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One-year and ultimate reserve risk in Mack Chain Ladder model

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Introduction

Introduction

Insurance companies are exposed, among others, to reserve risk, which is related to the adequacy of the current volumes of the claims reserves. We differ between the notion of ultimate and one-year risk. For reserve risk we use the following definitions:

- Ultimate reserve risk - the risk that the current loss reserves will not be adequate in ultimate horizon, i.e. after full run-off of the liabilities.
- One-year reserve risk - the risk that the current loss reserves will not be adequate after one year of realization.

Motivation:

- Ultimate risk is traditionally investigated by actuaries.
- One-year risk needs to be investigated by the companies for Solvency II risk capital.
- The question is "what is the relation between one-year risk and ultimate risk"?

Introduction

The approach to quantifying ultimate reserve risk is well established in claims reserving.

According to the work "A Practictioner's Introduction to Stochastic Reserving: The One-Year View" (Institute and Faculty of Actuaries, [Scarth et al (2020)]), there are 3 methods used in practice for the modelling of one-year reserve risk:

- Actuary-in-the-Box - method with a simulation of the run-off triangle and applying a ReReserving procedure after one year.
- Merz-Wüthrich formula - method, which provides an analytical estimation of the standard deviation of one-year risk.
- Emergence pattern - method with a linear scaling procedure which provides a switch from ultimate to one-year risk.

Introduction

Further comments on the methods:

- The Actuary-in-the-Box method is relatively computationally expensive and complex to implement. Additionally, it is more vulnerable to unstable results since, a priori, we cannot describe all rules for extreme situations.

- The Merz-Wüthrich formula only applies to the mean square error of prediction for Mack Chain Ladder model. If any alterations are made to the model (e.g. curve fitting, adding a tail factor), we consider a different risk measure, or Mack Chain Ladder model is not used, then the formulas no longer apply.

- Both above methods can be used to estimate parameters which, after review of reasonableness, can be fed into the main calculation kernel using the emergence pattern method.

Introduction

We investigate the relation between one-year reserve risk and ultimate reserve risk in Mack Chain Ladder model in a simulation study. In our work we have three goals:

- Validate the so-called linear emergence pattern formula, which maps the ultimate loss to the one-year loss, in case when we measure the risks with Value-at-Risk.
- Estimate the true emergence pattern of the ultimate loss, i.e. the conditional distribution of the one-year loss given the ultimate loss, from which we can properly derive a risk measure for one-year horizon from simulations of ultimate losses.
- Test if classical actuarial distributions can be used for modelling of the outstanding loss in Mack Chain Ladder models.

Mathematical framework

Mathematical framework - notation

Let $i = 1, \dots n$ denote the accident year and $j = 1, \dots, n$ denote the development year initiated with respect to the accident year. We consider a sequence of random variables:

- $\left(X_{i,j}\right)_{i,j \in \{1,\dots,n\}}$ denotes the cumulative payments made for the i-th accident year up to the j-th development year.
- $X_{i,n}$ denotes the ultimate loss for the i-th accident year.
- $BE_i(n) = E\left[X_{i,n} \middle| D(n)\right]$ denotes the best estimate of the ultimate loss at the end of year n for the i-th accident year.

Consequently, we consider the claims development result for reserve risk from the perspective of the end of the n-th calendar year, respectively, in one-year horizon and ultimate horizon:

- $CDR_i^{1Y}(n) = BE_i(n) - BE_i(n+1)$,
- $CDR_i^{ULT}(n) = BE_i(n) - X_{i,n}$.

We consider also sums over all accident years (we lose the subscript i).

Mathematical framework – emergence pattern

We follow two different approaches for the emergence pattern method:

- "EP_AY" approach, where we calculate an emergence pattern (α_i), which consists of values for each accident year separately:
  - $BE^{ep\ ay}(n+1) = \sum_{i=1}^{n} BE_i^{ep\ ay}(n+1) = \sum_{i=1}^{n}\left(\alpha_i X_{i,n} + (1-\alpha_i)BE_i(n)\right)$,
  - $CDR^{ep\ ay,1Y}(n+1) = \sum_{i=1}^{n}\left(\alpha_i CDR_i^{ULT}(n)\right)$.

