	Elliptical	t-Student
8	All data of the pair (0351,0378)	**EVC**	**Galambos**
9	All data of the pair (0118,0351)	Archimedean	Joe
10	All data of the pair (0520,0531)	Archimedean	Joe

RESULTS

Scenario	Details	Copula family	Copula	$\hat{\theta}$
1	All data of the pair (0531,0553)	**EVC**	**Gumbel**	1.23595 ± 0.008
2	90%-quantile in each of lines 0531 and 0553	**EVC**	**Husler-Reiss**	0.87226 ± 0.067
3	95%-quantile in each of lines 0531 and 0553	Elliptical	Gaussian	0.29163 ± 0.068
4	99.5%-quantile in each of lines 0531 and 0553	**EVC**	**Tawn**	0.88252 ± 0.313
5	90%-quantile in sum of lines 0531 and 0553	Elliptical	t-Student	-0.39272 ± 0.024
6	95%-quantile in sum of lines 0531 and 0553	Archimedean	Frank	-2.64214 ± 0.217
7	99.5%-quantile in sum of lines 0531 and 0553	Elliptical	t-Student	-0.37102 ± 0.130
8	All data of the pair (0351,0378)	**EVC**	**Galambos**	0.75320 ± 0.339
9	All data of the pair (0118,0351)	Archimedean	Joe	1.84144 ± 0.795
10	All data of the pair (0520,0531)	Archimedean	Joe	1.07774 ± 0.398

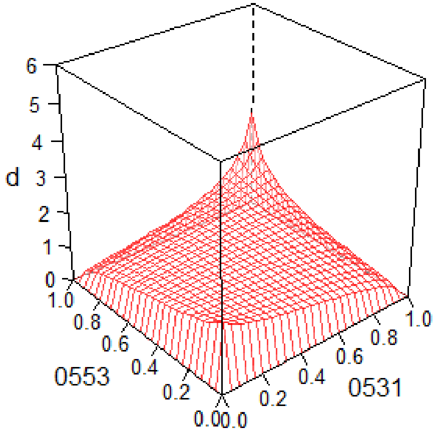
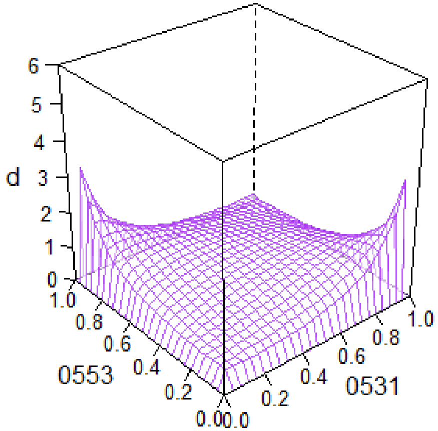
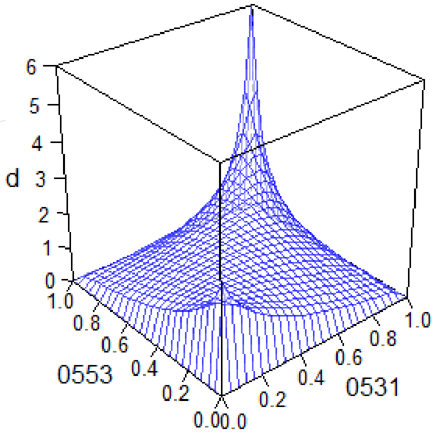
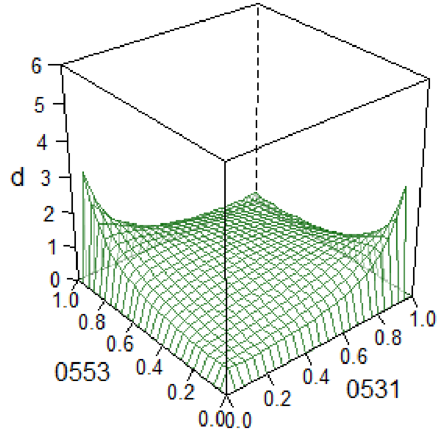
RESULTS

Joint probability density, for each of the 4 pairs

(0531,0553)	(0351,0378)	(0351,0118)	(0520,0531)
			

