Rebalancing the off-Balance Factor Using the Complement of Credibility

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Key Point of Presentation

- Off-balance from credibility arises when post-credibility (or ”after credibility process”) rates by class don’t weight to the total average rate.

- Current practice is to spread off-balance with factor multipied by all rates.

- **Key Point: You get more accurate and more reasonable rates when you just spread the off-balance from credibility across the 'complement of credibility' terms $(1 - Z) \times \ldots$**

Why is Using the Complement of Credibility Term Better?

- For fully credible or “nearly fully credible” data, the current process alters a rate that is known to be proper and then modifies it.
  - The current practice of spreading off-balance across the full rates **takes a rate that’s right and makes it wrong.**

- As we’ll see, the present process is also suboptimal

Example—Similar to Data Seen in Actual Practice

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	(Data)	(Data)	(2)/(1)	(Data)	(Data)	((3)/(5))-1.0	[Comb(6)] /[Wtd.(6)]	(1.0+(6))×(7)-1.0
Coverage	On-Level Earned Premium	Trended Ultimate Losses	Loss Ratio	Credibility	Permssble Loss Ratio	Indicated Changes	Off-Balance Factor	Revised Indicated Changes
A	$ 450,000	$ 900,000	200.0 %	25 %	65 %	52.3 %		63.0 %
B	$ 500,000	$ 500,000	100.0 %	27 %	65 %	14.3 %		22.3 %
C	$ 1,000,000	$ 800,000	80.0 %	38 %	65 %	8.7 %		16.3 %
D	$ 3,000,000	$ 1,400,000	46.7 %	65 %	65 %	-18.3 %		-12.6
E	$ 5,000,000	$ 3,800,000	76.0 %	100 %	65 %	16.9 %		25.1 %
Wtd. Average						6.9 %		14.4 %
Comb. Total	$ 9,950,000	$ 7,400,000	74.4 %	100 %	65 %	14.4 %	1.070	

Assigning Complement to the Overall Average Relativity, or something else that effectively multiplies by the overall rate, or rate level.

- As long as the complement is multiplied by something containing the overall claims costs—the same issue will inevitably arise.

General Results

- Present method takes rates that are right (fully credible rates) and makes them wrong.

- Especially when small classes have different loss experience than large classes, off-balance generated by smaller classes impacts large classes that did not generate off-balance.

- Off balance may be smaller, especialy when complement is close to experience data, but can also be large.

Why Allocate Across the Complement of Credibility When Using Classical Credibility?

- Maximum (optimum) Plausibility approach-Leave the Credible Data Alone
  - No change to fully credibile rates.
  - Assuming overall change is proper and must be matched, assign off-balance to rates where it is most plausible the rate is wrong (small classes).
  - Only two unbiased estimators of classes' losses to allocate across, overall rate or class rates, generates obvious bias. Instead allocate solely across the $(1 - Z)$ term.

Does Allocating the Balance Across the Complement Term Make a Difference?—Difference From First Example

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	(Data)	(Data)	(2)/(1)	(Data)	(Data)	((3)/(5))-1.0	(1.0-(4))*(5)	((1)*(7)	(1)×[Comb (6)] -[Wtd(6)]	(9)×(8) [Total (8)]	(6)+ [(10)/(1)]	Prev. Col.(8)
Coverage	On-Level Earned Premium	Trended Ultimate Losses	Loss Ratio	Credibility	Permssble Loss Ratio	Indicated Changes	Complmnt of Credibility Term	Dollar Amount of Complement	Dollar Deficiency in Base Calculation	Off-Balance Correction Pro-Rated by Dollars in Complement	Off-Balance Corrected Indications	Indications Under Old Off-Balance
A	$ 450,000	$ 900,000	200.0 %	25 %	65 %	52.3 %	48.6	$ 218,865		$ 105,474	75.7 %	63.0 %
B	$ 500,000	$ 500,000	100.0 %	27 %	65 %	14.3 %	47.8	$ 238,758		$ 115,061	37.3 %	22.3 %
C	$ 1,000,000	$ 800,000	80.0 %	38 %	65 %	8.7 %	40.6	$ 406,070		$ 195,691	28.2 %	16.3 %
D	$ 3,000,000	$ 1,400,000	46.7 %	65 %	65 %	-18.3 %	22.8	$ 682,500		$ 328,906	(7.4) %	(12.6) %
E	$ 5,000,000	$ 3,800,000	76.0 %	100 %	65 %	16.9 %	0.0	$ 0			16.9 %	25.1 %
Wtd Average						6.9 %					14.4 %	14.4 %
Comb Total	$ 9,950,000	$ 7,400,000	74.4 %	100 %	65 %	14.4 %		$ 1,546,192	$ 745,132	$ 745,132		

