ASTIN AFIR/ERM
Actuarial Colloquia 2022

Stable Value Fund Guarantee

An Asset Liability Model

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Stable Value Guarantees

Stabilizing Pension Fund Investment

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How to lower the risk of a pension fund investment?

- Business cycles and crisis.
- If buy and hold ✓, but what about emergency fund withdrawals?

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Why not to synthesize an existing fund and make it less risky using structured finance?

Problem Overview

Past Literature

Retain earnings in bull markets
Release past earnings in bear markets

(Miltersen and Persson, 2003) Optimal distribution schedule

Distribute Earnings $\Longleftrightarrow$ Provision Earnings for Crisis

(Kling et al., 2007) For fixed terms contracts.

Crediting Rate $\Longleftrightarrow$ Reserves Surplus

(Wang et al., 2005) investor's Value-at-Risk-optimal policy.
(Cheng et al., 2019) Optimal stopping time for rational investor (dynamic lapse model)
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This presentation focus: Stable Value Guaranteed investment structure

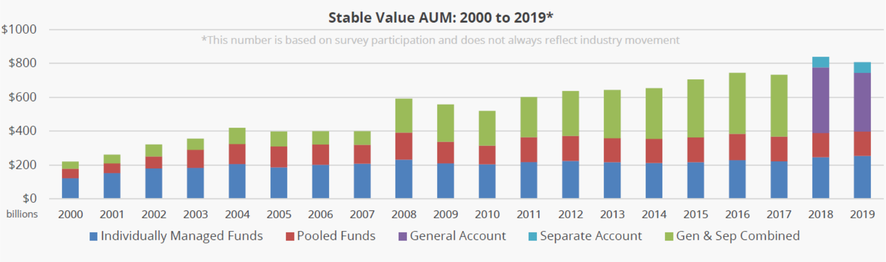
- **Study an existing structure with 800B$ of assets under management**
- **Provide practitioners a quantitative method to asses the risk of the guarantee**
- **Make recommendations to regulators if necessary**

Stable Value Product

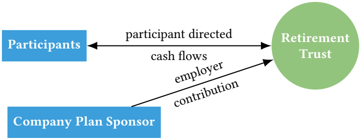
Stable Value Funds

Stable value is a retirement plan investment option regulated by US Employee Retirement Income Security Act (ERISA).

Market Growth Since 2000:



Stable Value Fund Mechanism



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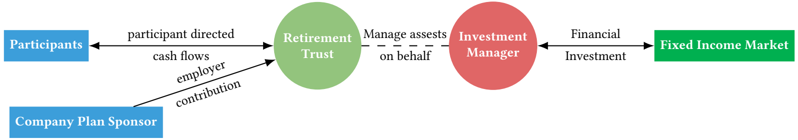

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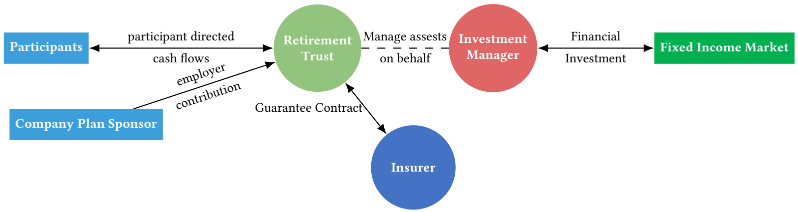

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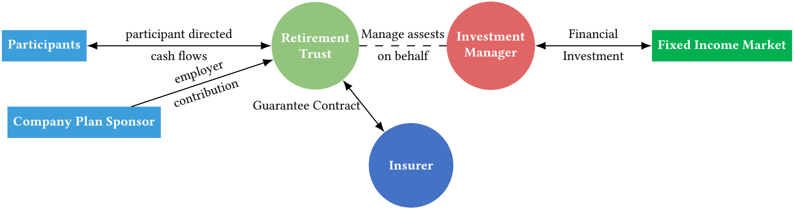
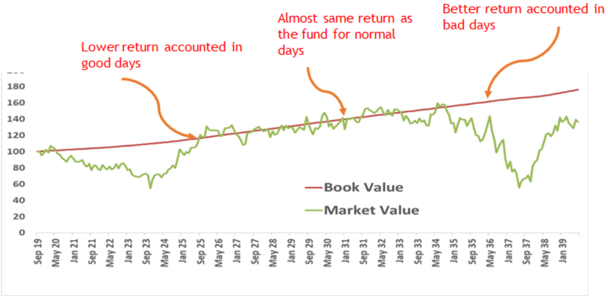


Stable value funds are synthesized version of a more volatile fund investment:

1. Return stabilization by Book Value accounting
2. The insurance company guarantees

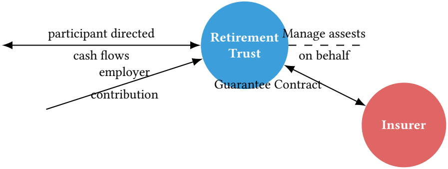
Return Stabilization: Book Value Accounting



$$dB_t = B_t \gamma_t dt + dC_t$$

C_t aggregated in/out cashflows.
There is a market standard formula for crediting rate γ_t.

