One-Year and Ultimate Reserving Uncertainties for the Bornhuetter-Ferguson Method

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Overview

1. Contribution to the ultimate view of reserving - New BF method
2. Contribution to the one-year view of reserving - new definition
3. Applications

Motivation

Disclaimer:
The opinions expressed in this presentation are those of the author and do not necessarily reflect the views of the company the author works for.

This research is mainly focused on **two practical problems** of stochastic reserving that were raised by the (re)insurance industry:

1. Provide an unified stochastic framework to combine both CL (older years) and BL (recent years) approaches and derive the closed-form formula for ultimate run-off uncertainty.
2. Derive the closed-formula for one-year run-off uncertainty for other than Mack CL reserving methods such as New BF for instance or other popular approaches.

New Bornhuetter-Ferguson method

- $u_i := E(C_{i,J})$, and its prior estimate $\hat{u}_i$ is taken from pricing (or market information).
- β_k - cumulative development pattern.
- $A_{i,k} := \{C_{i,1}, .., C_{i,k}\}$.
- $F_{i,k} = C_{i,k+1}/C_{i,k}$.

	Mack CL(1993)	**Mack BF(2008)**	**New BF**
H.1	$E(C_{i,k+1} \mid A_{i,k})$ $= f_k C_{i,k}$	$E(C_{i,k+1} - C_{i,k} \mid A_{i,k})$ $= (\beta_{k+1} - \beta_k) u_i$	$E(C_{i,k+1} - C_{i,k} \mid A_{i,k})$ $= (\beta_{k+1} - \beta_k) u_i$
H.2	$Var(C_{i,k+1} \mid A_{i,k})$ $= \sigma_k^2 C_{i,k}$	$Var(C_{i,k+1} - C_{i,k} \mid A_{i,k})$ $= \sigma_k^2 u_i$	$Var(C_{i,k+1} - C_{i,k} \mid A_{i,k})$ $= \sigma_k^2 u_i C_{i,k}$
H.3	$\{C_{i,1}, .., C_{i,J}\} \perp$ for all i	$\{C_{i,k+1} - C_{i,k}\} \perp$ for all i, k	$\{C_{i,1}, .., C_{i,J}\} \perp$ for all i
Est.	$\widehat{f_k} = \frac{\sum_{i=1}^{I-k} w_{i,k} F_{i,k}}{\sum_{i=1}^{I-k} w_{i,k}}$ $w_{i,k} := C_{i,k}$		$\widehat{f_k} = \frac{\sum_{i=1}^{I-k} w_{i,k} F_{i,k}}{\sum_{i=1}^{I-k} w_{i,k}}$ $w_{i,k} := C_{i,k}/u_i$ $\widehat{\beta_j} := \prod_{k=j}^{J-1} \frac{1}{\widehat{f_k}}$

Main Results and Limitations

- BF principle: $E[C_{i,k}] = \beta_k u_i \quad k \geq 1.$
- CL principle: $E(C_{i,j+1}|C_{i,1},..,C_{i,j}) = f_j E(C_{i,j}|C_{i,1},..,C_{i,j-1}).$
- **msep** for single accident year i:

$$\widehat{u}_i^2 \sum_{l=I+1-i}^{J-1} \widehat{\beta}_l \widehat{\sigma}_l^2 + \widehat{\beta}_{I-i+1}^2 \sum_{j=I-i+1}^{J-1} \frac{Var\left(\widehat{f}_j|D_I\right)}{\widehat{f}_j^2} + \left[Var(\widehat{\beta}_{I-i+1})) + (1-\widehat{\beta}_{I-i+1})^2\right] Var(\widehat{u}_i).$$

- **msep** over all accident years (only $Cov(\widehat{C}_{i,J}^{BF}, \widehat{C}_{j,J}^{BF})$ term)

$$\left[(1-\widehat{\beta}_{I-i+1})(1-\widehat{\beta}_{I-j+1}) + Cov(\widehat{\beta}_{I-i+1}, \widehat{\beta}_{I-j+1})\right] Cov(\widehat{u}_i, \widehat{u}_j) + \widehat{u}_i \widehat{u}_j Cov(\widehat{\beta}_{I-i+1}, \widehat{\beta}_{I-j+1}).$$

- **Limitations:** use of the first order Taylor's approximation
  - $E(1/X) \approx 1/E(X)$ [to show that $\hat{\beta}_k$ are unbiased]
  - $Var(1/X) \approx [1/E(X)]^4 Var(X)$ [to estimate $Var(\widehat{\beta}_k)$]
  - $Cov(1/X, 1/Y) \approx [1/E(X)]^2 [1/E(Y)]^2 Cov(X, Y)$ [to compute $Cov(\widehat{\beta}_k, \widehat{\beta}_l$]

MW formula- alternative derivation (1/2)

- MW formula for single accident year i, $b_j^{(I)} := \frac{C_{i_j,j}}{\sum_{k=i_0}^{i_j} C_{k,j}} \in (0,1)$.

