Stable Value Fund Guarantee

An Asset Liability Model

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Stéphane Loisel and Yahia Salhi (ISFA)
Stable Value Guarantees

Stabilizing Pension Fund Investment
How to lower the risk of a pension fund investment?

- **Business cycles and crisis.**
- If buy and hold ✔, but what about emergency fund withdrawals?
How to lower the risk of a pension fund investment?

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Why not to synthesize an existing fund and make it less risky using structured finance?

![Chart showing financial events such as dot com, financial crisis, 2015 fire sale, 2018 vol crisis, and COVID-19.](chart.png)
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Why not to synthesize an existing fund and make it less risky using structured finance?
Stabilizing Pension Fund Investment

Problem Overview

Past Literature

Retain earnings in bull markets
Release past earnings in bear markets

(Miltersen and Persson, 2003) Optimal distribution schedule

\[ \text{Distribute Earnings} \iff \text{Provision Earnings for Crisis} \]

(Kling et al., 2007) For fixed terms contracts.

\[ \text{Crediting Rate} \iff \text{Reserves Surplus} \]

(Wang et al., 2005) investor’s Value-at-Risk-optimal policy.
(Cheng et al., 2019) Optimal stopping time for rational investor (dynamic lapse model)
(Døskeland and Nordahl, 2008) Merton type Asset-Liability model
Stabilizing Pension Fund Investment

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This presentation focus: Stable Value Guaranteed investment structure

- Study an existing structure with 800B$ of assets under management
- Provide practitioners a quantitative method to assess the risk of the guarantee
- Make recommendations to regulators if necessary
Stable Value Guarantees

Stable Value Product
Stable value is a retirement plan investment option regulated by US Employee Retirement Income Security Act (ERISA).

**Market Growth Since 2000:**

![Stable Value AUM: 2000 to 2019*](image)

*This number is based on survey participation and does not always reflect industry movement*
Stable Value Product

Stable Value Fund Mechanism

Participants

Company Plan Sponsor

Retirement Trust

- participant directed cash flows
- employer contribution

Stable Value Funds are a synthesized version of a more volatile fund investment:
1. Return stabilization by Book Value accounting
2. The insurance company guarantees

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Stable Value Fund Guarantee
Stable Value Product

Stable Value Fund Mechanism

Participants

Company Plan Sponsor

participant directed
cash flows
employer
contribution

Retirement
Trust

Manage assets
on behalf

Investment
Manager

Financial
Investment

Fixed Income Market

Stable Value Fund Guarantee

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Stable Value Fund Guarantee
Stable Value Product

Stable Value Fund Mechanism

- Participants: participant directed cash flows
- Company Plan Sponsor
- Retirement Trust
- Manage assets on behalf
- Insurer
- Guarantee Contract
- Financial Investment
- Fixed Income Market
- Stable Value Fund Guarantee

Stable value funds are synthesized versions of more volatile fund investments:

1. Return stabilization by Book Value accounting
2. The insurance company guarantees participant directed cash flows
3. Employer contribution

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Stable value funds are synthesized version of a more volatile fund investment:

1. Return stabilization by Book Value accounting
2. The insurance company guarantees
\[ dB_t = B_t \gamma_t dt + dC_t \]

\( C_t \) aggregated in/out cashflows.
There is a market standard formula for crediting rate \( \gamma_t \).
The insurance covers asset liability mismatch at the last resort:

When the last participant requests its money back ($M_t$ reaches 0)

\[
\text{Insurance Payoff} = (B_\tau - M_\tau)_+ \\
\tau = \inf_t \{M_t = 0\}
\]
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Asset Model
A simple model with yield as the primary risk factor

\[ dM_t = M_t (y_t - \lambda) \, dt - M_t \theta \, dy_t + dC_t \]

\( \lambda \) is the default/impairement/credit quality migration rate of the fund’s portfolio. \( \theta \) is the duration.
A simple model with yield as the primary risk factor

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Yield - Duration - Market Value relationship

The yield \( y_t \) is the single valuation rate \( r \), that is solution to the duration formula:

\[
\left. \frac{\partial M_t(r)}{\partial r} \right|_y = M_t(y) \times \theta,
\]

where \( M(r) \) is the market value of the fund, \( r \) is the valuation rate.
Stable Value Guarantees

