Quantifying Reserve Risk Based on Volatility in Triangles of Estimated Ultimate Losses

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Agenda

- **Reserve Risk – Intro**
- The "Triangle Structure Problem"
- Feng-Robbin Method
- Comparison of Methods
- Questions

Which LOB's reserve is more volatile?

LOB A - Incurred Loss									
	12	24	36	48	60	72	Latest	SD	CV
2015	400	600	800	850	900	900	900	201	22.34%
2016	300	500	700	750	800		800	207	25.92%
2017	450	700	850	900			900	202	22.45%
2018	500	750	1000				1,000	250	25.00%
2019	350	500					500	106	21.21%
2020	600						600		0.00%

LOB B - Incurred Loss									
	12	24	36	48	60	72	Latest	SD	CV
2015	400	400	850	850	900	900	900	246	27.37%
2016	200	1000	700	750	800		800	297	37.08%
2017	100	700	900	900			900	379	42.07%
2018	500	900	1000				1,000	265	26.46%
2019	200	1000					1,000	566	56.57%
2020	700						700		0.00%

CVs of Case Incurred Loss TD by Row?

- Does not distinguish strongly enough between LOBs
- Confuses development in reported loss with volatility of loss development

Which LOB's reserve is more volatile?

LOB A - Estimated Ultimate Losses using Industry LDF Development									
	12	24	36	48	60	72	Latest	SD	CV
2015	924	866	924	893	900	900	900	22	2.40%
2016	693	722	809	788	800		800	52	6.44%
2017	1,040	1,011	982	945			945	40	4.28%
2018	1,155	1,083	1,155				1,155	42	3.61%
2019	809	722					722	61	8.49%
2020	1,386						1,386		0.00%

LOB B - Estimated Ultimate Losses using Industry LDF Development									
	12	24	36	48	60	72	Latest	SD	CV
2015	924	578	982	893	900	900	900	143	15.94%
2016	462	1,444	809	788	800		800	357	44.66%
2017	231	1,011	1,040	945			945	386	40.82%
2018	1,155	1,299	1,155				1,155	83	7.22%
2019	462	1,444					1,444	694	48.08%
2020	1,617						1,617		0.00%

CVs of Estimated Ultimate Loss by Row?

- Does distinguish strongly enough between LOBs

- Attempts to disentangle loss development from volatility of loss development

Omniscient Actuary

LOB A - Final Best Estimate of Ultimate Losses									
	12	24	36	48	60	72	Latest	SD	CV
2015	910	877	921	893	900	900	900	15	1.68%
2016	810	777	821	793	800		800	17	2.11%
2017	960	977	971	943			943	15	1.57%
2018	1,010	1,027	1,121				1,121	60	5.32%
2019	860	777					777	59	7.63%
2020	1,110						1,110		0.00%

LOB B - Final Best Estimate of Ultimate Losses									
	12	24	36	48	60	72	Latest	SD	CV
2015	900	900	900	900	900	900	900	-	0.00%
2016	800	800	800	800	800		800	-	0.00%
2017	950	950	950	950			950	-	0.00%
2018	1,100	1,100	1,100				1,100	-	0.00%
2019	1,250	1,250					1,250	-	0.00%
2020	1,200						1,200		0.00%

Ultimate Loss Development Factors

LOB A - Ultimate Age-to-Age Development factors					
	12	24	36	48	60
2015	0.9656	1.0518	0.9708	1.0084	1.0012
2016	0.9614	1.0584	0.9672	1.0094	
2017	1.0192	0.9956	0.9723		
2018	1.0183	1.0927			
2019	0.9058				
2020					

LOB B - Ultimate Age-to-Age Development factors					
	12	24	36	48	60
2015	1.0000	1.0000	1.0000	1.0000	1.0000
2016	1.0000	1.0000	1.0000	1.0000	
2017	1.0000	1.0000	1.0000		
2018	1.0000	1.0000			
2019	1.0000				
2020					

UDF Analysis – the Key to Reserve Risk

- UDF - Ultimate Loss Development Factor
  - Age-to-age factor in triangle of estimated ultimate loss amounts
- Reserve risk is buried in the volatility of age-to-age UDF
- UDF Volatility impacted by both inherent volatility of the data and the methods/parameters used to estimate ultimate.

