Modern Life-Care Tontines

Peter Hieber, AFIR/ERM, June 2022

joint work with:
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We want to thank Prof. Michel Denuit (UC Louvain) for many comments and discussions.
Belgium: LTC spending (in terms of GDP) increased from 1.7% in 2000 to 2.3% in 2018 (source: Eurostat).

United Nations projections: The number of elderly people, i.e. older than 65, is projected to triple from 2020 to 2080 to reach 2.2 billion. The global share of the elderly population is expected to rise from 9.4% in 2020 to 20.6% in 2080.
Definition: A mutual insurance company (tontine) is an insurance company owned entirely by its policyholders.

- This usually avoids risk charges and reduces administration, regulation.
What is mutual insurance?
What is mutual insurance?
Mutual life/pension insurance?
Mutual life/pension insurance?
Mutual insurance schemes in past...

Historic tontines (17th-19th century)

- Plans to rise government money.
- Predefined income stream is paid to survivors of a pool.

... start a revival today:

The New York Times

When Others Die, Tontine Investors Win

By Tom Verde
March 24, 2017

Living a long life is its own reward. But when you invest in a [tontine](https://en.wikipedia.org/wiki/Tontine), there's an added benefit: You collect money that would have gone to people who have died.

That is part of the macabre appeal of the tontine, a 350-year-old investment vehicle that fell into disfavor more than a century ago but is now getting fresh consideration as a way to help people receive steady income in retirement.
Modern tontines: Example Xianghubao

<table>
<thead>
<tr>
<th>Age group</th>
<th>Mild critical illness</th>
<th>Severe critical illness</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 days to 39-year-old</td>
<td>50,000 yuan</td>
<td>300,000 yuan</td>
</tr>
<tr>
<td>40- to 59-year-old</td>
<td>50,000 yuan</td>
<td>100,000 yuan</td>
</tr>
</tbody>
</table>

▶ Disability insurance.
▶ Based on an **app in China**, founded 2018.
▶ After 1 year, **100 million users**.

See also:
1. Motivation: Mutual insurance (tontines)

2. A fair, heterogeneous, modular mutual insurance scheme

3. Modern Life-Care Tontine
Related literature

Mutual (life) insurance schemes gain popularity in academic literature:

- **(Natural) tontines**: Milevsky, Salisbury [2015, 2016], Chen, Hieber, Klein [2019], Chen, Hieber, Rach [2020], Chen, Qian, Yang [2021], Bernhardt, Donnelly [2021], Denuit, Robert [2021], Bernhardt, Qu [2022], Winter, Planchet [2022], Denuit, Dhaene, Robert [2022]. (many more . . .)

- **Pooled annuities, P2P insurance, (tontines)**: (Sabin [2010]), Qiao, Sherris [2013], Donnelly, Guillén, Nielsen [2013, 2014], Denuit [2019]. (many more. . .)
This talk: Academic research

- Mutual risk-sharing schemes for **heterogeneous pools** (for example heterogeneous in age, health).

- **We pool mortality and morbidity** (long-term care) **risks**.


*Joint work with Nathalie Lucas (National Bank, Belgium), Michel Denuit (UC Louvain), Christian Y. Robert (ENSEA Paris)*
1. Motivation: Mutual insurance (tontines)

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Tontine products and surplus distribution

How do modern tontines work? Difference to pure financial investment?

We share insurance "gains":

- **mortality credits**: As in traditional insurance, accounts of deceased are (maybe only partially) distributed to survivors.

- **morbidity credits** (see later slides): Long-term care risks (more dependent people, longer time in dependency) are shared. This can be a surplus or a deficit!

All this comes on top of the regular financial return.
Multi-period heterogeneous tontine: Sketch/example

Fixed payoff (gray line) = “individual account”
Some notation

- Pool members $\mathcal{L}_0 = \{1, 2, \ldots, n\}$. Time in periods $t = 0, 1, 2, \ldots$.
- Individual $j \in \mathcal{L}_0$ contributes single premium $c_j(0)$ at time 0.
- Deterministic, risk-free rate $\delta_t$, $t \geq 0$.
- Remaining lifetimes $T_j, j \in \mathcal{L}_0$, are assumed to be independent.
- Death probability: $q_{x_j}$. Maximal age $\omega \in \mathbb{N}$.
- Individual account value, fixed payoff $s_j(t)$:

$$c_j(t) = \begin{cases} e^{\int_{t-1}^{t} \delta s ds} c_j(t - 1) - s_j(t), & j \in \mathcal{L}_t \\ 0, & \text{otherwise} \end{cases}$$

(1)

