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Model error in loss forecasts: What is it? Why should we care? How might it be measured?

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  - Benjamin Avanzi
  - Bernard Wong
  - David Yu

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Overview

- What is it?
- Internal and external model error
- Why should we care?
- Estimation of internal model error: ingredients
- Model distribution error
- Model structure error
  - Confidence sets
  - Worst case scenario
  - Bayesian model averaging

Terminology

- **Model uncertainty**
  - Uncertainty as to the correct algebraic form (but not parameters) of the model of the data under investigation
- **Forecast error**
  - The error contained in a forecast (i.e. difference from the ultimately observed value)
    - A random variable
- **Forecast risk**
  - A summary statistic of forecast error (e.g. standard deviation)

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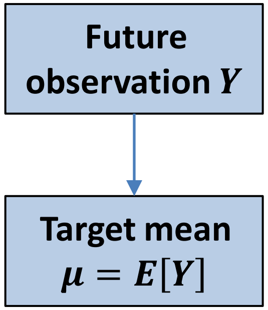
Framework for discussion

- Comments will relate to the general area of **statistical forecasting**
- Though the remarks in the presentation relate to this area generally, **forecast of a loss reserve** will be the usual application, for the sake of definiteness
- The **data input** to the loss reserve is unspecified and quite general
  - It may, but need not, take the form of the familiar **claim triangle**

What is it? The forecasting environment

Future observation Y

What is it? The forecasting environment



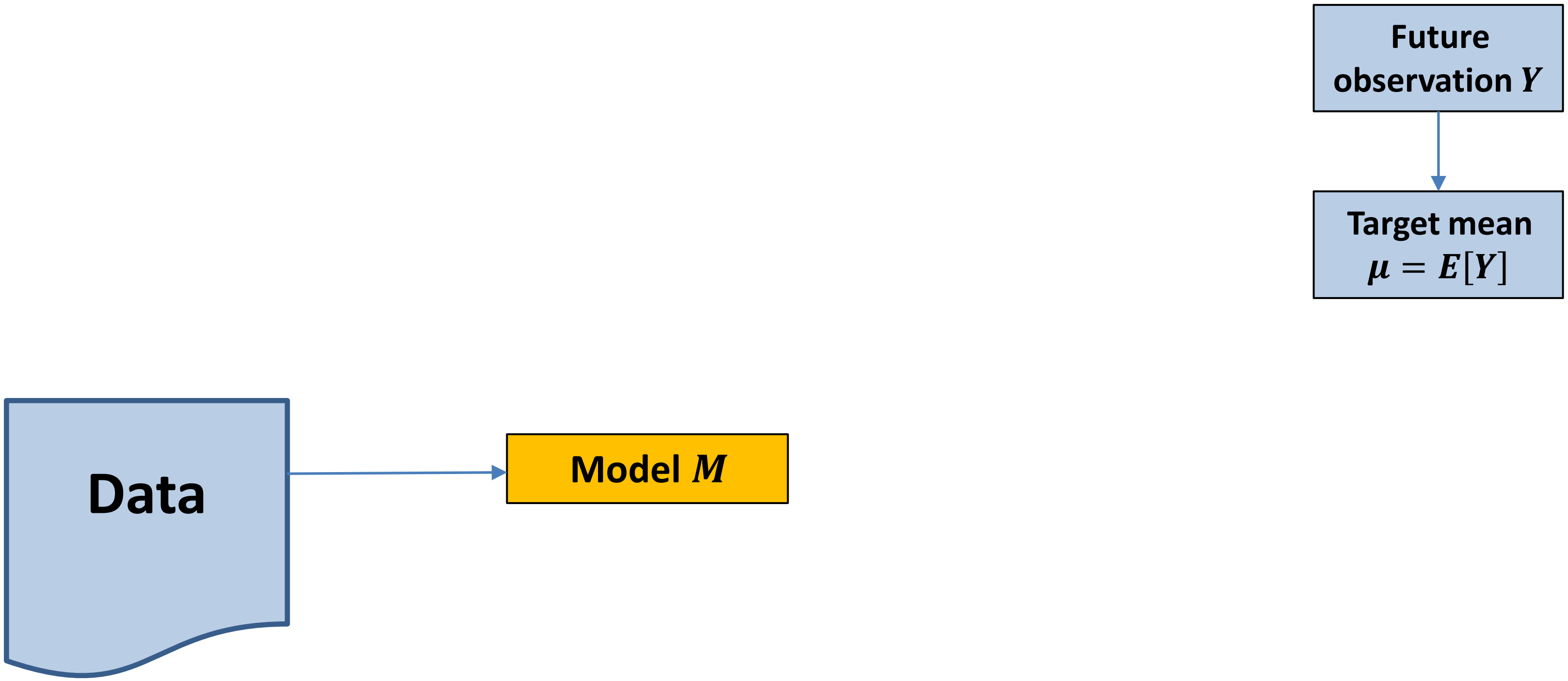
What is it? The forecasting environment

Future observation Y

Target mean $\mu = E[Y]$

Data

What is it? The forecasting environment



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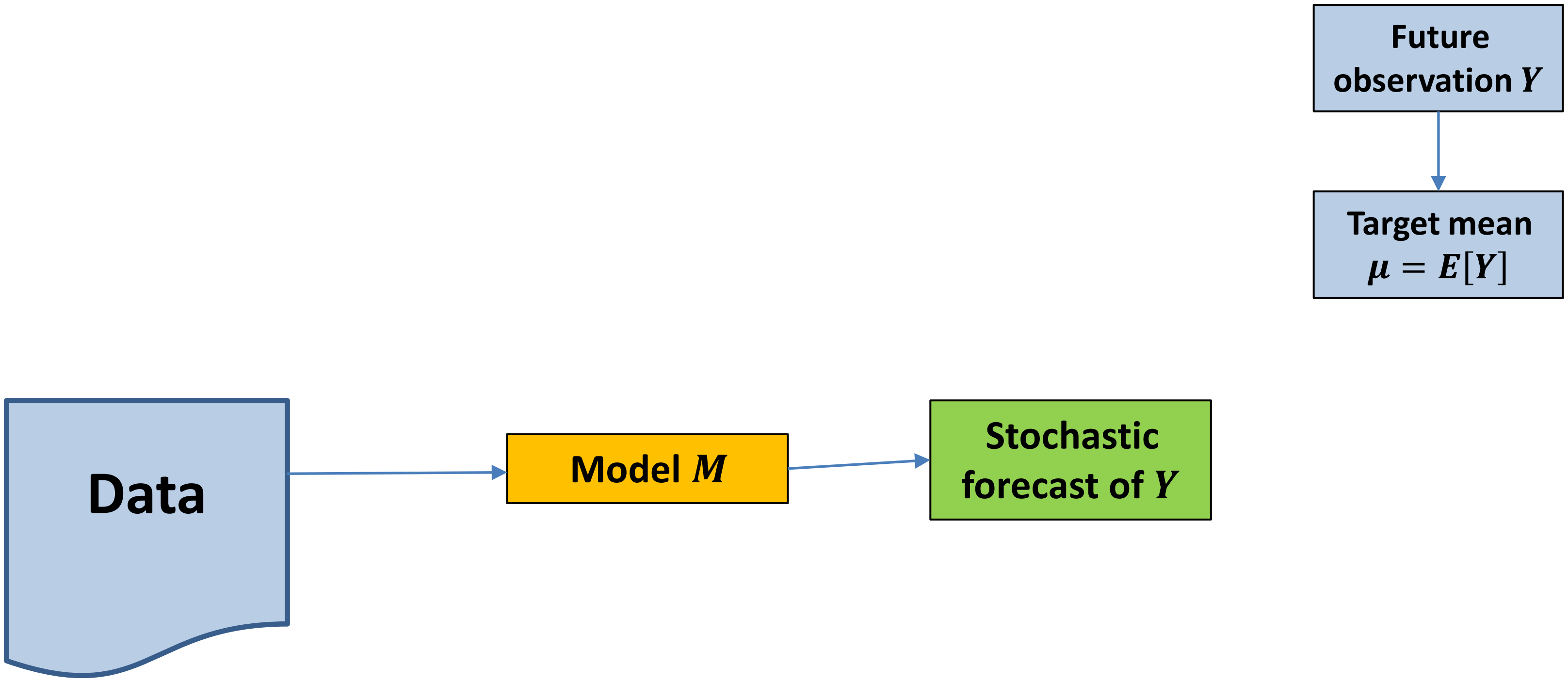