Insurance guarantee



The insurance covers asset liability mismatch at the last resort:

When the last participant requests its money back (M_t reaches 0)

$$\text{Insurance Payoff} = (B_\tau - M_\tau)_+$$
$$\tau = \inf_t \{M_t = 0\}$$

Stable Value Guarantees

Asset Model

Assets Modelling : Fixed income funds

A simple model with yield as the primary risk factor

$$dM_t = M_t\left(y_t - \lambda\right)dt - M_t\theta dy_t + dC_t$$

λ is the default/impairement/credit quality migration rate of the fund's portfolio. θ is the duration.

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Yield - Duration - Market Value relationship

The yield (y_t) is the single valuation rate r, that is solution to the duration formula:

$$\left. \frac{\partial M_t(r)}{\partial r} \right|_y = M_t(y) \times \theta,$$

where $M(r)$ is the market value of the fund, r is the valuation rate.

Stable Value Guarantees

Liability Cash Flow Model

Cash Flow (ζ_t) Properties

Historical data :

Period	1997 - Mar20
U.S.	$ 198 B$
# funds	288
# data	35,850

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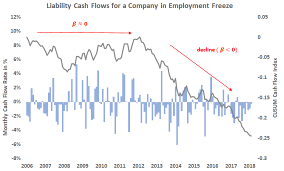

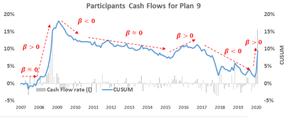

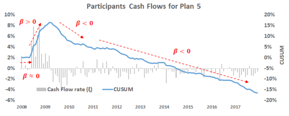

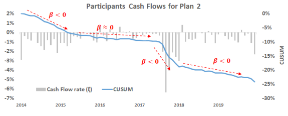

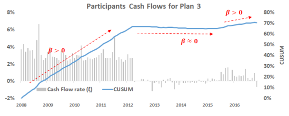

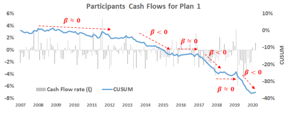

Cash Flow (ζ_t) Properties

Historical data :

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U.S.	$ 198 B$
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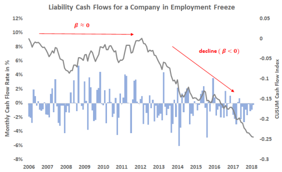
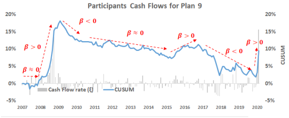
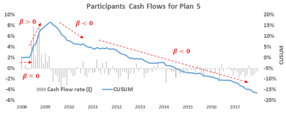
- Cash Flows Exhibit trends
- Each trend represent a regime

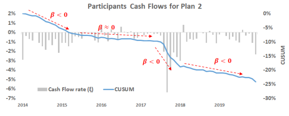
Trends are because of:

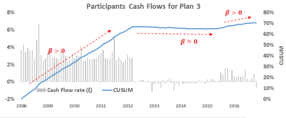
- Systemic financial market
- Back to safety (increase of influx) during crisis and decrease after
- Employer's financial health
- Reputational/bankruptcy issues

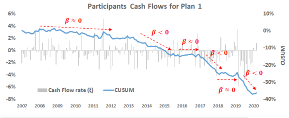








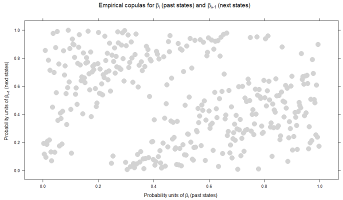




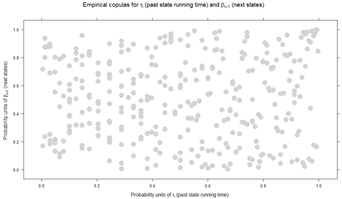
Cash Flows trends Properties

Let's call β_i as the i-th trend and θ_i itd duration:

New regimes (β_{i+1}) vs past regimes (β_i)



New regimes duration (τ_{i+1}) vs past regimes (β_i)



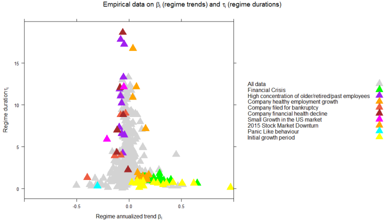
New regimes size and duration seems to be independent from the past regimes:

$$P[\beta_{i+1} \leq b, \tau_{i+1} \leq t | \beta_i, \tau_i] = P[\beta_{i+1} \leq b, \tau_{i+1} \leq t]$$

