On the study of premium renewal problem in non-life insurance based on two families of customer renewal probability through reinforcement learning

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Overview

1. Motivation
   - Problem and Context
   - Previous work
2. Reinforcement Learning
   - General aspect reinforcement learning
   - value functions and Bellman equation
3. Results / Contribution
   - Main results
   - Discussions

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Each end of year and based on the recorded claims history, Non-Life Insurance companies examine premiums of their products and strategies to get more tailored ones to the local economic environment. This helps them

- to be more adapted to customer's needs.
- to satisfy solvency requirements
- to increase their revenues.

Optimization Problem

Optimization Problem

Find $(r_i)_{i\in\mathcal{V}}$, $r_i \in \mathbb{R}$ maximizing:

$$\begin{cases} R(\mathcal{V}) = \sum_{i\in\mathcal{V}} \Phi_i(r_i) * P_i * r_i, \\ T(\mathcal{V}) = \frac{1}{N}\sum_{i\in\mathcal{V}} \rho_i(r_i). \end{cases}$$

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Literature overview

- Shapiro et al. (2003), Fuzzy clustering and non-linear integer programming for optimal ratio estimation.
- Dutang (2012), logistic regression for lapse rate estimation and deduces customer sensitivity as derivative of lapse rate.
- Dutang et al. (2013) , competition of market, multinomial model for lapse rate estimation, Nash and stackelberg equilibrium as concept solution for optimal ratio estimation.

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- Elena et al. (2019), dynamical approach through Q-learning model of reinforcement learning.

Overview

Reinforcement learning

definition

Dynamic study of an environment by an active learner for achieving certain goal.

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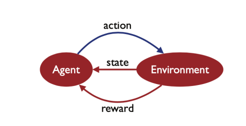
Reinforcement learning Process



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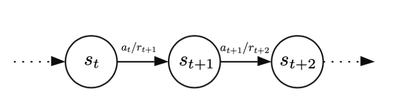

Overview

Reinforcement learning

$$V_\pi(s) = E_\pi(\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} | S_t = s) \tag{2.1}$$

$$Q_\pi(s,a) = E_\pi(\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} | S_t = s, A_t = a) \tag{2.2}$$

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Theorem

Any finite MDP admits an optimal deterministic policy.

Reinforcement learning

Bellman equation

$$\pi_*(s) = argmax_{a' \in \mathcal{A}} Q_*(s, a) \tag{2.3}$$

$$Q_*(s, a) = E(R_{t+1} | S_t = s, A_t = a) + \gamma \sum_{s' \in \mathcal{S}} P^a_{ss'} max_{a' \in \mathcal{A}} Q_*(s', a') \tag{2.4}$$

Stochastic approximation

Q-learning

$$q(S_t, A_t) \longleftarrow q(S_t, A_t) + \alpha(R_{t+1} + \gamma \max_{a'} q(S_{t+1}, a') - q(S_t, A_t)) \tag{2.5}$$

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Theorem

Consider a finite MDP. Assume that for all $s \in S$ and $a \in A$, $\sum_{t=0}^{+\infty} \alpha_t(s,a) = +\infty$ and $\sum_{t=0}^{+\infty} \alpha_t^2(s,a) < +\infty$ with $\alpha_t(s,a) \in [0,1]$. Then, the Q-learning algorithm converges to the optimal value q*.

