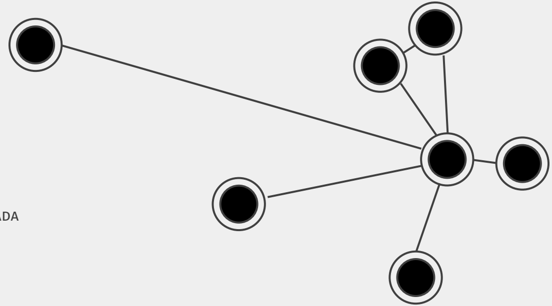
Collaborative Insurance Sustainablity And Network Structure

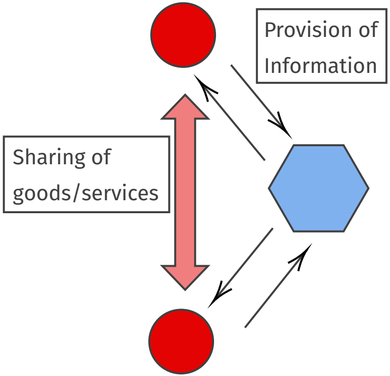
Arthur Charpentier[a]
Lariosse Kouakou[b]
Matthias Löwe[c]
Philipp Ratz[a]
Frank Vermet[b]

[a]Université du Québec à Montréal (UQAM), Montréal (Québec), Canada
[b]EURo Institut d'Actuariat (EURIA), Université de Brest, France
[c]University of Münster, Germany

19 05 2021



The Main Idea: Sharing deductibles with friends



- In a nutshell - we will consider different risk sharing models between peers in a network and analyze the robustness of the mechanisms
- We are essentially in a peer-to-peer (P2P) framework, which is characterized by decentralized transactions. In the context of insurance, there are indications that such a system might reduce moral hazard [2]
- For our approach, we will work with networks represented by graphs. The most important definitions are given before we move on to the setup of the studies and the results

In mathematical terms, networks are represented by graphs. Many interesting properties of graphs can be described using its *adjacency matrix*

Definition of a graph

A graph $\mathcal{G}$ is an ordered pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ represents a set of vertices (or nodes) and $\mathcal{E}$ a set of edges (or links) such that $\mathcal{E} \subseteq \{\{x, y\}|x, y, \in \mathcal{V}, x \neq y\}$. That is, an edge links two vertices together.

Adjacency matrix

The adjacency matrix A for the set of vertices $\mathcal{V} = \{v_1, ..., v_n\}$ is of size $n \times n$ and such that $A_{i,j} = 1$ if there is a edge from vertex v_i to v_j. In the following we will also assume that $A_{i,i} = 0$ (ie. a node does not have a connection to itself).

Note that edges can take many forms, such as directed or weighted. In this presentation we only consider unweighted and undirected graphs.

Degree distribution

The degree of a vertex is the number of edges that are incident to it. The average degree of a graph is denoted as $\overline{d}$. The degree vector for each vertex can be calculated by taking $A\mathbb{1} = d$. Note that the average degree of a network is:

$$\mathbb{E}(D) = \frac{1}{|\mathcal{V}|} \sum_{v \in \mathcal{V}} d(v) = \frac{2|\mathcal{E}|}{|\mathcal{V}|} = \frac{1}{|\mathcal{V}|} \|d\|_1$$

The degree distribution gives insights about the structure of the network. If the variance of the degrees is high, we have few nodes with many connections and many nodes only have a few. We will see later on that this has a major impact on the sustainability of an insurance system based on a network.

To investigate properties of different networks, we will work on *random graphs*. Here we assume that the degree of a random node in $\mathcal{V}$ is a random variable, with some distribution. Two popular methods are outlined below:

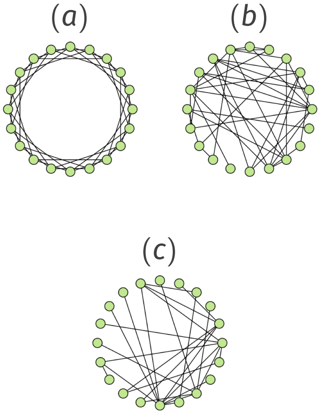
Erdős–Rényi

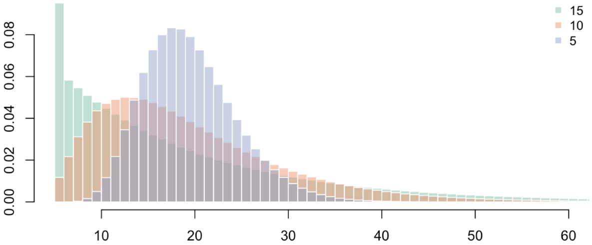
Each edge $i, j \in \mathcal{V} \times \mathcal{V}$ is included in the network with probability p independent from other edges. The Network then has a degree distribution that follows a binomial distribution $\mathcal{B}(n-1, p)$. See [5] for details.