Allocation of Off-Balance /using Complement of Credibility

Main Calculations for Class Rates

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
i	e_i	l_i	L_i	c_i	Z_i	$Z_iL_i + (1 - Z_i)M$	$R_i \times e_i$	$e_i(1 - Z_i)$	$e_i(1 - Z_i)M \times C$	$e_i \times r_i$	$final\ r_i$	$old\ r_i$
	(Data)	(Data)	(1)/(2)	(1)×C	Sqrt of ((4)/683)	(3)×(5) +A(1.0-(5))	(6)×(1)	((1)×(1.0-(5))) × A	(8)×H	(5)×(3)×(1) +(9)	(5)×(10)/(1)	Prev Col. 7
Class	Exposure	Losses	Raw Rate "L"	Expected Claim Count	Credibility of Class	Credibility Adjusted Rate	Losses in Adjusted Rates	Losses From Complement of Credibility	Complement Losses After Off-Balance Correction	Off-Balance Corrected Total Losses	Off-Balance Corrected (Rate	Indications Under Old (Off-Balance
A	25	$ 78,427	$ 3,137	65	31 %	$ 1,324	$ 33,097	$ 8,958	$ 15,465	$ 39,605	$ 1,584	$ 1564
B	30	$ 40,687	$ 1,356	78	34 %	$ 800	$ 24,012	$ 10,293	$ 17,771	$ 31,489	$ 1,050	$ 946
C	36	$ 65,073	$ 1,808	93	37 %	$ 994	$ 35,787	$ 11,752	$ 20,290	$ 44,324	$ 1,231	$ 1,174
						… D-L not shown …						
M	223	$ 69,726	$ 313	577	92 %	$ 329	$ 73,422	$ 9,338	$ 16,122	$ 80,206	$ 360	$ 389
N	267	$ 64,108	$ 240	692	100 %	$ 240	$ 64,108	-	-	$ 64,108	$ 240	$ 283
O	321	$ 86,197	$ 269	831	100 %	$ 269	$ 86,197	-	-	$ 86,197	$ 269	$317
Total	1,801	$ 932,211	$ 518	4,661		$ 438	$ 789,053	$ 197,065	$ 340,223	$ 932,211	$ 518	$ 518

Reference Values for All Classes

A =Needed Overall Average Rate	$518
B = Severity (Computed Outside the Table)	$200
C = A/B = Expected Claims/Exposure	2.588
D = Total Losses in Data	$932,211
E = Total Losses in Adjusted Rates	$789,053
F = D-E = Off-Balance in $	$143,158
G = Total Losses in $(1 - Z_i)$ Term	$197,065
H = 1.0+F/G = Off-Balance Factor	1.742

Is This Actually Better?

- Rates of fully credibile classes not distorted.
- Classes that generate the off-balance are most responsible for absorbing it.
- No method, that isn't an additional credibility method, and uses the two unbiased estimators for each class (overall rate and class loss data) generates more plausible results.
  - "This could happen." The calculations could actually hit the true underlying loss costs.

Do Large Off-Balances Actually Happen in Regular Actuarial Work?

- Not always described as “off-balance”, could be “effect of changes in relativity factor”, or some other adjustment (credibility- induced) needed to achieve overall rate level.
- Some off-balances are per data differences, e.g., relativities- per 5 years of countrywide data, in state indication. For data differences, old method likely best.
- If you say “my off-balance has never been over 0.5%", well ”If it’s immaterial, it’s immaterial how you handle it”.
  - If you may ever have to change, it’s easier to start when it doesn’t matter.

Bailey's Best Estimate $Z = P/(P + K)$ Credibiliy

- Excellent idea that is underutilized—Maybe because of processing limitations in 1945

- By design, gives the most accurate estimate of rate needs
  - Much attention currently given to removing overlap factor with GLMs, or adding new predictors to improve rate accuracy.
  - If just converted to best estimate credibility (example later) could perhaps get even better results.

Optimum Off-Balance Approach for Best Estimate Credibility

- This also requires allocating the off-balance using the complement of credibility term.
- Why? If you start with the post-credibility (credibility-weighted class loss rate and the overall loss rate) rates as the “right”, but require that the results weight to the overall average rate, “constrained optimization” shows that allocating the difference according to the complement of credibility produces the least expected square error.
- One example (second one) rerun with best estimate credibility on next page.