RESULTS

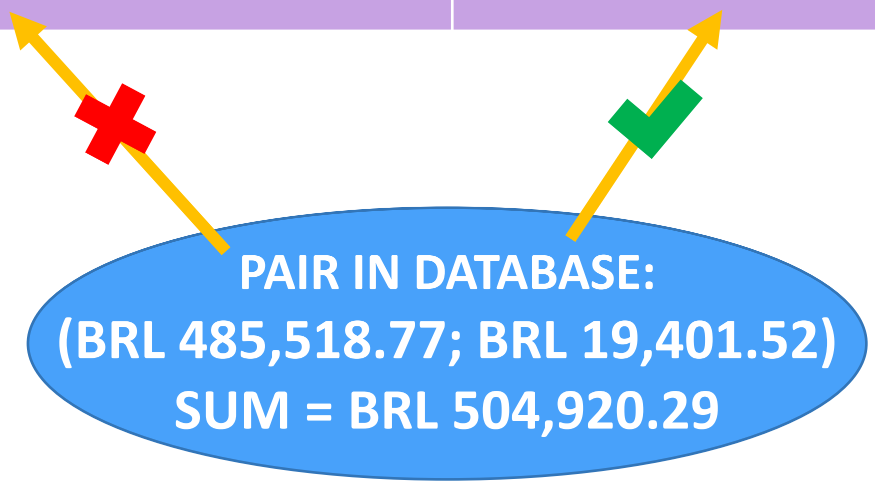
Scenario	Details
2 (EVC)	90%-quantile (individuals losses)
5 (t-Student)	90%-quantile (sum of losses)
4 (EVC)	99.5%-quantile (individuals losses)
7 (t-Student)	99.5%-quantile (sum of losses)

	Individuals losses	Sum of losses
Quantile: 90%		
Quantile: 99.5%		

RESULTS

Illustrating the difference between scenarios

	Individual loss LoB 0531	Individual loss LoB 0553	Sum of losses LoBs 0531 and 0553
99.5% quantile	BRL 137,725.95	BRL 55,449.86	BRL 145,728.54
	Scenario 4		Scenario 7

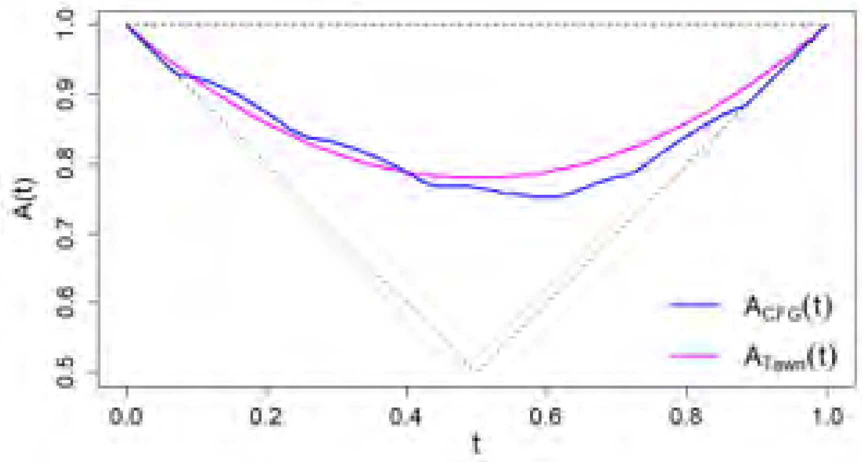


RESULTS

Scenario	Details	Upper tail coefficient λ_U
1	All data of the pair (0531,0553)	0.24789 (DF) / 0.24491 (MCFG)
2	90%-quantile in each of lines 0531 and 0553	0.25161 (DF) / 0.24163 (MCFG)
3	95%-quantile in each of lines 0531 and 0553	0
4	99.5%-quantile in each of lines 0531 and 0553	0.44126 (DF) / 0.46916 (MCFG)
5	90%-quantile in sum of lines 0531 and 055	
6	95%-quantile in sum of lines 0531 and 055	
7	99.5%-quantile in sum of lines 0531 and 05	
8	All data of the pair (0351,0378)	
9	All data of the pair (0118,0351)	
10	All data of the pair (0520,0531)	

+ 2133%

+ 3655%



THE BRAZILIAN INSURANCE REGULATORY AGENCY CASE

- SUSEP (Brazilian insurance regulatory agency) groups diferente LoB in “*business classes*”:

Classe de Negócio (k)	Nome da Classe de Negócio	Código do Ramo	Nome do Ramo
1	Residencial	0114	Compreensivo Residencial
2	Condominial	0116	Compreensivo Condomínio
3	Empresarial	0118	Compreensivo Empresarial
4	Patrimonial Demais	0111	Incêndio Tradicional (*run-off*)
		0112	Assistência – Bens em Geral
		0115	Roubo
		0141	Lucros Cessantes
		0167	Riscos de Engenharia
		0171	Riscos Diversos
		0173	Global de Bancos
		0196	Riscos Nomeados e Operacionais
		0542	Assistência e Outras Coberturas – Auto
		0711	Riscos Diversos – Financeiros
		0743	Stop Loss
5	Riscos Especiais	0234	Riscos de Petróleo (*run-off*)
		0272	Riscos Nucleares (*run-off*)
		0274	Satélites (*run-off*)
		1734	Riscos de Petróleo
		1872	Riscos Nucleares
		1574	Satélites
6	Responsabilidades	0351	R.C Geral
		0310	R.C. de Administradores e Diretores – D&O
		0313	R.C. Riscos Ambientais
		0378	R. C. Profissional