Preferential attachment

To start with, consider a small network, and create a new node at every time-step. During every time-step, the new node is connected to the existing nodes with a probability p that is proportional to the degree of an existing node d_i. This way, every new node is more likely to connect to existing "popular" nodes. See [1] or [6] for details. Here the degree distribution follows a power law.

Random Network Generation





Groups in a network often possess the *homophily* property. Extracting information based on the peers of a node may result in strange results.

Friendship paradox

Consider a finite network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Let $D : \mathcal{V} \to \mathbb{N}_0$ denote the function that assigns to each vertex v its degree $D(v)$ (the number of friends) Then, if we denote $n_{\mathcal{V}} = |\mathcal{V}|$

$$\frac{1}{n_{\mathcal{V}}} \sum_{v \in \mathcal{V}} \frac{1}{D(v)} \sum_{w:(v,w) \in \mathcal{E}} D(w) \geq \frac{1}{n_{\mathcal{V}}} \sum_{v \in \mathcal{V}} D(w)$$

For a proof see eg. [3]. In plain English this means that on average a person has fewer friends than her friends have. Note that the equality holds if and only if $\mathcal{G}$ is regular. We provide an extension to relate this to a possible insurance product.

We assume each node in a network can purchase an insurance contract with deductible s. Then, take the cost Y_i of a random loss and let F be its distribution. For simplicity also denote Z_i the indicator that node i claims a loss (ie. a Bernoulli with probability p). Further, let d_i denote the degree of node i and $\overline{d}$ the average degree.

Risk and reciprocal engagements

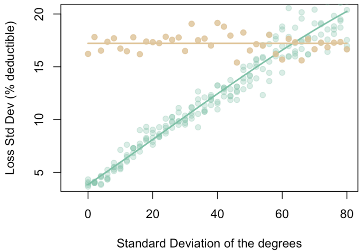
The random wealth of an insured at after the agreed time period is:

$$X_i = Z_i \cdot \min\{s, Y_i\} = \begin{cases} 0 \text{ if no claim occurred } (Z_i = 0) \\ \min\{s, Y_i\} \text{ if a claim occurred } (Z_i = 1) \end{cases}$$

If we set up the rule that we pay all our connections $\gamma = s/\overline{d}$ if they have a loss, we receive:

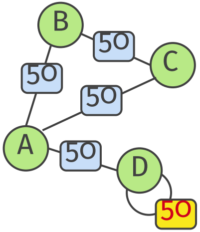
$$\xi_i = Z_i \cdot \min\{s, Y_i\} + \sum_{j \in V_i} Z_j \min\left\{\gamma, \frac{\min\{s, Y_j\}}{d_j}\right\} - Z_i \cdot \min\{d_i\gamma, \min\{s, Y_i\}\}$$

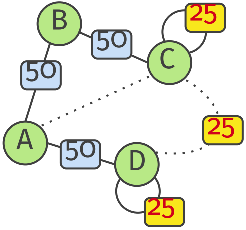
which can be shown to be a risk sharing principle (for a definition see eg. [4])



Assume that we generate a network with n nodes such that the distribution of the degrees is $D \stackrel{\mathcal{L}}{=} \min\{5 + [\Delta], n\}$, where $[\Delta]$ has a rounded Gamma distribution with mean $\overline{d} - 5$ and variance σ^2. For the application $\overline{d} = 20$, while the standard deviation parameter σ will be a parameters that will take values from 0 to $4\overline{d}$.

The graph to the left illustrates results of the simple risk sharing mechanism, where $\gamma = 50$, $\mathbb{E}(X) = 4.5\%$. We can see that for networks with low degree variance the mechanism works quite well, but in the extreme case with a large variance, the mechanism actually performs worse than no mechanism at all.