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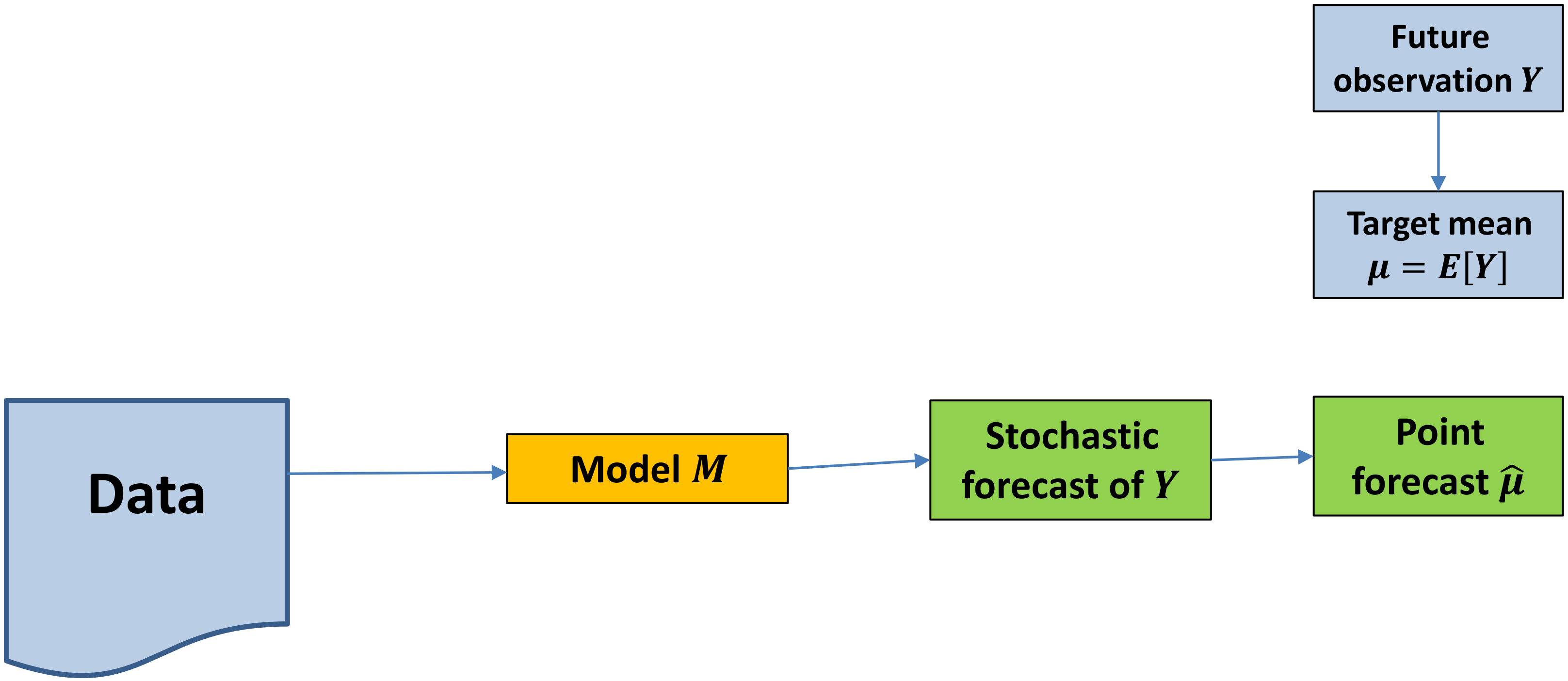

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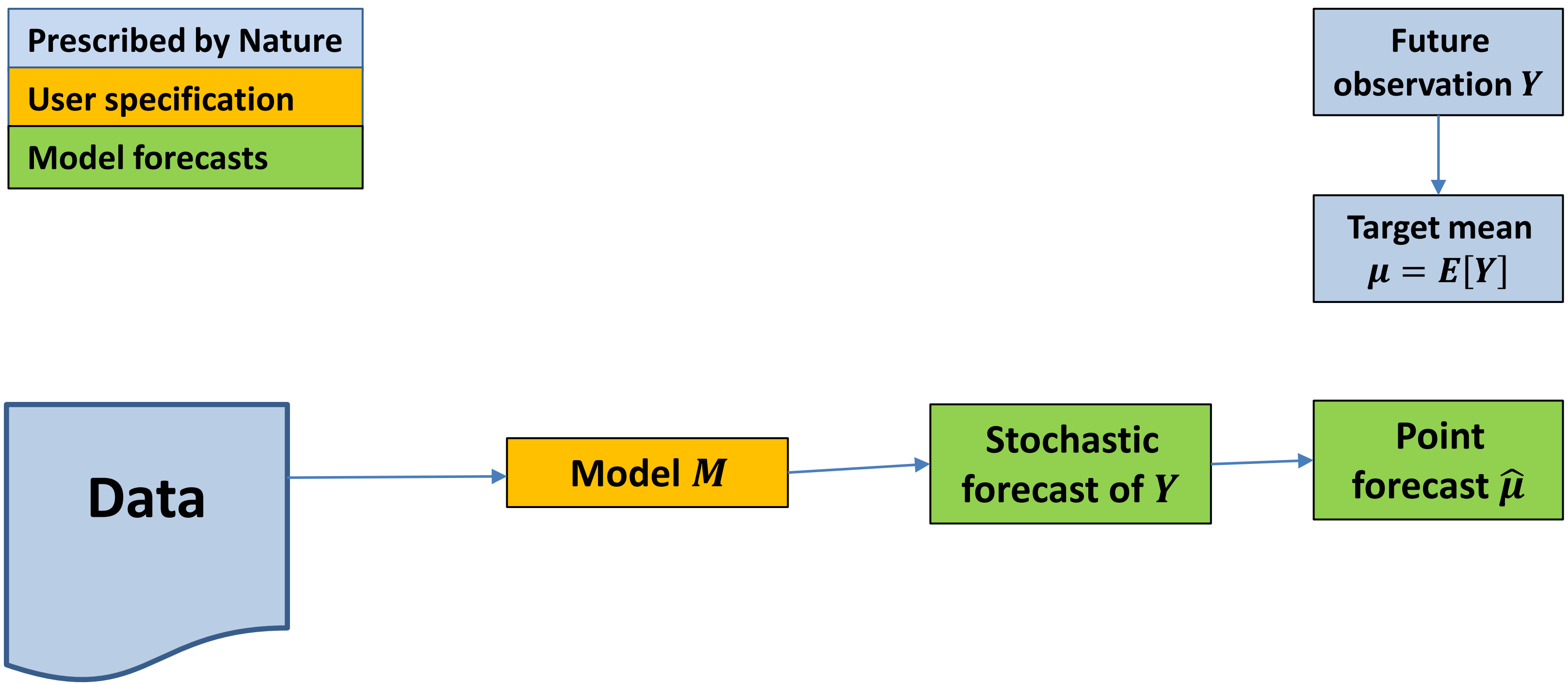