- "EP" approach, where we calculate an aggregated emergence factor α:
  - $BE^{ep}(n+1) = \alpha X_n + (1-\alpha)BE(n)$,
  - $CDR^{ep,1Y}(n+1) = \alpha CDR^{ULT}(n)$.

Mathematical framework – assumptions

In order to allow for a check of impact of different characteristics (e.g. duration or volatility) on the emergence pattern, we create 4 synthetic triangles based on characteristics observed in available data and we consider 3 different $F_{i,j}$ distributions for the simulation study.

In order to have a full structure of the triangle we set:

- The development factors f_i using duration and f_1.
- The sigma factors σ_i using CoV and $Skew$.
- The distribution of the individual development factors $F_{i,j} = X_{i,j+1}/X_{i,j}$.
- The exposure in different accident years.

Triangles and their parameters have been set based on loss triangles observed in practice to represent key characteristics of the claims development processes.

Conclusions of the simulation study

Conclusions – emergence pattern

For the emergence pattern analysis, we are interested in the following ratios:

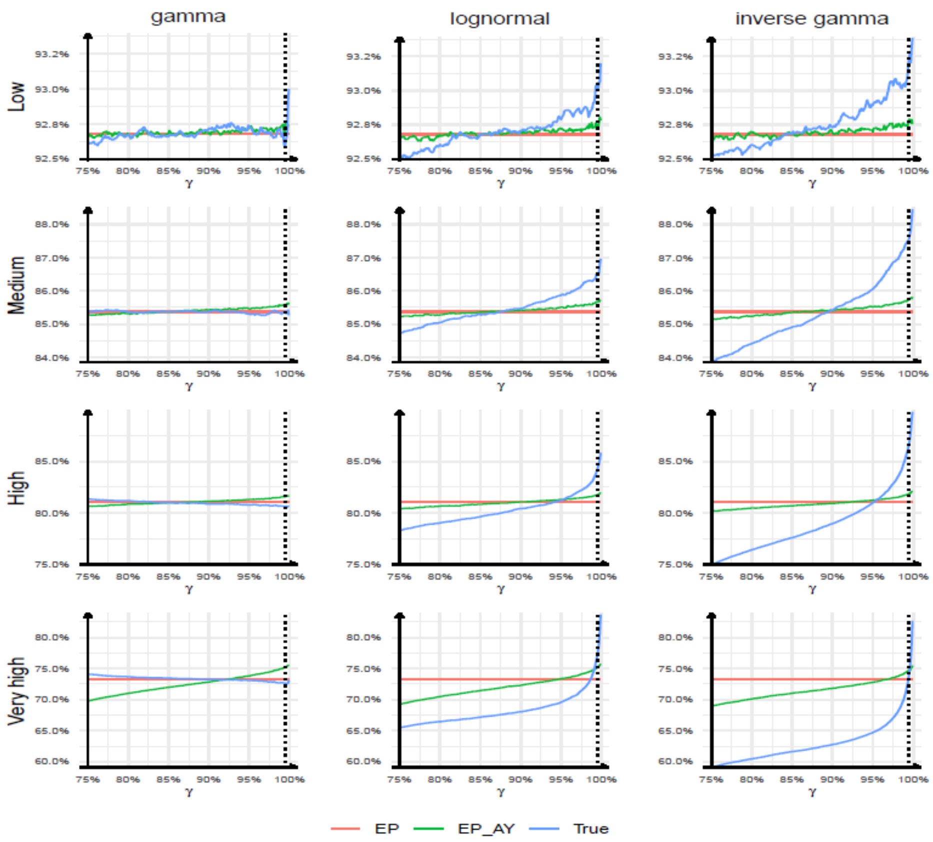
$$\frac{VaR_{\gamma}[-CDR^{1Y}(n)|D(n)]}{VaR_{\gamma}[-CDR^{ULT}(n)|D(n)]},$$