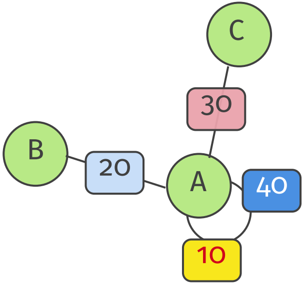


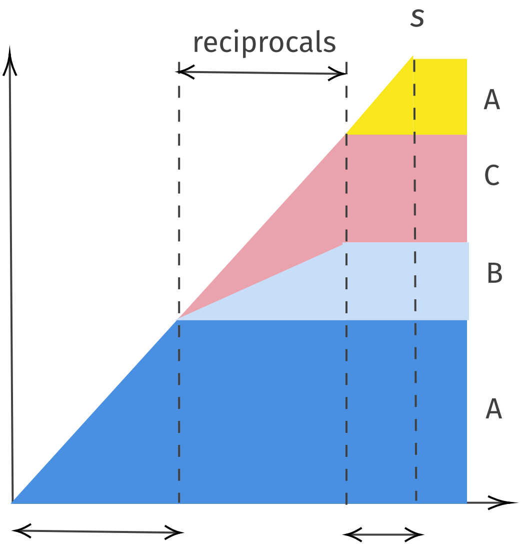


Already in the simple example, it becomes clear that the basic risk sharing mechanism needs some improvement, in the following we propose three such mechanisms

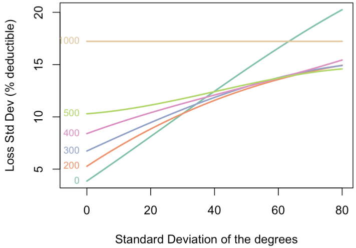
- Introducing self-contribution to ensure fairness in the mechanism and alleviate the variance issue
- Calculate optimal reciprocal engagements subject to some constraint - to take into account the structure of the network. This can be combined with engagements which are restricted to a subgraph (intuitively - we can cut some edges by setting the engagement to zero)
- Extending the reciprocal engagements to friends-of-friends to expand the set of possible edges for nodes with only a few links

Reciprocal engagement with self-contribution





Risk sharing with self-contribution

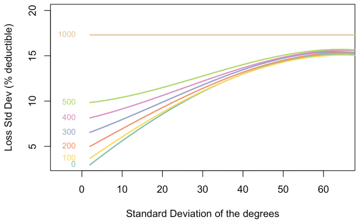


- It can be shown that in many cases, it is likely that the nodes who actually claim a loss will end up paying *less* than its adjacent vertices. The system seems fair ex-ante but ex-post (after the occurrence of the claim) it might seem unfair
- Self contribution makes a "fair" ex-post outcome more likely
- On the left the results when the simple risk sharing mechanism from above is augmented with different levels of self-contribution

$$\max \left\{ \sum_{(i,j)\in\mathcal{E}} \gamma_{(i,j)} \right\}$$

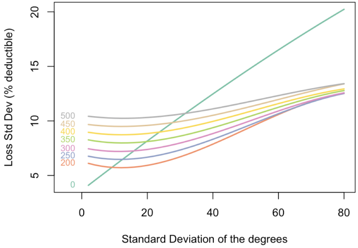
$$\text{s.t. } \gamma_{(i,j)} \in [0, \gamma], \ \forall (i,j) \in \mathcal{E}$$

$$\sum_{j\in\mathcal{V}_i} \gamma_{(i,j)} \leq s, \ \forall i \in \mathcal{V}$$



- Instead of a fixed contribution as in the simple example, contributions can also be globally optimized. Especially in the cases where the degree variance is high, many engagements with the central nodes will only have a small or zero amount
- The optimization can be conveniently formulated as a linear programming problem. Alternatively, a sparse(er) solution to the problem can be found via mixed-integer programming
- On the left the results for the risk sharing mechanism with personalized contributions and different levels of self-contribution

Risk sharing with Friends-of-Friends



- A further simple extension is to consider not only the adjacent nodes but also their adjacent nodes (friends-of-my-friends). This will naturally increase the number of connections
- The key restriction here is to proceed in two stages, first, the optimal reciprocal engagement with friends is found and only then the risk sharing is extended to friends-of-friens for the residual risk
- On the left the results, when the residual risk of the first stage can be shared with friends of friends

References I

A. Barabási and R. Albert.
Emergence of scaling in random networks.
Science, 286(5439):509–512, 1999.

Christian Biener, Martin Eling, Andreas Landmann, and Shailee Pradhan.
Can group incentives alleviate moral hazard? the role of pro-social preferences.
European Economic Review, 101:230–249, 2018.

Josh Brown Kramer, Jonathan Cutler, and A. J. Radcliffe.
The multistep friendship paradox.
Amer. Math. Monthly, 123(9):900–908, 2016.

Michel Denuit and Jan Dhaene.
Convex order and comonotonic conditional mean risk sharing.
Insurance: Mathematics and Economics, 51(2):265–270, 2012.

References II

P. Erdős.
Some remarks on the theory of graphs.
Bulletin of the American Mathematical Society, 53(4):292–294, 1947.

D. Watts and S. Strogatz.
Collective dynamics of 'small-world' networks.
Nature, 398:440–442, 1998.
`https://doi.org/10.1038/30918`.