($c_j(t)$ is the “individual account”, the gray line!)
In case of death, the pool shares the remaining account value

\[
X(t) := \sum_{j=1}^{n} \mathbb{1}_{j \in D_t} \cdot e^{\int_{t-1}^{t} \delta_{sds}} c_j(t - 1).
\]

An individual \( j \in L_{t-1} \) receives a payoff of:

\[
W_j(t) = \begin{cases} 
  s_j(t) + \beta_j(X(t)), & \text{if } j \in L_t \\
  \beta_j(X(t)), & \text{if } j \in D_t 
\end{cases}
\]

(2)

decomposed of

- \( s_j(t) \): individual, **fixed withdrawal amount**, 
- \( \beta_j(X(t)) \): **collective part** of the benefits, i.e. the mortality credits.
Examples: Sharing rules

Share linearly according to (1) amount invested and (2) death probability.

Example (Linear risk sharing rule)

At time $t$, each individual $j \in \mathcal{L}_{t-1}$ receives the mortality credit (respectively death benefit):

$$
\beta_j(X(t)) = \frac{q_{x_j + t - 1} \cdot c_j(t - 1)}{\sum_{j \in \mathcal{L}_{t-1}} q_{x_j + t - 1} \cdot c_j(t - 1)} \cdot X(t). \quad (3)
$$

(see, e.g., Donnelly, Guillén, Nielsen [2013, 2014], Schumacher [2018])
Actuarial fairness: Insurer’s view

For each $t = 0, 1, \ldots$, the premium equivalence holds: (pool view)

$$\sum_{j=1}^{n} c_j(t) = \sum_{j=1}^{n} \omega - x_j \sum_{s=t+1} e^{-\int_t^s \delta u \, du} W_j(s).$$

- Right hand side: random (big letter!)
- Left hand side: deterministic.
Actuarial fairness: Individual’s view

For each $t = 0, 1, \ldots$, the contract is 

**fully-funded**: (individual view)

\[
\begin{align*}
  c_j(t) &= \mathbb{E}_t \left[ \sum_{s=t+1}^{\omega - x_j} e^{-\int_t^s \delta u \, du} W_j(s) \right].
\end{align*}
\]

The expected present value of future benefits equals the current account value.
2. A fair, heterogeneous, modular mutual insurance scheme

The diagrams illustrate the payoff process for a 65-year cohort over time. The plots show:

- **$s_j(t)$** as a solid line, representing the average payoff over time.
- **$W_j(t)$**: average payoff with a 95% confidence interval, depicted as a dashed line.
- **Mortality credits** are shown as a dotted line, decreasing over time.
- **Fixed payoff** is represented by a gray line, which remains constant over time.

The x-axis represents elapsed time, ranging from 0 to 30 years, while the y-axis shows the payoff. The diagrams effectively demonstrate the dynamics of payoff and confidence intervals under different scenarios.
1. Motivation: Mutual insurance (tontines)

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3. Modern Life-Care Tontine
Why pool mortality and morbidity (long-term care) risks?

- People moving into dependency need more money but have a reduced life expectancy!
  \[\Rightarrow \text{Natural hedge, diversification!}\]

- Individuals in bad health cannot receive long-term care insurance!
  \[\Rightarrow \text{Combined product gives access to insurance for a larger share of the population!}\]

- Cost reduction due to \textbf{reduced adverse selection!}
  \[\Rightarrow \text{Combined product is attractive for people in bad health...}\]
Life-Care Tontine: semi-Markov model

- \( p_{x_j}^{aa} \)
- \( p_{x_j}^{ai} \)
- \( p_{x_j}^{ad} = q_{x_j}^{(a)} \)
- \( p_{x_j; z}^{id} = q_{x_j; z}^{(i)} \)

z: time spent in dependency.
Modern Life-Care Tontine

We move in two steps:

1. A natural, actuarial fair increase in payments in dependency: Higher payments “compensated” by lower life expectancy.

2. The increase in dependency is fixed a priori. Any gains / deficits are shared within the pool of active individual (“morbidity credits”).

We use the notation $\alpha(T^{(a)})$ where $T^{(a)}$ is the time where the individual moves into dependency to account for the increase in payments: $b_j(t)$ as an active; $\alpha(T^{(a)}) \cdot b_j(t)$ as a dependent person.
Modern Life-Care Tontine

“Natural increase”: French mortality/disability data shows actuarially fair values for $a(T^{(a)})$:

Mortality credits of a dependent person depend on the death probability

$$q_{x_j+t-1}^{(i)} > q_{x_j+t-1}^{(a)}.$$
Discussion and conclusion

▶ It is beneficial to pool mortality and long-term care (morbidity) risks.