Classe de Negócio (k)	Nome da Classe de Negócio	Código do Ramo	Nome do Ramo
8	Automóvel	0520	Acidentes Pessoais de Passageiros – APP
		0523	Resp. C. T. Rodoviário Interestadual e Internacional (*run-off*)
		0524	Garantia Estendida / Extensão de Garantia – Auto
		0525	Carta Verde
		0526	Seguro Popular de Automóvel Usado
		0531	Automóvel – Casco
		0544	RC T. Viagem Intern. – Pes. Trans. ou não (*run-off*)
		0553	Responsabilidade Civil Facultativa Veículos – RCFV
		0623	Resp. C. T. Rodoviário Interestadual e Internacional – RC ÔNIBUS
		0628	Responsabilidade Civil Facultativa Veículos – RCFV-Ônibus
		0644	R. C. Transp. Em Viagem Internacional pessoas transportadas ou não – Carta Azul
		1428	Responsabilidade Civil Facultativa para Embarcações – RCF
		1528	Responsabilidade Civil Facultativa para Aeronaves – RCF

THE BRAZILIAN INSURANCE REGULATORY AGENCY CASE

- SUSEP quantifies the (linear) dependence that exists among *business classes*...

Tabela 1

Matriz de Correlação – Risco de Emissão/Precificação ($\rho_{i,j}^{prem}$)

i \ j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1,00	0,50	0,45	0,06	-0,12	0,48	0,24	0,35	0,46	0,44	0,18	-0,03	-0,01	0,33	0,04	0,18	0,24
2	0,50	1,00	0,31	0,24	0,04	0,32	-0,04	0,05	0,11	0,39	0,18	0,33	-0,07	0,05	-0,29	0,31	0,06
3	0,45	0,31	1,00	-0,33	-0,06	0,27	0,12	0,14	0,31	0,44	0,22	-0,03	0,07	-0,01	0,00	0,17	0,01
4	0,06	0,24	-0,33	1,00	0,24	0,03	0,19	0,09	0,07	0,01	-0,05	0,16	0,09	0,21	-0,15	-0,15	-0,03
5	-0,12	0,04	-0,06	0,24	1,00	0,03	-0,20	-0,09	-0,05	-0,18	0,23	0,17	-0,05	0,08	0,06	0,37	0,02
6	0,48	0,32	0,27	0,03	0,03	1,00	0,10	0,05	0,32	0,43	0,32	-0,09	-0,19	0,02	-0,09	-0,19	0,09
7	0,24	-0,04	0,12	0,19	-0,20	0,10	1,00	0,17	0,22	0,23	-0,04	0,10	0,16	0,02	-0,20	-0,28	-0,09
8	0,35	0,05	0,14	0,09	-0,09	0,05	0,17	1,00	0,39	0,26	0,19	-0,22	0,21	0,32	0,11	0,22	0,15
9	0,46	0,11	0,31	0,07	-0,05	0,32	0,22	0,39	1,00	0,13	0,14	0,00	0,24	0,25	0,22	-0,05	0,14
10	0,44	0,39	0,44	0,01	-0,18	0,43	0,23	0,26	0,13	1,00	0,11	0,01	0,08	0,20	-0,28	0,04	0,08
11	0,18	0,18	0,22	-0,05	0,23	0,32	-0,04	0,19	0,14	0,11	1,00	0,19	0,03	-0,36	-0,32	0,12	0,16
12	-0,03	0,33	-0,03	0,16	0,17	-0,09	0,10	-0,22	0,00	0,01	0,19	1,00	0,30	-0,44	-0,65	-0,21	0,03
13	-0,01	-0,07	0,07	0,09	-0,05	-0,19	0,16	0,21	0,24	0,08	0,03	0,30	1,00	-0,10	-0,11	-0,12	-0,17
14	0,33	0,05	-0,01	0,21	0,08	0,02	0,02	0,32	0,25	0,20	-0,36	-0,44	-0,10	1,00	0,45	0,30	0,13
15	0,04	-0,29	0,00	-0,15	0,06	-0,09	-0,20	0,11	0,22	-0,28	-0,32	-0,65	-0,11	0,45	1,00	0,24	0,22
16	0,18	0,31	0,17	-0,15	0,37	-0,19	-0,28	0,22	-0,05	0,04	0,12	-0,21	-0,12	0,30	0,24	1,00	0,10
17	0,24	0,06	0,01	-0,03	0,02	0,09	-0,09	0,15	0,14	0,08	0,16	0,03	-0,17	0,13	0,22	0,10	1,00

… but does **not** do so for LoB of the same *business class*!

FINAL REMARKS AND CONTRIBUTIONS

- SUSEP does **not** predict neither quantify the dependence among every pair of Brazilian lines of business in calculating the insurance company's **risk capital**;
- This study sh … lculation of **solvency capi** …
- Some of the … by SUSEP is **higher** than th …
- SUSEP metho … ctly predict **extreme events**;
- The regulator must be aware of these facts when dimensioning the **minimum capital requirement**.

Thank you very much for your attention!

Contact details:

Thiago Dutra de Araújo / João Vinícius de França Carvalho

- *mail:* thiago.dutra.araujo@usp.br / jvfcarvalho@usp.br
- *web:* www.fea.usp.br