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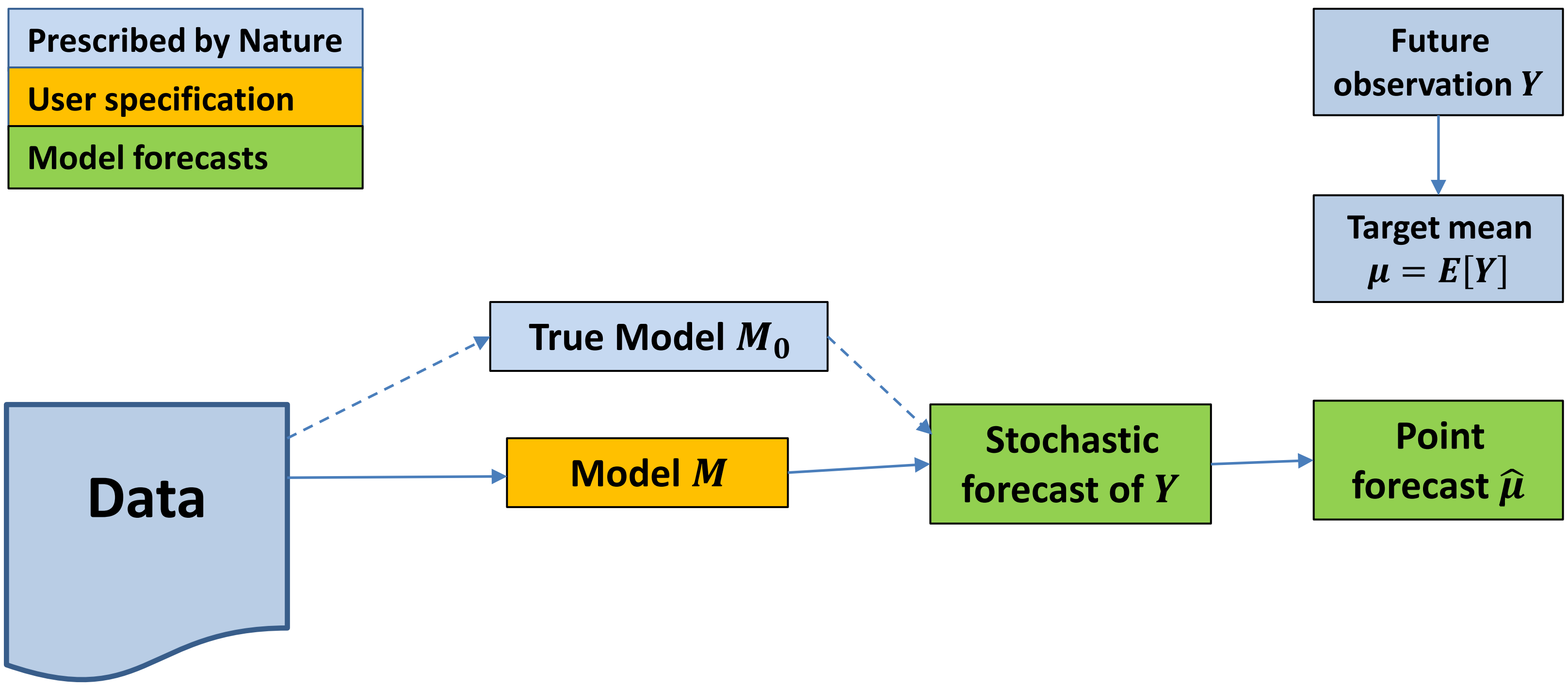

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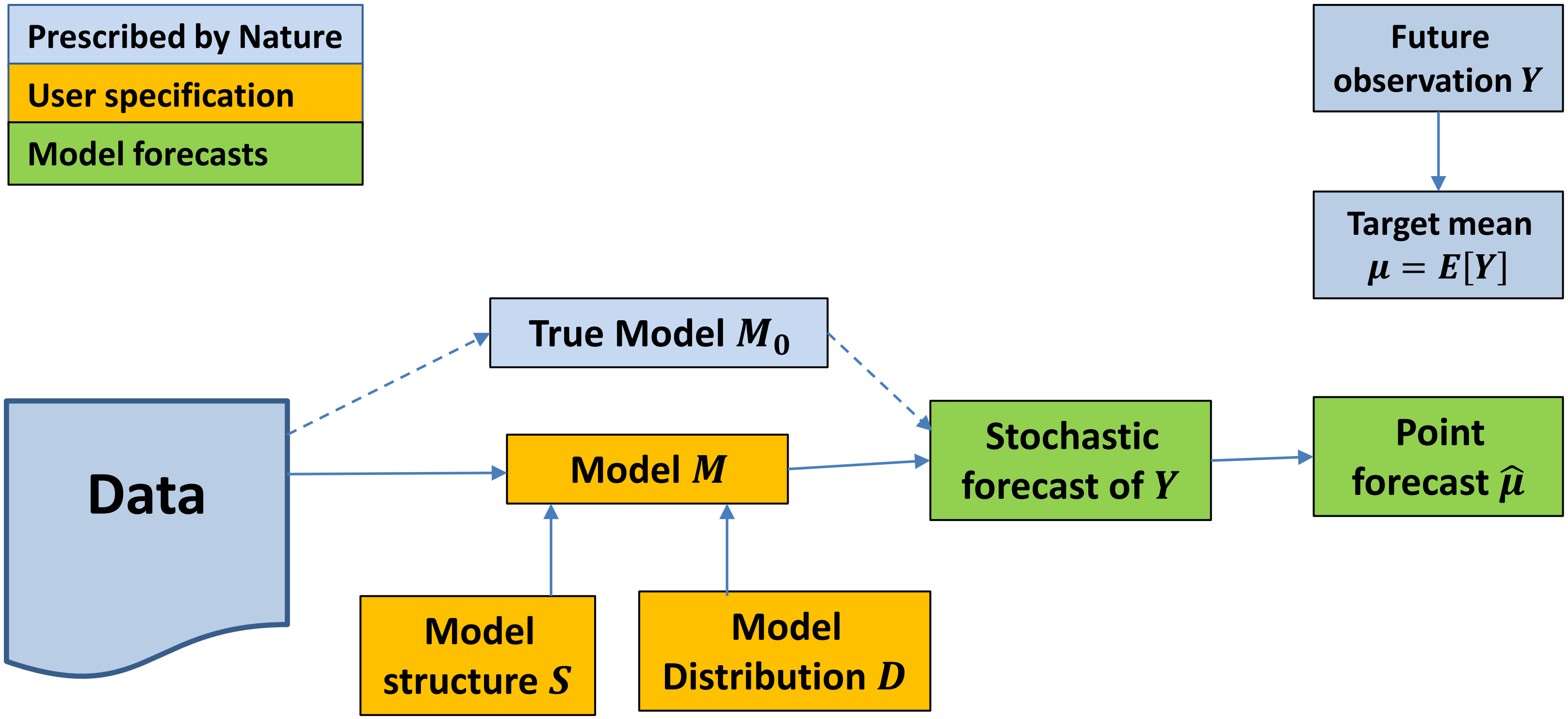

