Stable dividends are optimal under linear-quadratic optimization

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Agenda

- Motivation
- LQ optimization
- Classical optimization
- Compare and benchmarks

Motivation

Stability criteria

- Expected present value of dividends until ruin. [de Finetti(1957)]

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Motivation

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- Expected present value of dividends. [de Finetti(1957)]

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- Stable dividends. [Avanzi, Tu, Wong (2016)]

  - Optimal strategies rarely have this property.

Motivation

Stable dividends - Affine dividend strategy.

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- Not optimal for maximizing expected present value of dividends.

- Optimal in the linear quadratic optimal control problem. [Cairns(2000)]

Setup

Surplus model

$$dX(t) = c\ dt + \sigma\ dW(t) - l(t, X(t))\ dt,$$

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$$dX(t) = c\,dt + \sigma\,dW(t) - l(t, X(t))\,dt,$$

- c deterministic variation.
- $W(t)$ Brownian motion.
- $l(t, X(t))$ continuous dividend process.

Linear-Quadratic objective

Value function

$$V(t, X(t)) = \min_{l} \mathbb{E}_{t,x}\Bigg[\frac{1}{2}\int_t^T e^{-\delta(s-t)}\Big(l(s, X(s)) - l_0 - l_1 X(s)\Big)^2 ds + \frac{1}{2}\int_t^T e^{-\delta(s-t)}\gamma\Big(X(s) - x_0\Big)^2 ds + \lambda\Big(X(T) - k\Big)^2\Bigg]$$

Linear-Quadratic objective

Value function

$$V(t, X(t)) = \min_{I} \mathbb{E}_{t,x}\Bigg[\frac{1}{2}\int_t^T e^{-\delta(s-t)}\Big(I(s, X(s)) - I_0 - I_1 X(s)\Big)^2 \mathrm{d}s + \frac{1}{2}\int_t^T e^{-\delta(s-t)}\gamma\Big(X(s) - x_0\Big)^2 \mathrm{d}s + \lambda\Big(X(T) - k\Big)^2\Bigg]$$

Satisfy Hamilton-Jacobi-Bellman equation.

Linear-Quadratic objective

Value function

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Satisfy Hamilton-Jacobi-Bellman equation.

Optimal dividend strategy $l^*(t, x) = l_0 + l_1 x + V_x(t, x)$.

Linear-Quadratic objective

Guess

$$V(t,x) = q(t)x^2 + p(t)x + r(t),$$

where the functions q, p, r satisfy ODEs.

Linear-Quadratic objective

Guess

$$V(t,x) = q(t)x^2 + p(t)x + r(t),$$

where the functions q, p, r satisfy ODEs.

Optimal dividend strategy $l^*(t,x) = l_0 + p(t) + \big(l_1 + 2q(t)\big)x$.

Classical objective

Dividend payouts to shareholders

$$\mathbb{E}_{t,x}\left[\int_t^{\tau_x} e^{-\tilde{\delta}(s-t)}\mathrm{d}D(s)\right],$$

where D is dividends and τ_x is the time of ruin.

Classical objective

Dividend payouts to shareholders

$$\mathbb{E}_{t,x}\left[\int_t^{\tau_x} e^{-\tilde{\delta}(s-t)} \mathrm{d}D(s)\right],$$

where D is dividends and τ_x is the time of ruin.

Optimal dividend strategy is the barrier strategy.

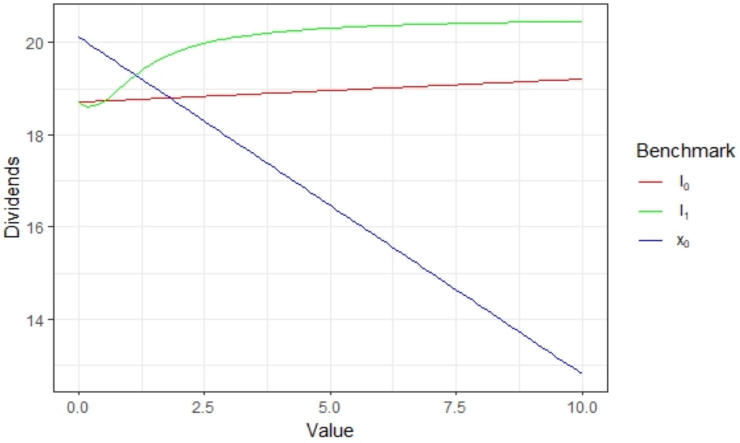
Comparing objectives

Comparing the classical optimization problem and the LQ problem.

- Understanding different objectives.

- Optimal dividend strategies.