Reserve Risk

- Definition: the potential for adverse development of the estimate of ultimate
- Volatility of data (inherent error)
  - Reserve Risk in triangle is 0 if all UDFs =1
- Different estimates of ultimate have different risk.

Many Ways to Quantify Reserve Risk

- Based on paid and incurred triangle
  - ICRFS/Zenwirth – Trends of log-normal increments
  - URS/Alex - Stochastic Decay Model
  - Ohlsson and Lauzenings – Diagonal simulation and defined reserving method
  - Mack – closed-form chain ladder
  - Merz Wuthrich – one-year risk
  - MCMC Methods
- Based on ultimate loss triangles
  - Rehman Klugman
  - Siegenthaler
  - Feng Robbin

Feng-Robbin Estimates

- Based on ultimate loss triangles
- Uses Variance-Covariance of UDF
- Derives Variance of one year and ultimate reserve conditional on current estimated ultimates.
- Includes development age covariance terms
- Parameter risk quantified as deviation from 1
- One estimate among many

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- **The “Triangle Structure Problem”**
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Triangle Structure Problem

- Hidden lack of positive semi-definiteness
  - Covariance of UDF factor columns use vectors of different sizes
  - Resulting variance-covariance matrix may not be positive semi-definite
  - Could lead to negative calculated variance
- Tail-driven instability
  - Tail UDFs based on small sample sizes
  - Leveraged impact
  - Could lead to unstable risk estimates

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Rehman & Klugman (2010)

- Computes ultimate reserve risk
- Assumes UDFs (*g* factors) follow log-normal distribution
- Uses sample mean and variance as estimators
- Uses first order Taylor Series approximations

Feng & Robbin (2021)

- Computes both one-year and ultimate reserve risk
- Provides more detailed treatment of covariance
- Addresses the triangle structure problem
- Includes a parameter error term

Feng Robbin Method Steps

- Step 1 – Obtain Ultimate Triangles
- Step 2 – Calculate ATA UDF Factors
- Step 3 – Take Logarithm of UDF Factors
- Step 4 – Calculate Var-CoVar Matrix
- Step 5 – Calculate Weight factors
- Step 6 – Compute One-year and Ultimate standard errors

Step 1 – Obtain Ultimate Triangles

AY\Age	$d=1$	2	3	4	5	$D=6$
$y=1$	900	877	921	1,250	901	900
2	900	777	821	788	800	
3	900	977	971	945		
4	900	1,027	1,121			
5	900	777				
$Y=6$	900					

Step 2 – Calculate ATA UDF Factors

AY\Age	1-2	2-3	3-4	4-5	5-6
1	0.9744	1.0502	1.3572	0.7208	0.9989
2	0.8633	1.0566	0.9598	1.0152	
3	1.0856	0.9939	0.9732		
4	1.1411	1.0915			
5	0.8633				

Step 3 – Log UDF Factors, Mean, Var

AY\Age	1-2	2-3	3-4	4-5	5-6
1	(0.025888)	0.048953	0.305439	(0.327394)	(0.001110)
2	(0.146954)	0.055083	(0.041025)	0.015114	
3	0.082092	(0.006160)	(0.027142)		
4	0.132002	0.087579			
5	(0.146954)				

The estimated unadjusted mean and variance of each log UDF factor is computed below:

Age	d=1	2	3	4	5
$\hat{\mu}_d$	(0.021140)	0.046364	0.079091	(0.156140)	(0.001110)
$\hat{\sigma}_d{}^2$	0.016448	0.001513	0.038473	0.058656	n/a

Step 4 – Variance – Covariance Matrix*

Age\Age	$d=1$	2	3	4	5
$d=1$	0.016448	(0.000063)	0.001086	(0.010367)	-
2	(0.000063)	0.001513	0.002090	0.000606	-
3	0.001086	0.002090	0.038473	(0.041955)	-
4	(0.010367)	0.000606	(0.041955)	0.058656	-
5	-	-	-	-	-