Calculation of Best Estimate Credibillity and Best Estimate Allocation of the Off=Balance

- Calculations not too bad-allocating the off-balance is the easiest part.
- Not that hard to do the top variance estimates in R, although they do require work.
- It was different in Bailey's time.

Non-Bailey Situations Where Off-Balance Should be Allocated Using $1-Z$

- Bailey formula = $Z = P/(P+K)$; K =(Expected process variance)/(Variance of hypothetical means).

- Allocatiion across $(1-Z)$'s still optimal when expected process variance differs among classes.

- When variance of hypothetical means varies among classes allocate by
  - (Variance of hypothetical means for this class)$\times(1-Z)$.

Bühlmann’s Complement of Credibility

- Not new, but was ‘new to me”
- Standard ratemaking uses exposure-weighted average of loss rates, Bühlmann’s complement of credibility = loss rates weighted with the credibility of each class.
- Then post-credibility rates weight exactly to overall rate—no off-balance allocation needed.
- Results exactly equal results of allocating off-balance across the complement of credibiity.

Consequences of Matching Bühlmann's Complement of Credibility

- Given all the data for all the classes, each final Bühlmann credibility weighted rate, or $(1 - Z)$ off-balance allocated rate is the <u>expected</u> loss rate for the class under certain assumptions.

- If there is enough data for Central Limit Theorem to govern the distribution, mean of normal from above = maximum likelihood estimate.

- Allocating off-balance gives (in context) minimum expected squared difference from post credibility rates, mean estimate of loss costs, and (often) maximum likelihod estimate.

Why Not Just Use Bühlmann's Complement of Credibility

- Doesn't work for capped data

- Presentation of calculations is less intuitive.

  – Our audience usually includes more than actuaries: company managers, underwriters, financial staff, and once in my case, regulators and judges.

  – Helpful to make the presentation as intuitive as possible.

Test Correction

- Test Correction vs. Off-Balance Correction: Off-balance correction spreads off-balance resulting from credibility, test correction spreads that off-balance plus the off-balance resulting from capping.
- Often requires multiple iterations- classes fall in and out of capping
- Allocating off-balance by $(1 - Z)$ is “scalable ”. You can just increase the amount multiplied by $(1-Z)$’s to offset capping, while still getting first type of optimum estimate.
- Bühlmann’s complement of credibility, although extremely elegant, is not consistent with test correction process.

Test Correction Example - Best Estimate Data-First Step Follows

First Step: Calculations Using Uncapped Rates

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	(Data)	Prev. Table col.(5)	Prev. Table col.(10)	(1)*(3)	(Data)	.85*(5)	1.15*(5)	(3) within (6),(7)	(1)*(8)	Y=Knocked Out
Class	Exposures	Credibility Adjusted Rate	Pre-Cap Test Corrected Rate (Set 0)	Losses in Pre-Cap Test Corrected Rate	Present Rate	Cap Below	Cap Above	Capped Rates (Set 0)	Total Losses in Capped Rates	Is Rate Knocked Out of TCF by Capping?
A	25	$ 1,907	$ 2,038	$ 50,945	$ 2,000	$ 1,700	$ 2,300	$ 2,038	$ 50,945	
B	30	$ 1,000	$ 1,118	$ 33,550	$ 1,500	$ 1,275	$ 1,725	$ 1,275	$ 38,250	Y
C	36	$ 1,317	$ 1,422	$ 51,207	$ 1,200	$ 1,020	$ 1,380	$ 1,380	$ 49,680	Y
... D-L not shown ...										
M	223	$ 331	$ 356	$ 79,448	$ 350	$ 298	$ 403	$ 356	$ 79,448	
N	267	$ 261	$ 282	$ 75,473	$ 250	$ 213	$ 288	$ 282	$ 75,473	
O	321	$ 285	$ 303	$ 97,110	$ 300	$ 255	$ 345	$ 303	$ 97,110	
Total	1,801	$ 478	$ 518	$ 932,211	$ 489	$	$	$ 513	$ 923,362	