Trends hump shape behaviour

Data observation

Regime duration is shorter for larger or smaller regime trends



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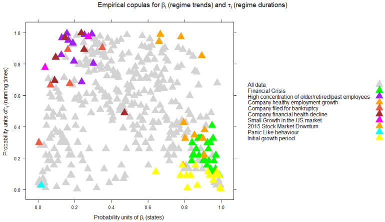
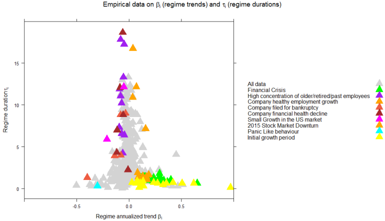


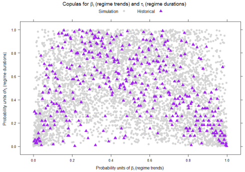


Modelling

Hump shape copula, inspired by Nelsen (2007) geometric method.

$$P[\beta_i \leq b, \tau_i \leq t] = C(\Phi(b), \Lambda(t))$$

$$C(u,v) = \begin{cases} \Psi \times C_L\left(\frac{u}{\Psi}, v\right) & u < \Psi \\ (1-\Psi) \times \left(v - C_R\left(\frac{1-u}{1-\Psi}, v\right)\right) & u > \Psi \end{cases}$$



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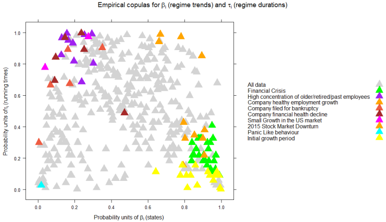

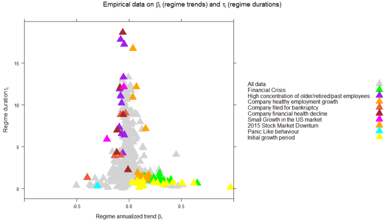

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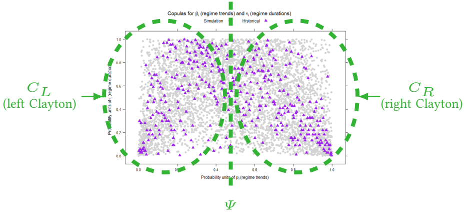
C_L and C_R (left and right) assumed as Clayton:



Cash Flows (ζ_t) Modelling

$$dC_t = B_t d\zeta_t$$

$$d\zeta_t = \overbrace{g\left(\gamma_t\right)dt}^{\text{Product competitiveness}} + \overbrace{\beta_t dt}^{\text{Regime trend}} + \overbrace{\epsilon_t}^{\text{Idiosyncratic randomness}} + \overbrace{dE_t}^{\text{Employer intervention}}$$

$g(\cdot)$ the crediting rate competitiveness to other alternative investment options.

Model assumption

New regimes size and duration are independent from the past regimes:

$$P[\beta_{i+1} \leq b, \tau_{i+1} \leq t | \beta_i, \tau_i] = P[\beta_{i+1} \leq b, \tau_{i+1} \leq t]$$

The joint distribution of the regime and their trends follow a hump shape copula function

$$P[\beta_i \leq b, \tau_i \leq t] = C(\Phi(b), \Lambda(t))$$

Stable Value Guarantees

Results and conclusions

Tail Risk Estimation

The Risk is deep in the Tail

Loss frequency is $< 0.5\%$. But the risk also depends on the assumption of the rate competitiveness components

NAIC capital C3 formulaic	90% CTE of loss	99% CTE of loss	99.5% VaR of loss	Loss frequency	Avg. loss size
0%	0.070%	0.70%	0%	0.49%	1.40%

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NAIC formulaic approach does not capture the tail risk

1. The NAIC formulaic approach gives almost always 0, which may infer an incomplete picture of U.S. insurers actuarial risk.
2. C3 Phase III principal based methodology is using the 90%CTE, but not compulsory for Stable Values

S&P Methodology

Standard & Poors (2007) paper on C3 (Asset Liability Mismatch) : different C3 if the insurance guarantee covers for asset defaults.

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Better methodology: consider the crediting migration risk

Using Caouette et al. (1998) Naranayan's transition table and assets to be sold when migrated to a lower bucket.

Avg OAS	Assets avg. S&P grading	Migration haircut λ	99% CTE of loss	99.5% VaR of loss	NAIC capital C3 formulaic	Loss frequency	Avg. loss size
1.5%	AA	0.05%	0.70%	0%	0%	0.49%	1.58%
2.5%	A	0.20%	2.18%	0.39%	0%	0.61%	3.57%
3.5%	BBB	0.50%	4.00%	1.96%	0%	0.70%	5.71%

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To consider credit migration risk rather than default

Conclusions

Assets	Liabilities
A simple and novel model for fixed income funds with yield as the primary risk.	Analyzed 200B$ of fund's data and developed a cash flow model fitting to the observations

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Discussion and Conclusions

- An asset-liability model for a 800 B$ structure
- The risk is deep in the tail but NAIC RBC formulaic approach cannot capture the tail risk
- The rating agencies to consider the asset migration risk instead of the default risk.

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