$$\frac{VaR_{\gamma}[-CDR^{ep\ ay,1Y}(n)|D(n)]}{VaR_{\gamma}[-CDR^{ULT}(n)|D(n)]},$$

$$\frac{VaR_{\gamma}[-CDR^{ep,1Y}(n)|D(n)]}{VaR_{\gamma}[-CDR^{ULT}(n)|D(n)]}.$$

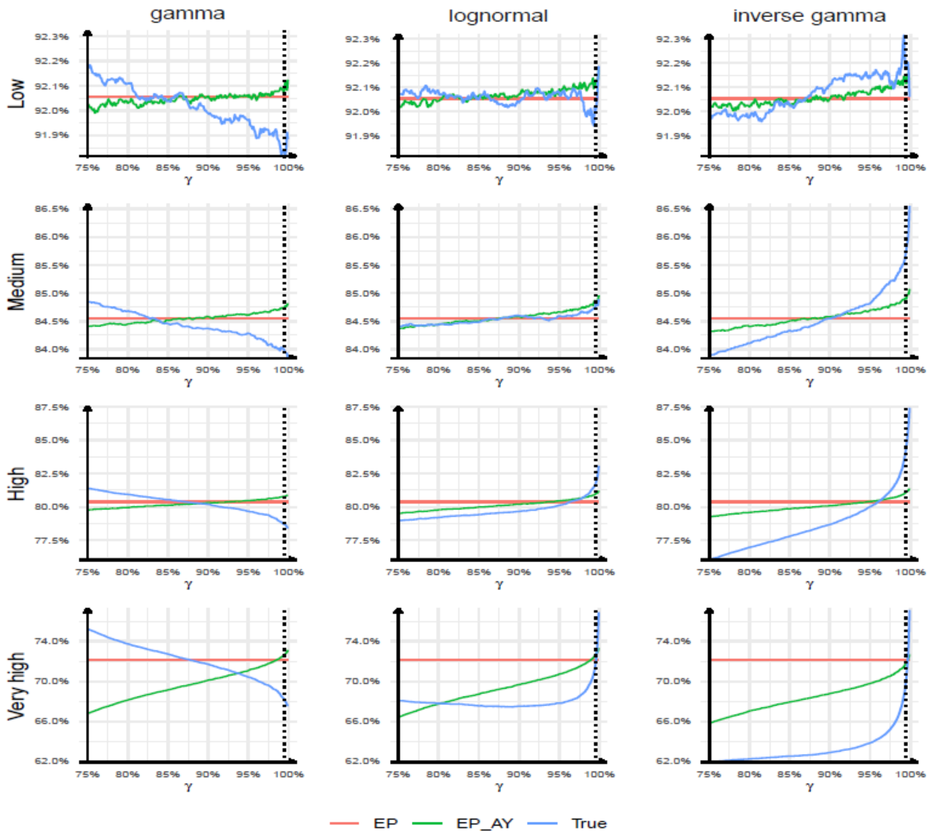
We consider the case both with and without the estimation error.

Conclusions – emergence pattern



The ratios of Value-at-Risk measures for the one-year risk and the ultimate risk as a function of the confidence level - **the case without the estimation error.**

Conclusions – emergence pattern



The ratios of Value-at-Risk measures for the one-year risk and the ultimate risk as a function of the confidence level - **the case with the estimation error.**

Conclusions – emergence pattern

Conclusions:

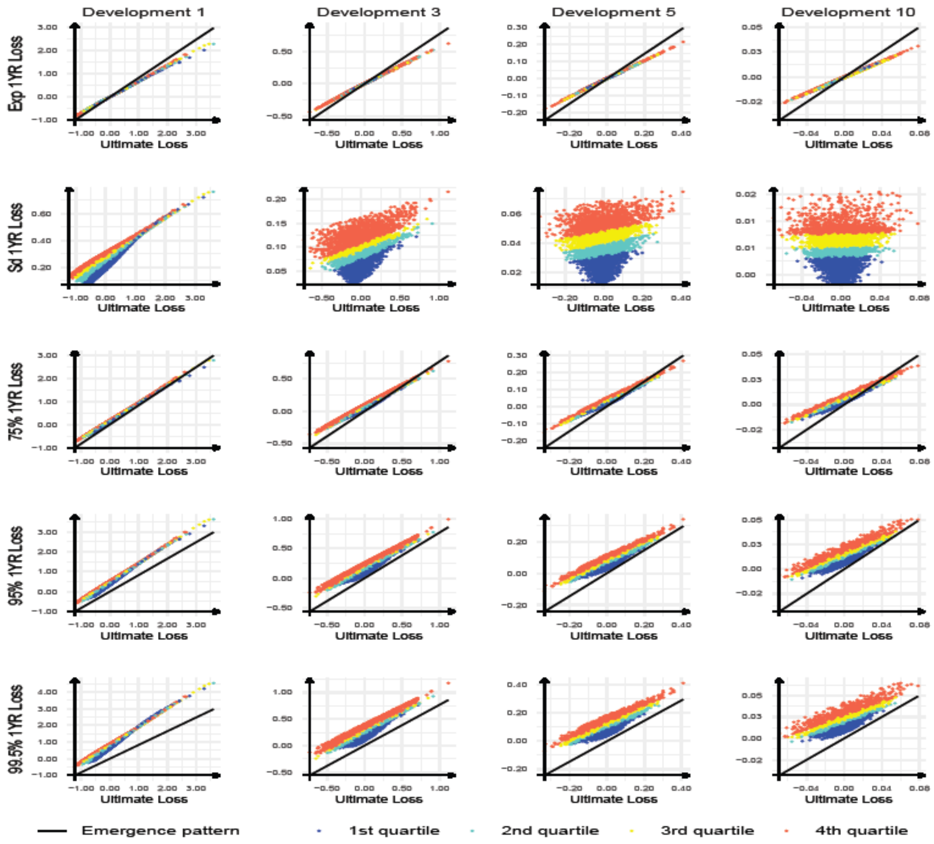
- The ratios can be significantly different and, in general, depend on the confidence level, the distribution and the triangle type. The higher the volatility, the higher the range of the ratios.
- Two emergence pattern approaches provide similar results.
- Emergence pattern formula provides a good approximation for low and medium volatility triangles. The higher the volatility, the worse the approximation provided by emergence pattern formula.
- In most of the presented cases, except for Gamma distribution where the behaviour is reverse, emergence pattern formula tends to underestimate high quantiles and overestimate low quantiles.
- In case with the estimation error, the misestimation error increases for the triangles with gamma distribution and decreases for the triangles with lognormal and inverse gamma distributions.

Conclusions – neural networks

We simulate scenarios of the revaluations of the best estimate of the ultimate loss $\left(BE(1), \ldots, BE(n)\right)$ in consecutive development years from the claims development model specified for the *high* triangle with lognormal distribution of development factors $F_{i,j}$. Clearly, $BE(n)$ gives the ultimate loss X_n.

We fit the conditional distributions of $(BE(k+1)|k, BE(n), BE(k))_{k=1,\ldots,n-2}$ to the simulated data using neural networks methods. The response is $BE(k+1)$ and the explanatory variables are given by $\left(k, BE(k), BE(n)\right)$. We consider the case without the estimation error.

Conclusions – neural networks



The true emergence pattern and the linear emergence pattern of **the one-year loss for the case without the estimation error.**

Conclusions – neural networks

Conclusions:

- The true emergence pattern of the ultimate loss in our Mack Chain Ladder model is significantly different from the linear emergence pattern.
- The higher the current best estimate of the ultimate loss, the higher the conditional standard deviation of the one-year loss.
- The higher the ultimate loss and the higher the current best estimate of the ultimate loss, the higher the quantile of the conditional distribution of the one-year loss at high confidence levels.