Second Step: Test Correction Step Post Capping

	(11)	(12)	(13)	(14)	(15)	(16)	(17)
	Prev. Table col.(7)	(11)) Less Knockouts	F*(10)	(4)+(13)	(14)/(1)	(15) within (5),(6)	Y=Knocked Out
Class	Original Test Correction Basis	Test Correction Basis Less Knockouts	Additional Losses for Test Correction	Revised Test Corrected Total Losses	Test Corrected Rate (Set 1)	Capped Rates (Set 1)	Is Rate Knocked Out of TCF by Capping?
A	$ 12	$ 12	$ 598	$ 51,543	$ 2,062	$ 2,062	
B	$ 13	$	$ 648	$ 34,198	$ 1,140	$ 1,275	Y
C	$ 14	$	$ 698	$ 51,905	$ 1,442	$ 1,380	Y
... D-L not shown ...							
M	$ 20	$ 20	$ 1,025	$ 80,473	$ 361	$ 361	
N	$ 20	$ 20	$ 1,041	$ 76,513	$ 286	$ 286	
O	$ 21	$ 21	$ 1,054	$ 98,164	$ 306	$ 306	
Total	$ 257	$ 174	$ 13,091	$ 945,302	$ 525	$ 517	

Reference Values for All Classes

A = (Prev. Table) Overall Average Rate in Data	$518
B = Total Loss in Set 0 Rates	$923,632
C = Total Losses in Data	$932,362
D = C-B = Shortfall	$8,849
E =Total Test Correction Basis on Non-Capped Classes (12)	$174
F = D/E = Test Correction Factor	50.92

Test Correction Example - Best Estimate Data-Last Step Follows

First Step: Calculations Using Rates from First Iteration

Class	Exposures	Credibility Adjusted Rate	Pre-Cap Test Corrected Rate (Set 1)	Losses in Pre-Cap Test Corrected Rate	Present Rate	Cap Below	Cap Above	Capped Rates (Set 1)	Total Losses in Capped Rates	Is Rate Knocked Out of TCF by Capping?
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	(Data)	Prev. Table col.(5)	Prev. Table col.(14)	(1)*(3)	(Data)	.85*(5)	1.15*(5)	(3) within (6),(7)	(1)*(8)	Y=Knocked Out
A	25	$ 1,907	$ 2,062	$ 51,543	$ 2,000	$ 1,700	$ 2,300	$ 2,062	$ 51,543	
B	30	$ 1,000	$ 1,140	$ 34,198	$ 1,500	$ 1,275	$ 1,725	$ 1,275	$ 38,250	Y
C	36	$ 1,317	$ 1,442	$ 51,905	$ 1,200	$ 1,020	$ 1,380	$ 1,380	$ 49,680	Y
				... D-L not shown ...						
M	223	$ 331	$ 361	$ 80,473	$ 350	$ 298	$ 403	$ 361	$ 80,473	
N	267	$ 261	$ 286	$ 76,513	$ 250	$ 213	$ 288	$ 286	$ 76,513	
O	321	$ 285	$ 306	$ 98,164	$ 300	$ 255	$ 345	$ 306	$ 98,164	
Total	1,801	$ 478	$ 1,978	$ 945,302	$ 489			$ 517	$ 931,786	

Second Step: Test Correction Step Post Capping

Class	Original Test Correction Basis	Test Correction Basis Less Knockouts	Additional Losses for Test Correction	Revised Test Corrected Total Losses	Test Corrected Rate (Set 2)	Capped Rates (Set 2)	Is Rate Knocked Out of TCF by Capping?
	(11)	(12)	(13)	(14)	(15)	(16)	(17)
	Prev. Table col.(7)	(11)) Less Knockouts	F*(10)	(4)+(13)	(14)/(1)	(15) within (5),(6)	Y=Knocked Out
A	$ 12	$ 12	$ 32	$ 51,575	$ 2,063	$ 2,063	
B	$ 13	$	$ 35	$ 34,233	$ 1,141	$ 1,275	Y
C	$ 14	$	$ 38	$ 51,943	$ 1,443	$ 1,380	Y
				... D-L not shown ...			
M	$ 20	$ 20	$ 55	$ 80,528	$ 361	$ 361	
N	$ 20	$ 20	$ 56	$ 76,569	$ 286	$ 286	
O	$ 21	$ 21	$ 57	$ 98,221	$ 306	$ 306	
Total	$ 257	$ 155	$ 704	$ 946,006	$ 525	$ 518	

Reference Values for All Classes

A = (Prev. Table) Overall Average Rate in Data	$518
B = Total Loss in Set 1 Rates	$931,786
C= Total Losses in Data	$932,362
D = C-B = Shortfall	$424
E =Total Test Correction Basis on Non-Capped Classes (12)	$155
F = D/E = Test Correction Factor	2.74

???