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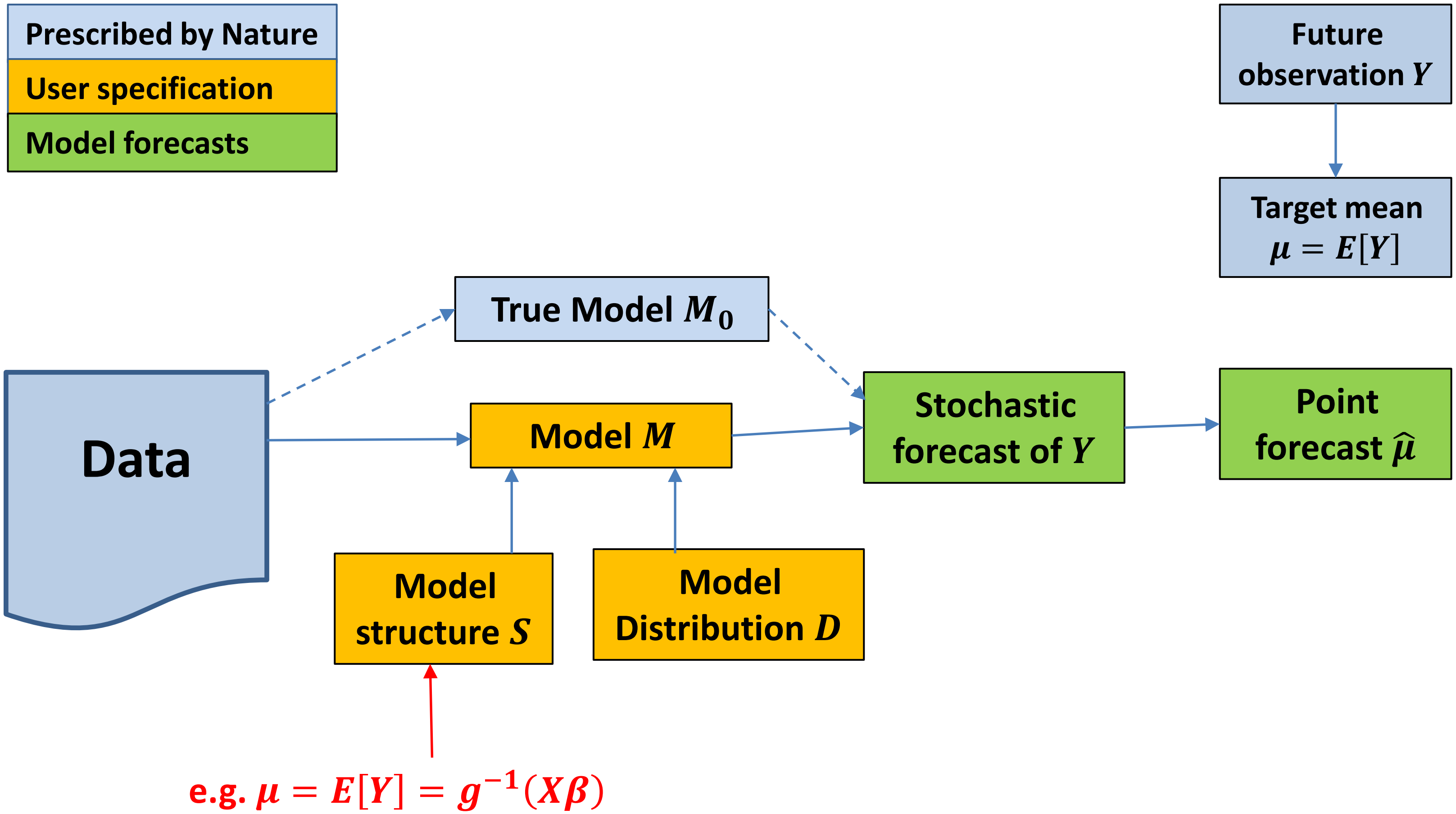