Step 5 – Calculate Weight Factors

AY	Y=6	5	4	3	2	y=1
r_y	0.1653	0.1428	0.2060	0.1736	0.1470	0.1653

Development Age	d=1	2	3	4	5	D=6
Corresponding AY	Y=6	5	4	3	2	y=1
R_d	0.1653	0.3081	0.5141	0.6877	0.8347	1.0000

Step 6 – Ultimate Reserve Risk

$$\text{StDev}\left(\sum_{y=1}^{Y} U_{y,D}\right) = \sum_{y=1}^{Y} U_{y,D-y+1} \times \sqrt{\left(e^{2\widehat{\omega}+2\widehat{\lambda}^2} - e^{2\widehat{\omega}+\widehat{\lambda}^2}\right)}$$

Latest ultimate	Mean parameter	Variance parameter	Standard error
$\sum_{y=1}^{Y} U_{y,D-y+1}$	$\widehat{\omega}$	$\hat{\lambda}^2$	$\text{StDev}\left(\sum_{y=1}^{Y} U_{y,D}\right)$
5,443	(0.056854)	0.007575	450

Step 6 – One-Year Reserve Risk

$$\text{StDev}\left(\sum_{y=1}^{Y} U_{y,D-y+2}\right) = \sum_{y=1}^{Y} U_{y,D-y+1} \times \sqrt{\left(e^{2\hat{\alpha}+2\hat{\theta}^2} - e^{2\hat{\alpha}+\hat{\theta}^2}\right)}$$

Latest ultimate	Mean parameter	Variance parameter	Standard error
$\sum_{y=1}^{Y} U_{y,D-y+1}$	$\hat{\alpha}$	$\hat{\theta}^2$	$\text{StDev}\left(\sum_{y=1}^{Y} U_{y,D-y+2}\right)$
5,443	(0.007860)	0.000509	122

Triangle Structure Problem

Age\Age	d=1	2	3	4	5
d=1	0.016448	(0.000073)	0.001536	(0.020733)	-
2	(0.000073)	0.001513	0.002559	0.001050	-
3	0.001536	0.002559	0.038473	(0.059333)	-
4	(0.020733)	0.001050	(0.059333)	0.058656	-
5	-	-	-	-	-

Sum is Negative!

Triangle Structure - Fixes

- Fill-in procedure
  - Var-coVar matrix using the filled-in square
- Diagonal calibrated procedure
  - Start from the fill-in Var-coVar matrix
  - Replace the diagonal with sample variance
- Full adjustment procedure
  - Start from the fill-in Var-coVar matrix
  - Factor adjustments to the entire matrix

Fill-In Procedure

AY\Age	1-2	2-3	3-4	4-5	5-6
1	(0.025888)	0.048953	0.305439	(0.327394)	(0.001110)
2	(0.146954)	0.055083	(0.041025)	0.015114	(0.001110)
3	0.082092	(0.006160)	(0.027142)	(0.156140)	(0.001110)
4	0.132002	0.087579	0.079091	(0.156140)	(0.001110)
5	(0.146954)	0.046364	0.079091	(0.156140)	(0.001110)

Revised Variance – Covariance Matrix*

Age\Age	d=1	2	3	4	5
d=1	0.016448	(0.000063)	0.001086	(0.010367)	-
2	(0.000063)	0.001513	0.002090	0.000606	-
3	0.001086	0.002090	0.038473	(0.041955)	-
4	(0.010367)	0.000606	(0.041955)	0.058656	-
5	-	-	-	-	-

Sum is Positive

Parameter Risk

- Measures difference between the average UDF and 1 for each age
- Essentially the same as Siegenthaler's formula

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Advantages of R-K, F-R, and Siegenthaler

- Can work with any actuarial method or mix of methods
- Can quantify risk at any stage in reserving process:
  - For a specific method only
  - For the actuarial best estimate
  - For management booked estimate
- Works well for lines with sparse activity in early years
- Fast and spreadsheet friendly
- Rewards accurate IBNR estimates
  - If ultimate loss estimates are historically stable and accurate along each row, these method shows low volatility.

Disadvantages

- Need to construct historical ultimate triangles
- Favorable development translates to higher reserve risk?
- Does not capture future changes in reserve methodology not present in ultimate triangle
- Gives standard errors but not simulation results
- Need additional assumptions to get 99.5% VaR

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