Liability Cash Flow Model
### Liability Cash Flow Model

#### Cash Flow ($\zeta_t$) Properties

<table>
<thead>
<tr>
<th>Historical data:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period</td>
</tr>
<tr>
<td>U.S.</td>
</tr>
<tr>
<td># funds</td>
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<td># data</td>
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Cash Flows exhibit trends:
- Each trend represents a regime.
- Trends are due to:
  - Systemic / financial market
  - Back to safety (increase of influx) during crisis and decrease after
  - Employer's / financial health
  - Reputational / bankruptcy issues
Liability Cash Flow Model

Cash Flow ($\zeta_t$) Properties

Historical data:
- Period: 1997 - Mar20
- U.S. funds: $198 B$
- # funds: 288
- # data: 35,850

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Liability Cash Flow Model

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- Cash Flows Exhibit trends
- Each trend represents a regime

Trends are because of:
- Systemic financial market
- Back to safety (increase of influx) during crisis and decrease after
- Employer’s financial health
- Reputational/bankruptcy issues
Let’s call $\beta_i$ as the $i$-th trend and $\theta_i$ itd duration:

New regimes ($\beta_{i+1}$) vs past regimes ($\beta_i$)  
New regimes duration ($\tau_{i+1}$) vs past regimes ($\beta_i$)

New regimes size and duration seems to be independent from the past regimes:

$$P[\beta_{i+1} \leq b, \tau_{i+1} \leq t | \beta_i, \tau_i] = P[\beta_{i+1} \leq b, \tau_{i+1} \leq t]$$
**Data observation**

Regime duration is shorter for larger or smaller regime trends
Data observation

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Modelling

Hump shape copula, inspired by Nelsen (2007) geometric method.

\[
P[\beta_i \leq b, \tau_i \leq t] = C(\Phi(b), \Lambda(t))
\]

\[
C(u, v) = \begin{cases} 
\Psi \times C_L \left( \frac{u}{\Psi}, v \right) & u < \Psi \\
(1 - \Psi) \times \left( v - C_R \left( \frac{1-u}{1-\Psi}, v \right) \right) & u > \Psi
\end{cases}
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\end{cases} \]

\( C_L \) and \( C_R \) (left and right) assumed as Clayton:
Liability Cash Flow Model

Cash Flows ($\zeta_t$) Modelling

\[ dC_t = B_t d\zeta_t \]

\[ d\zeta_t = g(\gamma_t) dt + \beta_t dt + \epsilon_t + dE_t \]

\( g(\cdot) \) the crediting rate competitiveness to other alternative investment options.

**Model assumption**

New regimes size and duration are independent from the past regimes:

\[ P[\beta_{i+1} \leq b, \tau_{i+1} \leq t | \beta_i, \tau_i] = P[\beta_{i+1} \leq b, \tau_{i+1} \leq t] \]

The joint distribution of the regime and their trends follow a hump shape copula function

\[ P[\beta_i \leq b, \tau_i \leq t] = C(\Phi(b), \Lambda(t)) \]
Stable Value Guarantees

Results and conclusions
The Risk is deep in the Tail

Loss frequency is $< 0.5\%$ . But the risk also depends on the assumption of the rate competitiveness components

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<th>NAIC capital C3 formulaic</th>
<th>90% CTE of loss</th>
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Results and conclusions
Tail Risk Estimation

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NAIC formulaic approach does not capture the tail risk

1. The NAIC formulaic approach gives almost always 0, which may infer an incomplete picture of U.S. insurers actuarial risk.
2. C3 Phase III principal based methodology is using the 90%CTE, but not compulsory for Stable Values

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Stable Value Fund Guarantee
Results and conclusions

Asset’s Credit Risk

S&P Methodology

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Standard & Poors (2007) paper on C3 (Asset Liability Mismatch): different C3 if the insurance guarantee covers for asset defaults. Their methodology is too simplistic: it fails to consider the fact that managers can sell the assets before any default occurs.
Results and conclusions

Asset’s Credit Risk

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**Better methodology: consider the crediting migration risk**

Using Caouette et al. (1998) Naranayan’s transition table and assets to be sold when migrated to a lower bucket.

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To consider credit migration risk rather than default
## Results and conclusions

### Conclusions

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Discussion and Conclusions

- An asset-liability model for a 800 B$ structure
- The risk is deep in the tail but NAIC RBC formulaic approach cannot capture the tail risk
- The rating agencies to consider the asset migration risk instead of the default risk.