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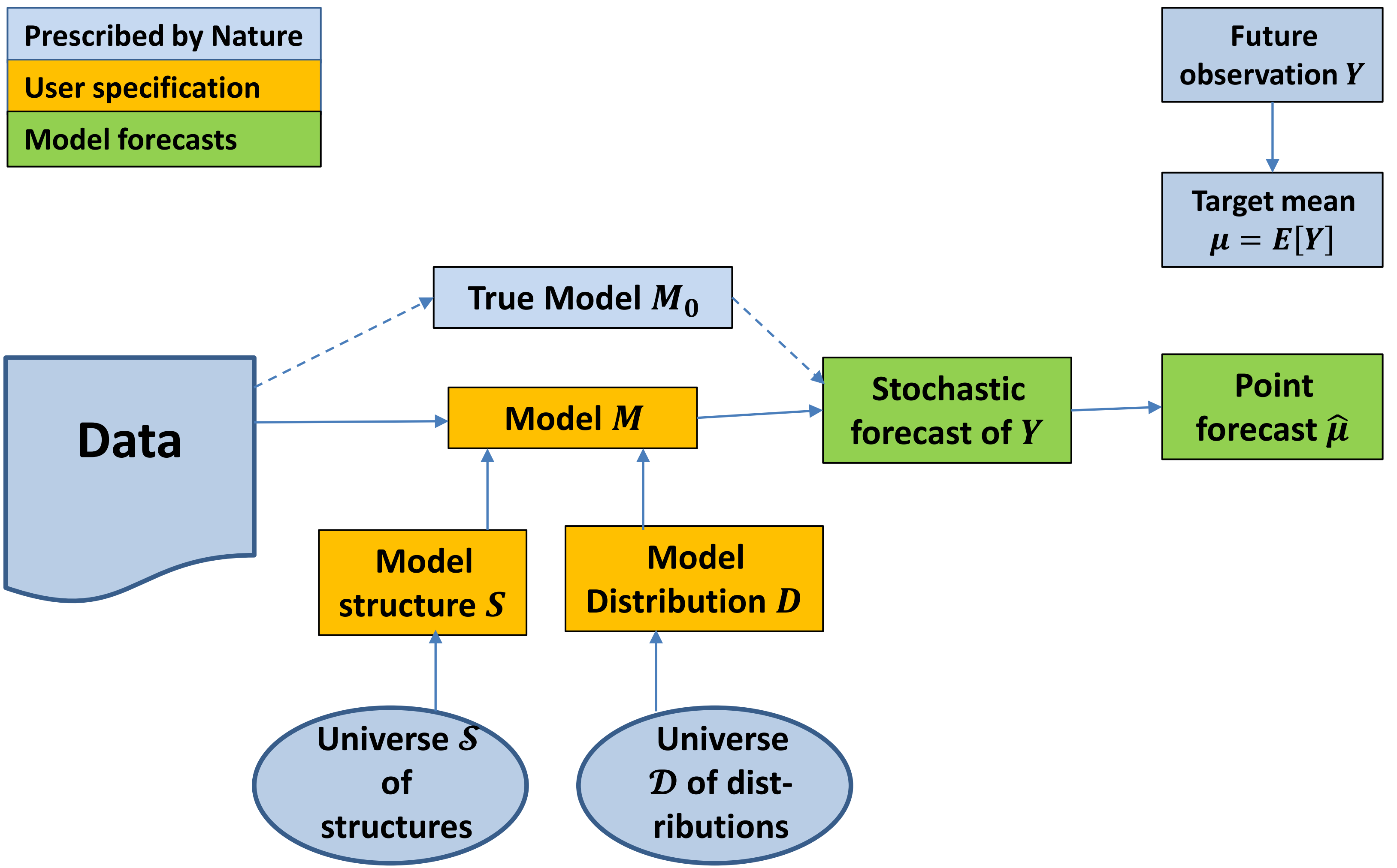
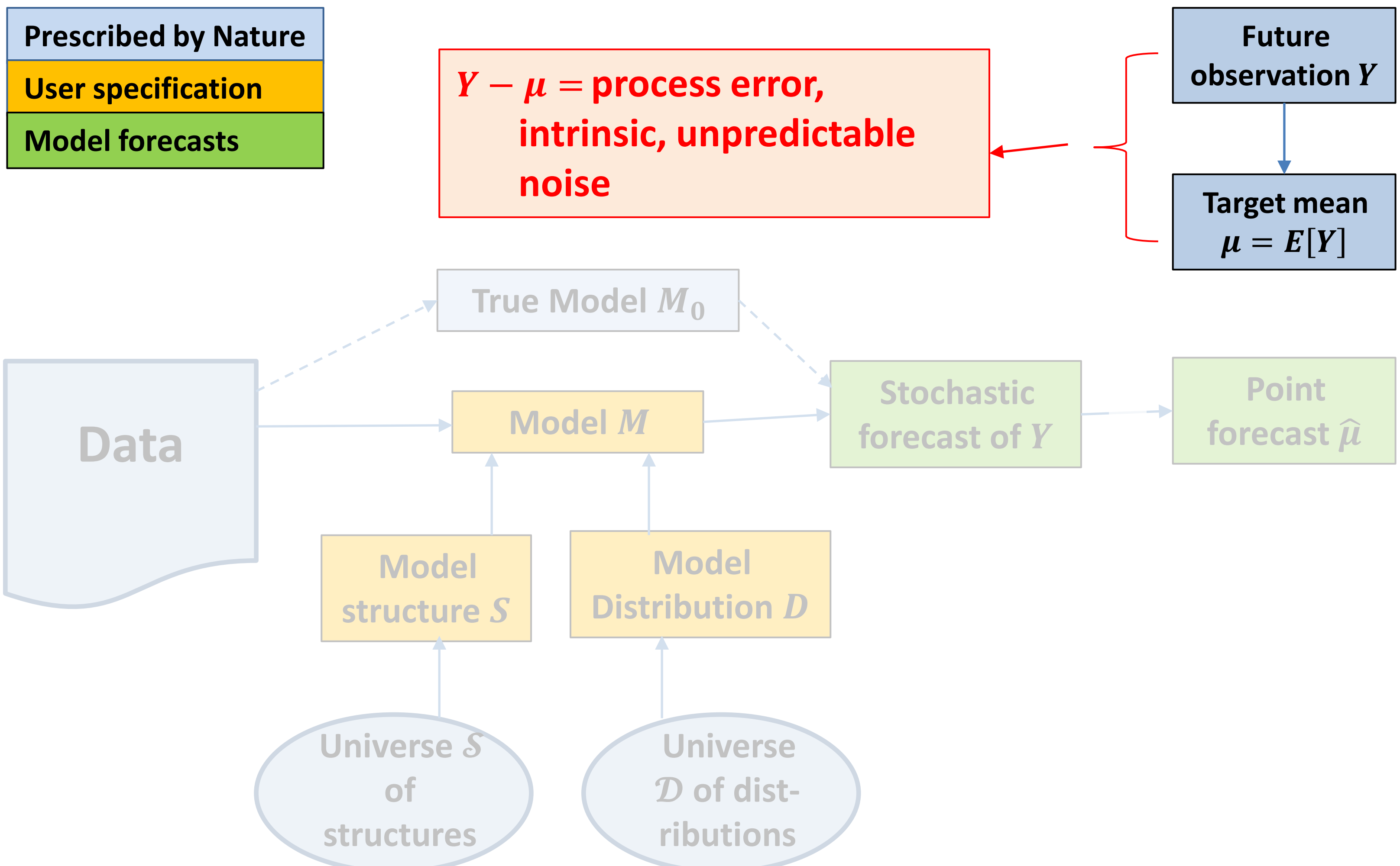
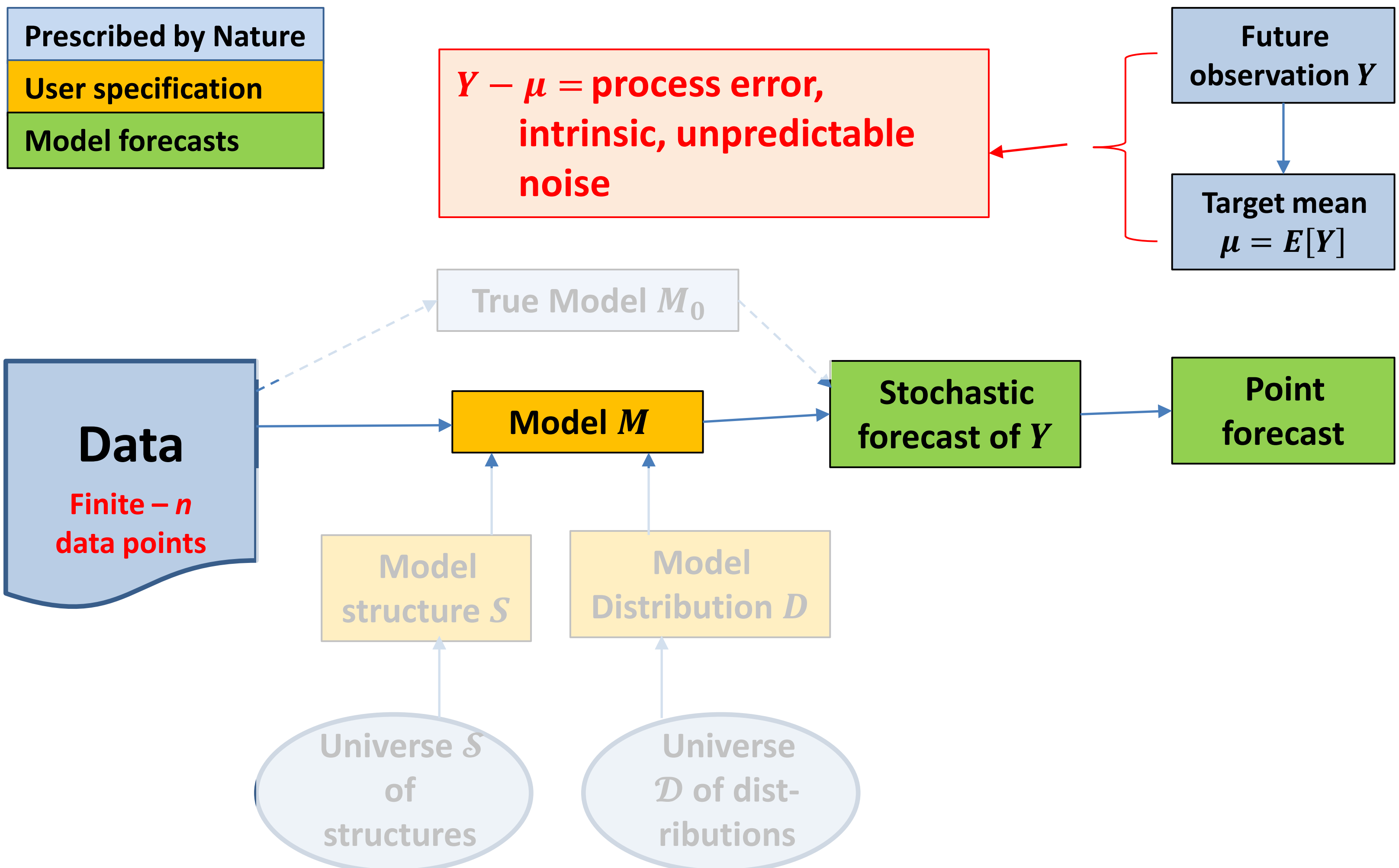