▶ We show how this scheme can be adapted to a life-care tontine introducing the concept of morbidity credits.

▶ The scheme allows to pool different age cohorts.

▶ It is fully-funded at all times, allowing individuals to later join the scheme!
Thank you!


Definition (Fair distribution rule: mortality credits)

A fair distribution rule \( \beta_j(X(t)) \) satisfies:

- **Self-sufficiency property**: \( \sum_{j \in \mathcal{L}_{t-1}} \beta_j(X(t)) = X(t). \)
- **Positivity property**: \( \beta_j(X(t)) \geq 0. \)
- **Fairness property**: 
  \[
  \mathbb{E}_{t-1} \left[ \beta_j(X(t)) \right] = \mathbb{E}_{t-1} \left[ \mathbf{1}_{j \in \mathcal{D}_t} \right] \cdot e^{\int_{t-1}^{t} \delta_s \, ds} \cdot c_j(t-1),
  \]

where \( \mathbb{E}_t := \mathbb{E}[ \cdot | \mathcal{F}_t ] \) is an expectation conditional on the information \( \mathcal{F}_t := \sigma(\mathcal{L}_t) \).
Example (Linear risk sharing rule)

At time $t$, each individual $j \in L_{t-1}$ receives the mortality credit (respectively death benefit):

$$
\beta_j(X(t)) = \frac{q_{x_j+t-1} \cdot c_j(t - 1)}{\sum_{j \in L_{t-1}} q_{x_j+t-1} \cdot c_j(t - 1)} \cdot X(t).
$$

(see, e.g., Donnelly, Guillén, Nielsen [2013, 2014], Schumacher [2018])

Example (Linear regression rule)

At time $t$, each individual $j \in L_{t-1}$ receives the mortality credit (respectively death benefit):

$$
\beta_j(X(t)) = \mathbb{E}_{t-1}[X_j(t)] + \frac{\text{Cov}_{t-1}[X_j(t), X(t)]}{\text{Var}_{t-1}[X(t)]} (X(t) - \mathbb{E}_{t-1}[X(t)]).
$$
Example (Conditional mean risk sharing rule)

At time $t$, each individual $j \in \mathcal{L}_{t-1}$ receives the mortality credit (respectively death benefit):

$$
\beta_j(X(t)) = \mathbb{E}_{t-1}[X_j(t) | X(t)] .
$$

(9)

(see, e.g., Denuit and Dhaene [2012], Denuit [2019])
Individual $j \in \mathcal{L}_t$’s time-$t$ account value is given by:

$$c_j(t) = \sum_{u=t+1}^{\omega-x_j} e^{-\int_u^t \delta_s ds} s_j(u).$$  \hspace{1cm} (10)$$

How do we choose $s_j(u)$, $u = 1, 2, \ldots, \omega - x_j$?

For example, choose the average payoff to be constant, equal to $b_j > 0$:

$$\mathbb{E}_{t-1}[W_j(t) \mid j \in \mathcal{L}_t] = \mathbb{E}_{t-1}[\mathbb{1}_{j \in \mathcal{L}_t} \cdot s_j(t) + \mathbb{1}_{j \in \mathcal{L}_{t-1}} \cdot \beta_j(X(t)) \mid j \in \mathcal{L}_t]$$

$$= s_j(t) + \mathbb{E}_{t-1}[\beta_j(X(t))]$$

$$= s_j(t) + q_{x_j+t-1} e^{\int_{t-1}^t \delta_s ds} c_j(t-1) \overset{!}{=} b_j. \hspace{1cm} (11)$$

\((13) \text{ is a system of equations backwards in time!}\)
Theorem (Backwards iteration)

If an individual \( j \in L_t \) aims for an average payoff \( b_j(t) \), the fixed payoff is given by:

\[
s_j(t) = \begin{cases} 
\frac{b_j(t)}{1+q_{\omega-1}}, & \text{for } t = \omega - x_j \\
\frac{b_j(t) - q_{x_j + t - 1} \sum_{u=t+1}^{\omega-x_j} e^{-\int_t^\mu \delta_s ds} s_j(u)}{1+q_{x_j + t - 1}}, & \text{for } t = \omega - x_j - 1, \omega - x_j - 2, \ldots, 1
\end{cases}
\]  

(12)

We derive the individual’s account value as

\[
c_j(t) = \sum_{u=t+1}^{\omega-x_j} e^{-\int_t^\mu \delta_s ds} s_j(u)
\]  

(13)

and the initial single premium as \( c_j(0) \).