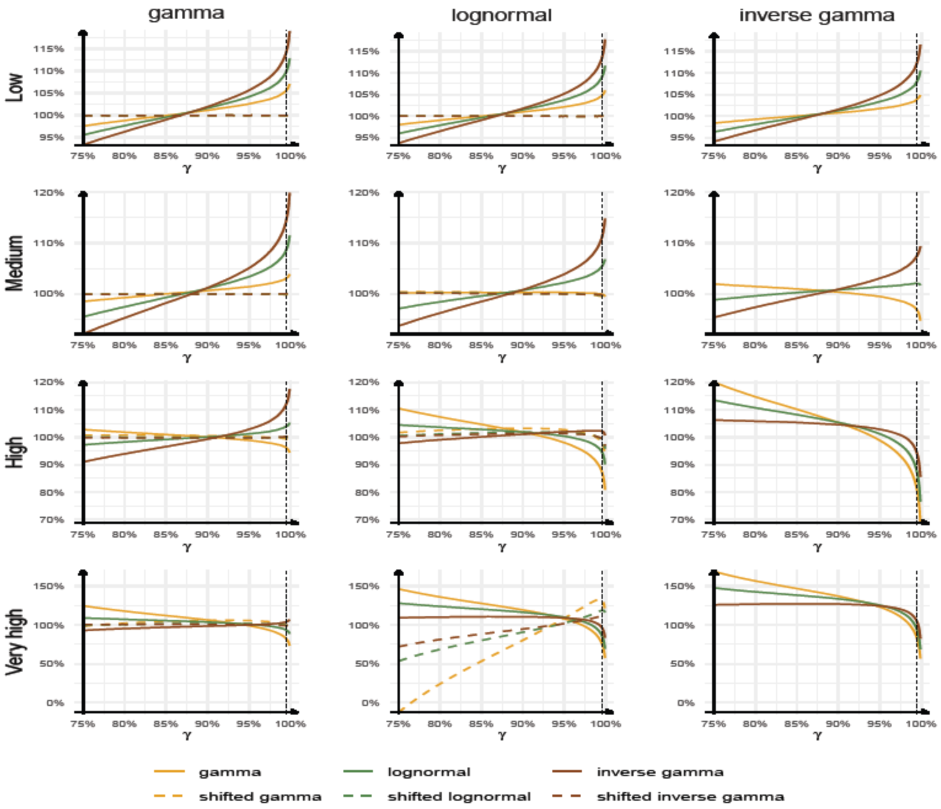
Conclusions – outstanding loss

We analyse the quality of our approximations by studying the ratios:

$$\frac{VaR_{\gamma}\left[\hat{R}(n)\middle|D(n)\right]}{VaR_{\gamma}[R(n)|D(n)]},$$

where $VaR_{\gamma}\left[\hat{R}(n)\middle|D(n)\right]$ is calculated using the estimated parametric distribution of the outstanding loss, hence this term describes our approximation, and $VaR_{\gamma}[R(n)|D(n)]$ is estimated using the simulated sample of the outstanding losses, hence this term describes the true quantile of the outstanding loss.

Conclusions – outstanding loss



The ratios of Value-at-Risk measures for the outstanding loss estimated with parametric distribution and empirical distribution as a function of the confidence level - **the case with the estimation error and the one-year perspective**.

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Conclusions – outstanding loss

Conclusions:

- Quality of the approximation and the choice of the best distribution depend on the triangle type and confidence level.
- The non-shifted distributions provide a good approximation for *low*, *medium*, *high* triangles if the best distribution is chosen. The approximation error increases for more volatile triangles.
- The shifted distributions outperform the non-shifted versions, increasing the quality of approximation.

The results are similar in case of the outstanding loss from the ultimate perspective.

Conclusions

In this paper we have investigated the relation between one-year reserve risk and ultimate reserve risk in Mack Chain Ladder model in a simulation study.

- We have demonstrated that the linear emergence pattern formula may misestimate the one-year risk.
- We have derived the true emergence pattern of the ultimate loss in a Mack Chain Ladder model and presented that it may differ from the linear emergence pattern formula.
- We have found that two-parameter loss distributions may not be sufficient to model the outstanding loss and goodness-of-fit can be improved by fitting shifted versions of classical loss distributions.

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Thank you for your attention.

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