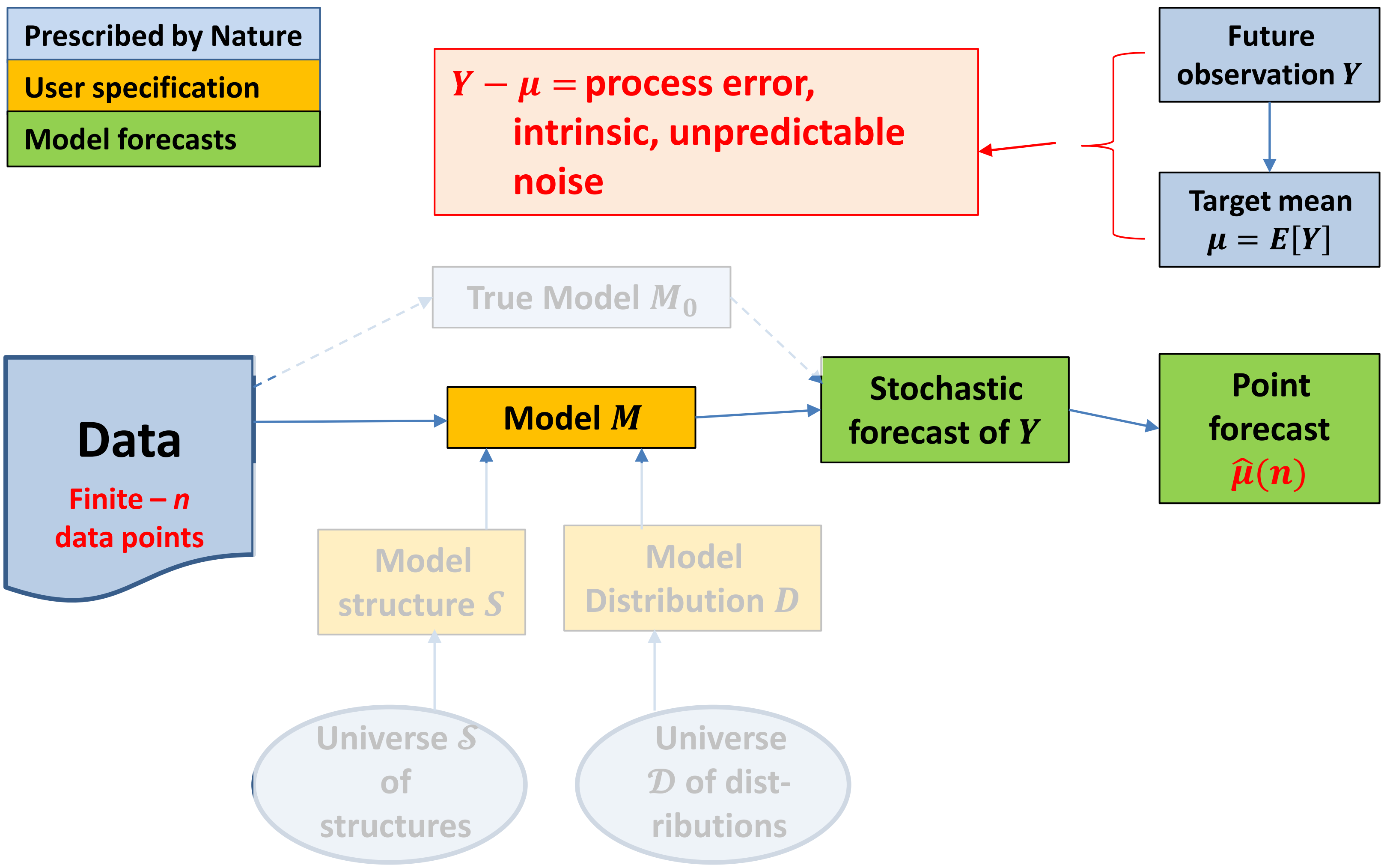
Process error



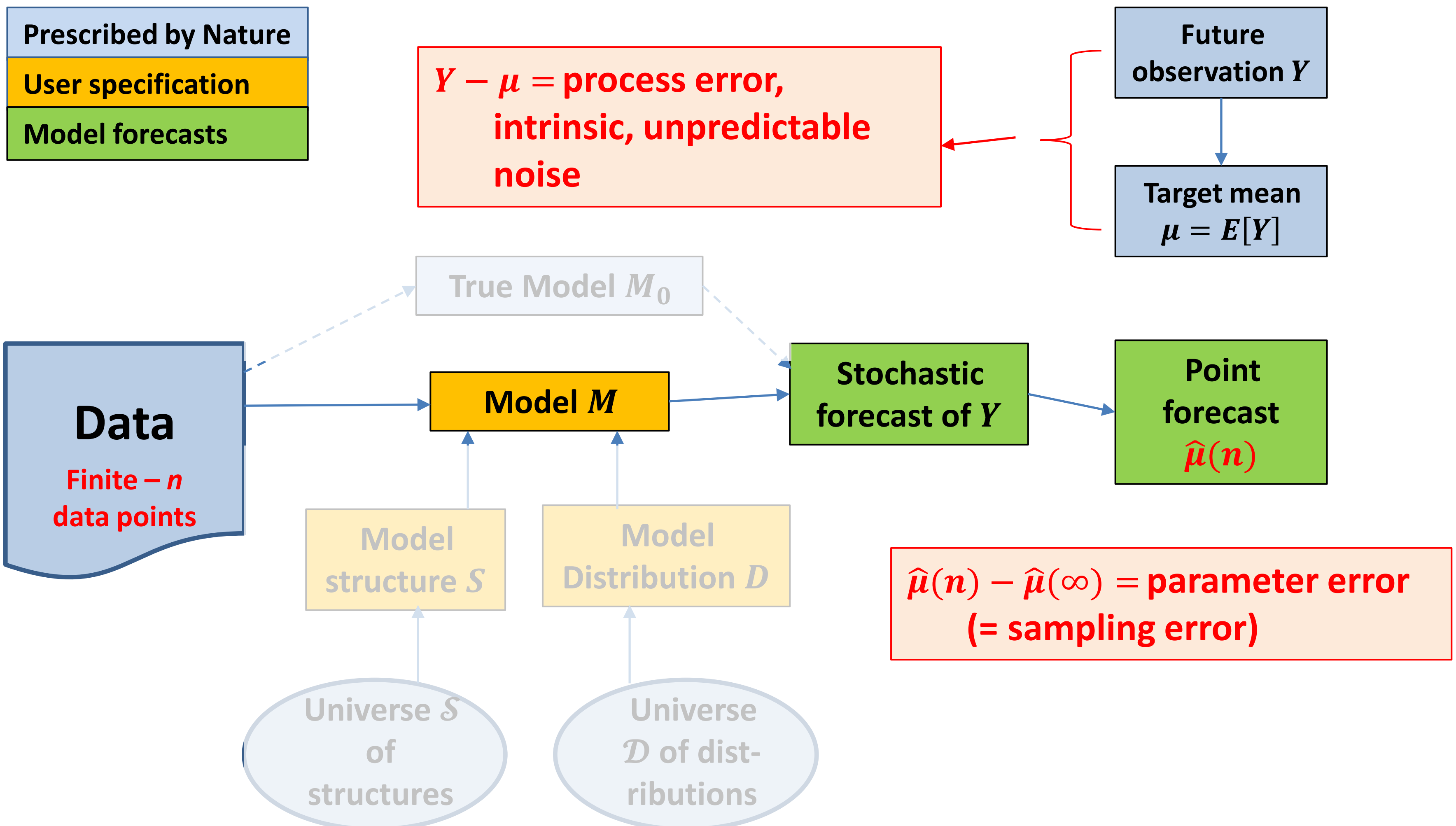
Parameter error



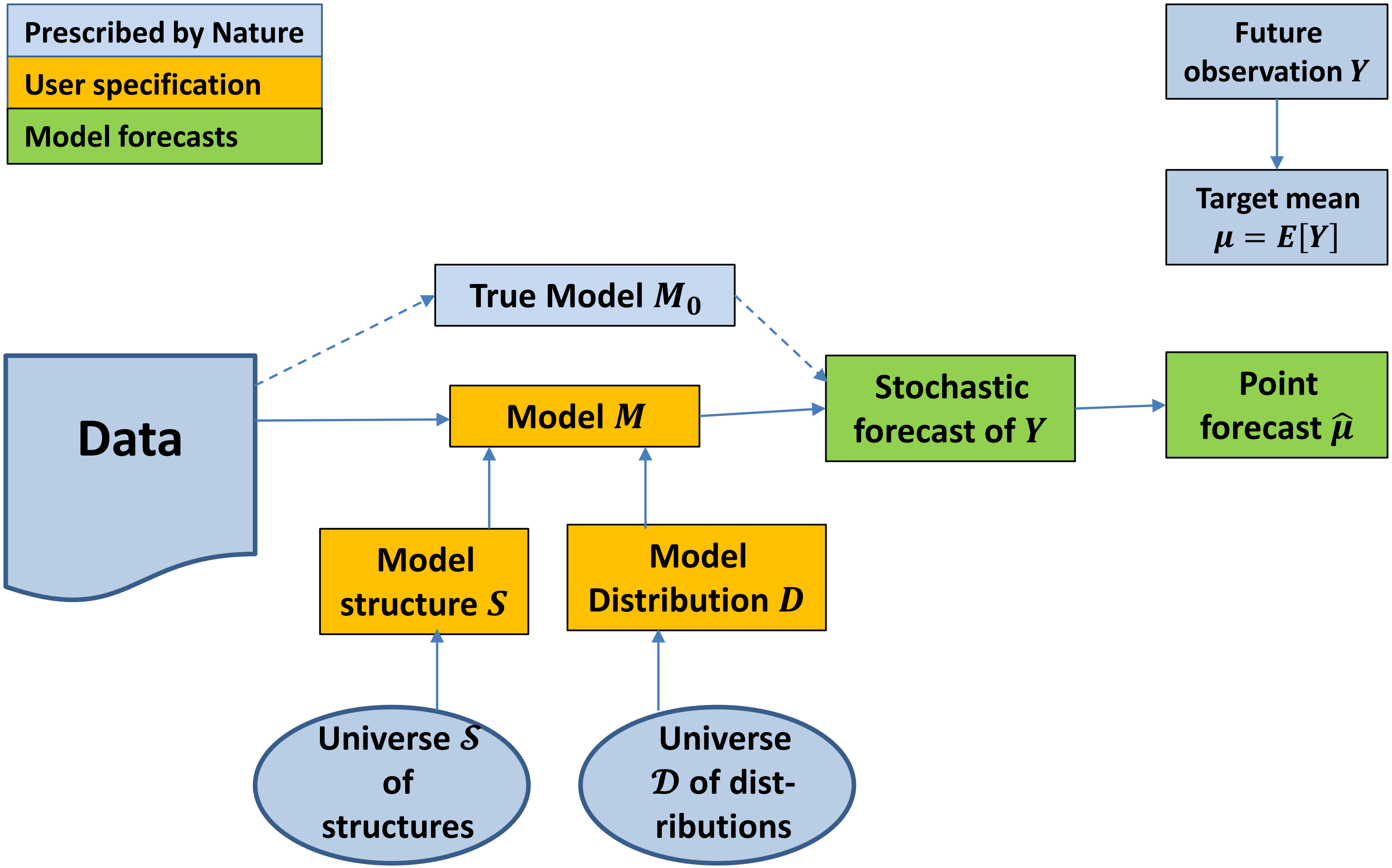
Parameter error



Parameter error