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SARSA model

$$q(S_t, A_t) \longleftarrow q(S_t, A_t) + \alpha(R_{t+1} + \gamma q(S_{t+1}, A_{t+1}) - q(S_t, A_t))$$

Forward and backward approach

SARSA forward

$$
\begin{cases}
G_t^{(n)} = R_{t+1} + \gamma R_{t+2} + ... + \gamma^n q(S_{t+n}, A_{t+n}), \\
G_t^{\lambda} = (1-\lambda) \sum_{n=1}^{\infty} \lambda^{n-1} G_t^{(n)}, \\
q(S_t, A_t) \longleftarrow q(S_t, A_t) + \alpha(G_t^{\lambda} - q(S_t, A_t)).
\end{cases}
$$

Forward and backward approach

SARSA forward

$$\begin{cases} G_t^{(n)} = R_{t+1} + \gamma R_{t+2} + \dots + \gamma^n q(S_{t+n}, A_{t+n}), \\ G_t^{\lambda} = (1-\lambda)\sum_{n=1}^{\infty} \lambda^{n-1} G_t^{(n)}, \\ q(S_t, A_t) \longleftarrow q(S_t, A_t) + \alpha(G_t^{\lambda} - q(S_t, A_t)). \end{cases}$$

SARSA backward

$$\begin{cases} e_0(s,a) = 0, \forall s \in \mathcal{S}, \forall a \in \mathcal{A}, \\ e_t(s,a) \longleftarrow \lambda\gamma e_{t-1}(s,a) + 1_{S_t=s, A_t=a}, \\ \delta_t = R_{t+1} + \gamma q(S_{t+1}, A_{t+1}) - q(S_t, A_t), \\ q(S_t, A_t) \longleftarrow q(S_t, A_t) + \alpha\delta_t e_t(S_t, A_t) \end{cases}$$

Overview

Portfolio segmentation

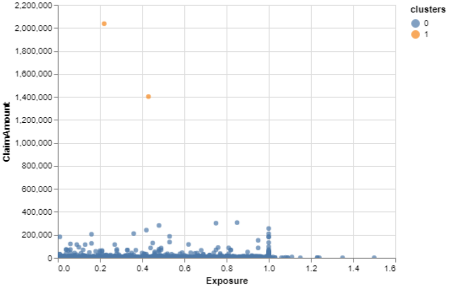


Figure 1: Clusters visualization

Renewal probability

$$p_\mu = \begin{cases} 1, \text{if, } r\pi \leq \pi + \mu\pi \\ 0, \text{else.} \end{cases} \quad r \in [1,2], \text{and} \mu \in [0,1]$$

Results

(a) Evolution of cumulative rewards using Logistic model

(b) Evolution of cumulative rewards using intuitive probability model

Figure 2: SARSA results

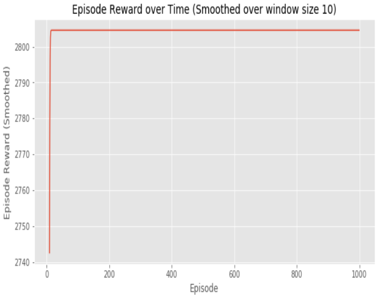
Reinsurance

Renewal probability with reinsurance

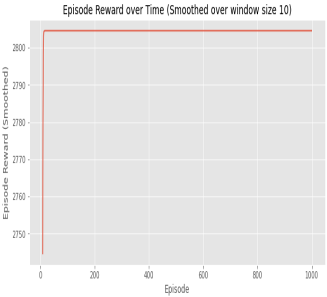
Let C_a the claims amount of a customer i. By considering an excess of loss contract a Xs b, the new model for estimating renewal probability is defined for customer i by:

$$\rho_\mu = \begin{cases} 1, \text{if}, r\pi \leq \pi + \mu\pi \text{ or } C_a > b, \\ 0, \text{else}. \end{cases}$$

Results with reinsurance considerations



(a) Evolution of cumulative rewards using intuitive probability model



(b) Evolution of cumulative rewards considering reinsurancel

Figure 3: SARSA results with reinsurance considerations

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Summary

- Optimization's curse in Q-learning convergence
- Rapid convergence of Backward SARSA got from new probability
- Estimation of sensitivity and new definition of strategy concept
- Contribution of reinsurance in renewal optimal strategies

Conclusion and discussion

In conclusion our model, improved by reinsurance considerations, found for each customer the best renewal ratio solving the trade-off between revenue and retention.

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Thank you for your attention!