Numerical



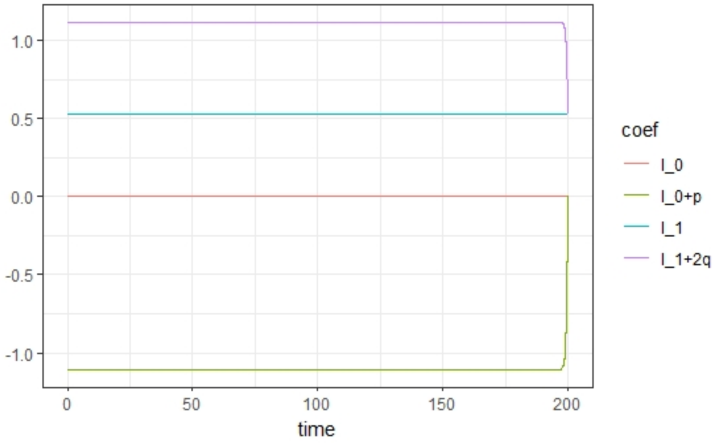
Affine dividends

[Avanzi and Wong(2012)] and [Albrecher and Cani(2017)] maximize expected present value of affine dividends until ruin

$$\max_{\tilde{l}_0,\tilde{l}_1} \mathbb{E}_{t,x}\left[\int_t^{\tau_x} e^{-\tilde{\delta}(s-t)} \left(\tilde{l}_0 + \tilde{l}_1 X(s)\right) \mathrm{d}s\right],$$

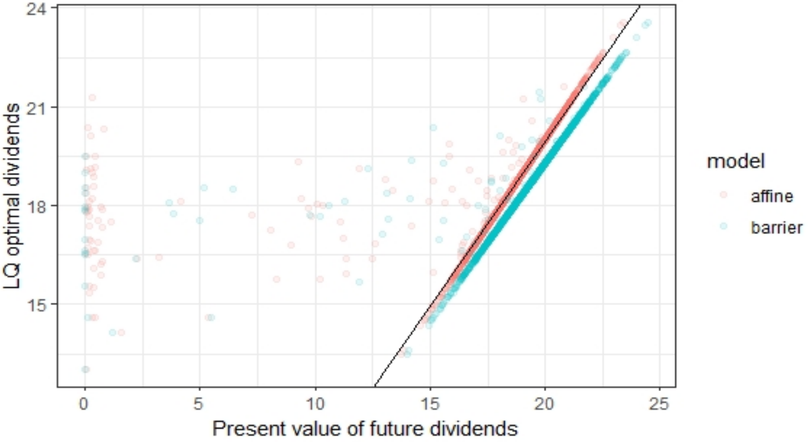
and X is mean reverting around $\frac{c-\tilde{l}_0}{\tilde{l}_1}$.

Numerical



Numerical

c	σ	δ	$X(0)$	l_0	l_1
1	0.5	0.05	$0.5b^*$	0	$\tilde{l}_1^{\,*}$



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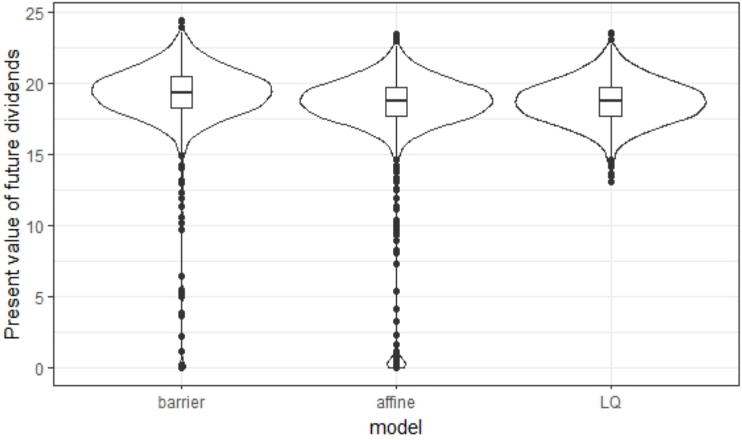
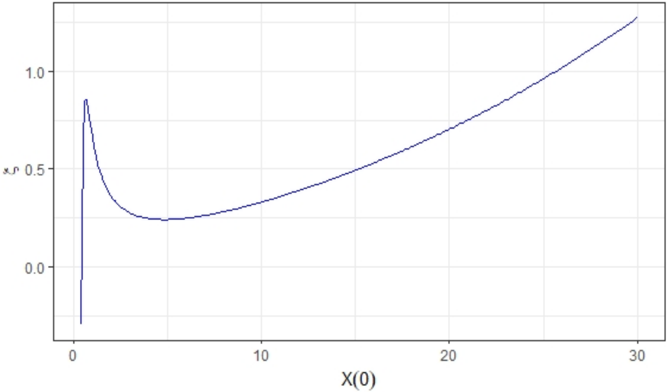


25
20
15
10
5
0
Present value of future dividends
barrier
affine
LQ
model

Relative difference in initial surplus

ξ - Relative difference in initial surplus for present value of future dividends to be equal

$$V^{LQ}(t, X(t)(1+\xi)) = V^{b}(t, X(t))$$



References

Hansjörg Albrecher and Arian Cani.
Risk theory with affine dividend payment strategies.
In *Number Theory – Diophantine Problems, Uniform Distribution and Applications*, pages 25–60. Springer International Publishing, Cham, 2017.

Benjamin Avanzi and Bernard Wong.
On a mean reverting dividend strategy with Brownian motion.
Insurance: Mathematics and Economics, 51(2):229–238, 2012.

Benjamin Avanzi, Vincent W. Tu, and Bernard Wong.
A note on realistic dividends in actuarial surplus models.
Risks, 4(4):37, 2016.

Andrew Cairns.
Some notes on the dynamics and optimal control of stochastic pension fund models in continuous time.
ASTIN bulletin, 30(1):19–55, 2000.