What is it? Model error



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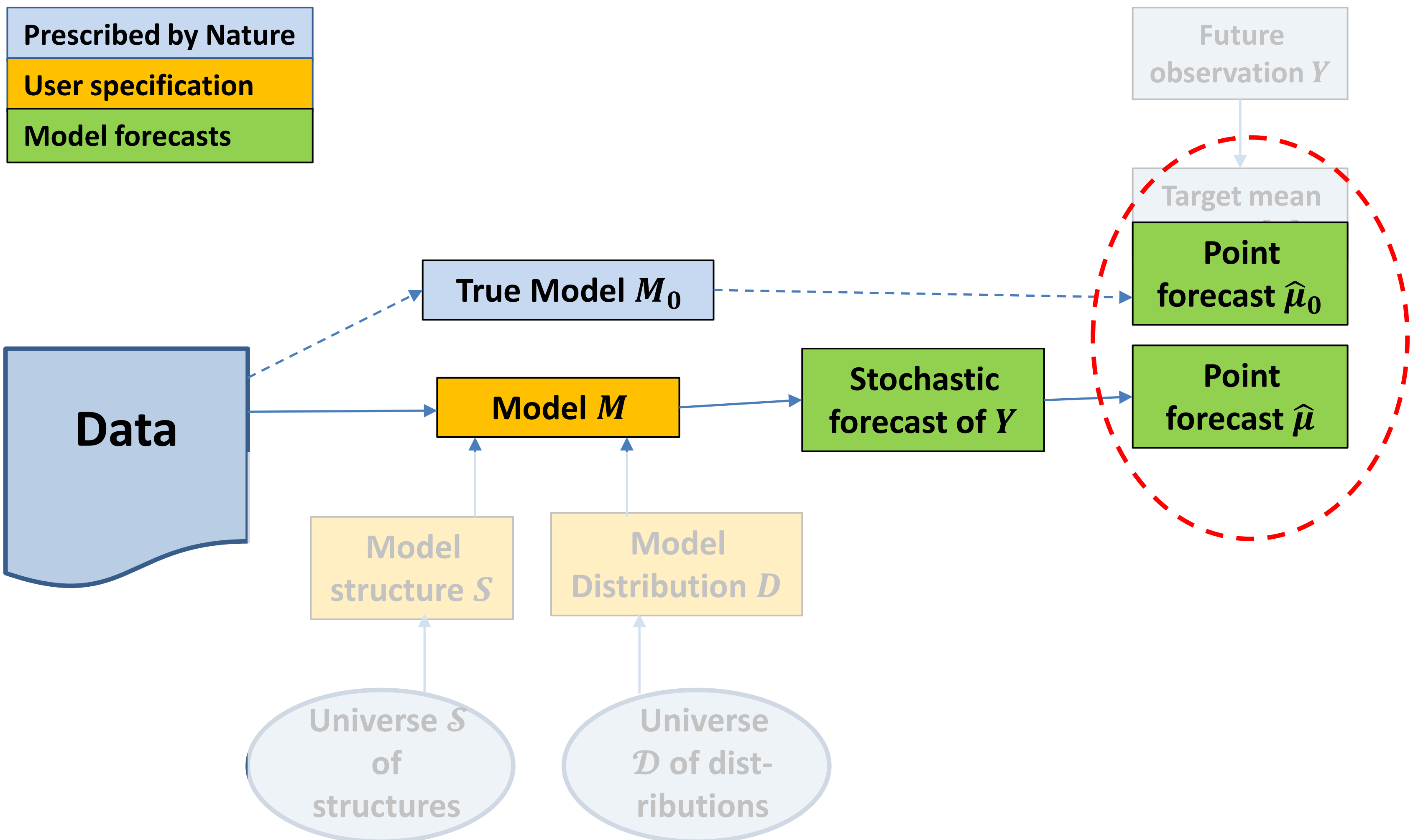

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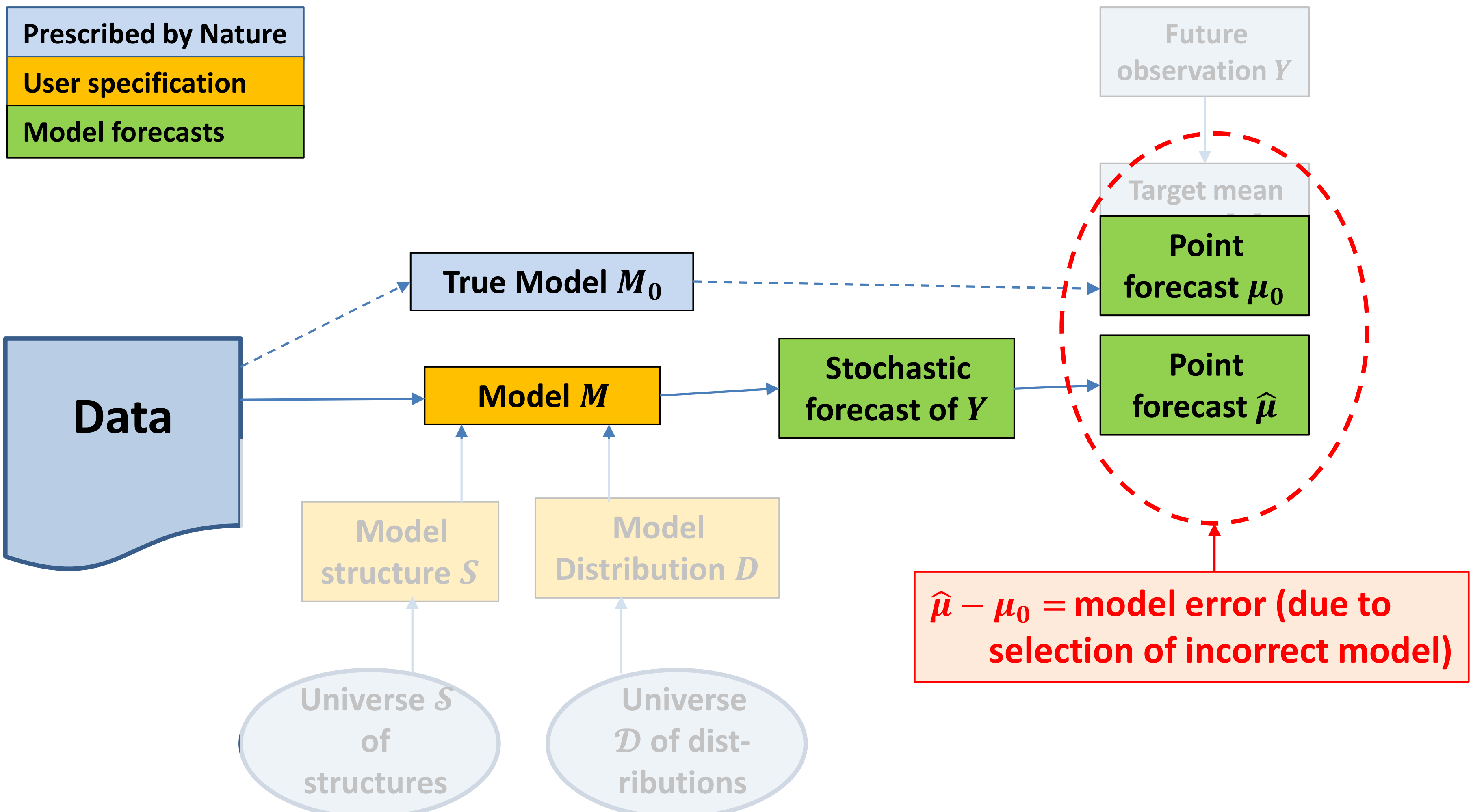