$$\widehat{msep}^{MW}_{CDR_i^{(I+1)}|D_I}(0) \approx \left(\widehat{C}_{i,J}\right)^2 \frac{\widehat{\sigma}^2_{j_i}}{\widehat{f}^2_{j_i}} \left(\frac{1}{\widehat{C}_{i,j_i}} + \frac{1}{\sum_{k=i_0}^{i_j-1} C_{k,j_i}} \right) + \underbrace{\sum_{j=j_i+1}^{J-1} \frac{\widehat{\sigma}^2_{j}}{\widehat{f}^2_{j}} \left(\alpha_j^{(I)} \frac{1}{\sum_{k=i_0}^{i_j-1} C_{k,j_i}} \right)}_{\text{extra term}}$$

- Alternative derivation of MW formula:

$$\widehat{msep}^{MW}_{CDR_i^{(I+1)}|D_I} \approx \widehat{msep}_{C_{i,J}|D_I}(\widehat{C}^{(I)}_{i,J}) - \widehat{msep}_{C_{i,J}|D_{I+1}}(\widehat{C}^{(I+1)}_{i,J}).$$

MW formula- alternative derivation (2/2)

- General formula for futures accounting years:

$$\widehat{msep}^{MW}_{CDR_i^{(I+k+1)}|D_I} \approx \left(\widehat{C}_{i,J}\right)^2 \left\{ PV^{Diag(I+k)}_{i,ult} + EE^{Diag(I+k)}_{i,ult} + REE_i^{(I+k\to I+k+1)} \right\},$$

where

$$REE_i^{(I+k\to I+k+1)} := \underbrace{\frac{Var\left(\widehat{f}^{(I+k)}_{j_i+k}|B^{(I+k)}_{j_i+k}\right) - Var\left(\widehat{f}^{(I)}_{j_i+k}|B^{(I)}_j\right)}{\left(\widehat{f}^{(I)}_j\right)^2}}_{(\leq 0)\quad \text{'drop' of volatility}\quad D_I \to D_{I+k}} + \underbrace{\sum_{j=j_i+k+1}^{J-1} \frac{Var\left(\widehat{f}^{(I+k)}_j|B^{(I+k)}_j\right) - Var\left(\widehat{f}^{(I+k+1)}_j|B^{(I+k+1)}_j\right)}{\left(\widehat{f}^{(I)}_j\right)^2}}_{(\geq 0)\quad \text{'rise' of volatility}\quad D_{I+k+1} \to D_{I+k}}.$$

- New (general) definition of one-year run-off uncertainties:

$$msep^{(I+k)}_{\widehat{C}^{(I+k)}_{i,J}|D_I}(\widehat{C}^{(I+k+1)}_{i,J}) := msep_{C_{i,J}|D_{I+k}}(\widehat{C}^{(I+k)}_{i,J}) - msep_{C_{i,J}|D_{I+k+1}}(\widehat{C}^{(I+k+1)}_{i,J})$$

Applications - New BF (ultimate view)

- Comparison of results from Mack BF, New BF and Mack CL:
  - Process Variance (PV): significantly higher estimation by New BF (+100%) due to the link to the variability of $F_{i,k}$ via (conditional) variance assumption
  - Estimation Error (EE): this term is of the same order
  - Covariance term (CoV): higher in Mack BF (+34%) and explained by Dirichlet distribution assumption in estimation of $Cov(\hat{\beta}_i, \hat{\beta}_j)$.

Mack BF						
AY	Reserve	EE	PV	CoV	MSEP	Vco
Total	**873 932**	**36 656**	**37 152**	**54 242**	**75 274**	**8,6%**

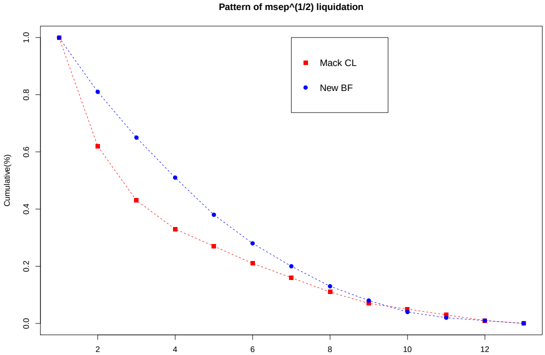
New BF						
AY	Reserve	EE	PV	CoV	MSEP	Vco
Total	**906 892**	**37 882**	**75 299**	**40 601**	**93 560**	**10,3%**

Mack CL						
AY	Reserve	EE	PV	CoV	MSEP	Vco
Total	**1 002 818**	**247 191**	**129 054**	**111 991**	**300 500**	**30,0%**

Applications - New BF (one-year view)

- SCR computation for reserve risk (ratio of one-year over ultimate run-off volatility)
- Risk Margin computation in Solvency 2 (or Risk Adjustment in IFRS 17)
  - Development pattern based on the $msep^{1/2}$, $\rho_k := a_k/a_0$, where

$$a_k := \sum_{l=k+1}^{J} \left\{ msep^{(I+l)}_{\widehat{C}^{(I+l)}_{tot,J}|D_I} \left(\widehat{C}^{(I+l+1)}_{tot,J} \right) \right\}^{1/2}.$$



Future research and Q&A session

- Conjecture on classical 'hybrid' approach between CL and BF
- BF standalone approach (β_k independent of f_k)
- Q&A session:

Thank You!