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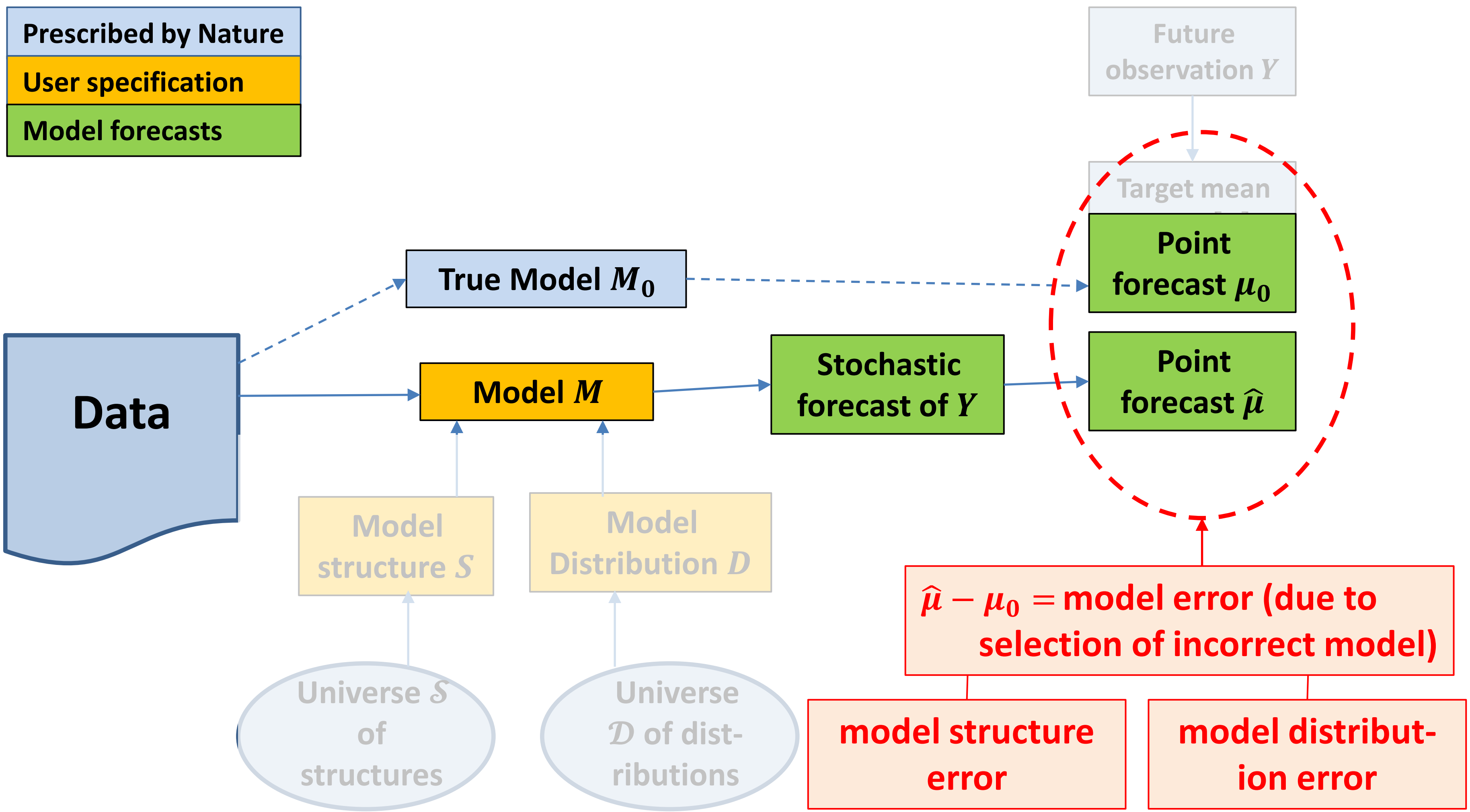

Summary of forecast error

$$\underbrace{Y-\mu_0}_{\text{Forecast error}} = \underbrace{\left(Y-\hat{\mu}(n)\right)}_{\text{Process error}} + \underbrace{\left(\hat{\mu}(n)-\hat{\mu}(\infty)\right)}_{\text{Parameter error}} + \underbrace{\left(\hat{\mu}(\infty)-\mu_0\right)}_{\text{Model error}}$$

Model structure error

Model distribution error

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- **Internal and external model error**
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  - Worst case scenario
  - Bayesian model averaging

Internal and external model error (1)

- Model error can be estimated from data only to the extent that data features are present in (**internal to**) the data
- Forecasting involves extrapolation of model to the future
  - **Extrapolation** carries its own risk (e.g. neural networks)
  - Risk of **structural breaks** in the future
    - Example 1: Government decree suddenly changes the quantum of existing liability
    - Example 2: Shift to a different regime of superimposed inflation
  - These risks are **external** to the data
- Hence **internal model error** and **external model error**

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Summary of forecast error (augmented)

$$\underbrace{Y-\mu_0}_{\text{Forecast error}} = \underbrace{\left(Y-\hat{\mu}(n)\right)}_{\text{Process error}} + \underbrace{\left(\hat{\mu}(n)-\hat{\mu}(\infty)\right)}_{\text{Parameter error}} + \underbrace{\left(\hat{\mu}(\infty)-\mu_0\right)}_{\text{Model error}}$$

- Model error
  - Model structure error
    - Int-ernal
    - Ext-ernal
  - Model distribution error
    - Int-ernal
    - Ext-ernal

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Internal and external model error (2)

- Remainder of presentation will concern estimation of **internal model error** from data
- Necessary to estimate external model error **exogenously**
  - Possibly **subjectively** (e.g. O'Dowd et (2005))

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Model error: why should we care?

- Various statutory solvency requirements are defined in terms of percentiles of estimated liability value
  - So, technically, inclusion of all components of forecast error, including model error, is a matter of black letter law
- Is it material?
- An example
  - A particular insurer made estimates of process, parameter and model errors for the loss reserve of its entire portfolio on the basis of O'Dowd et al (2005) (very round figures below)
    - Parameter + process error x%
    - "Internal" model error 2.5x%
    - Parameter + process error + total model error 4x%

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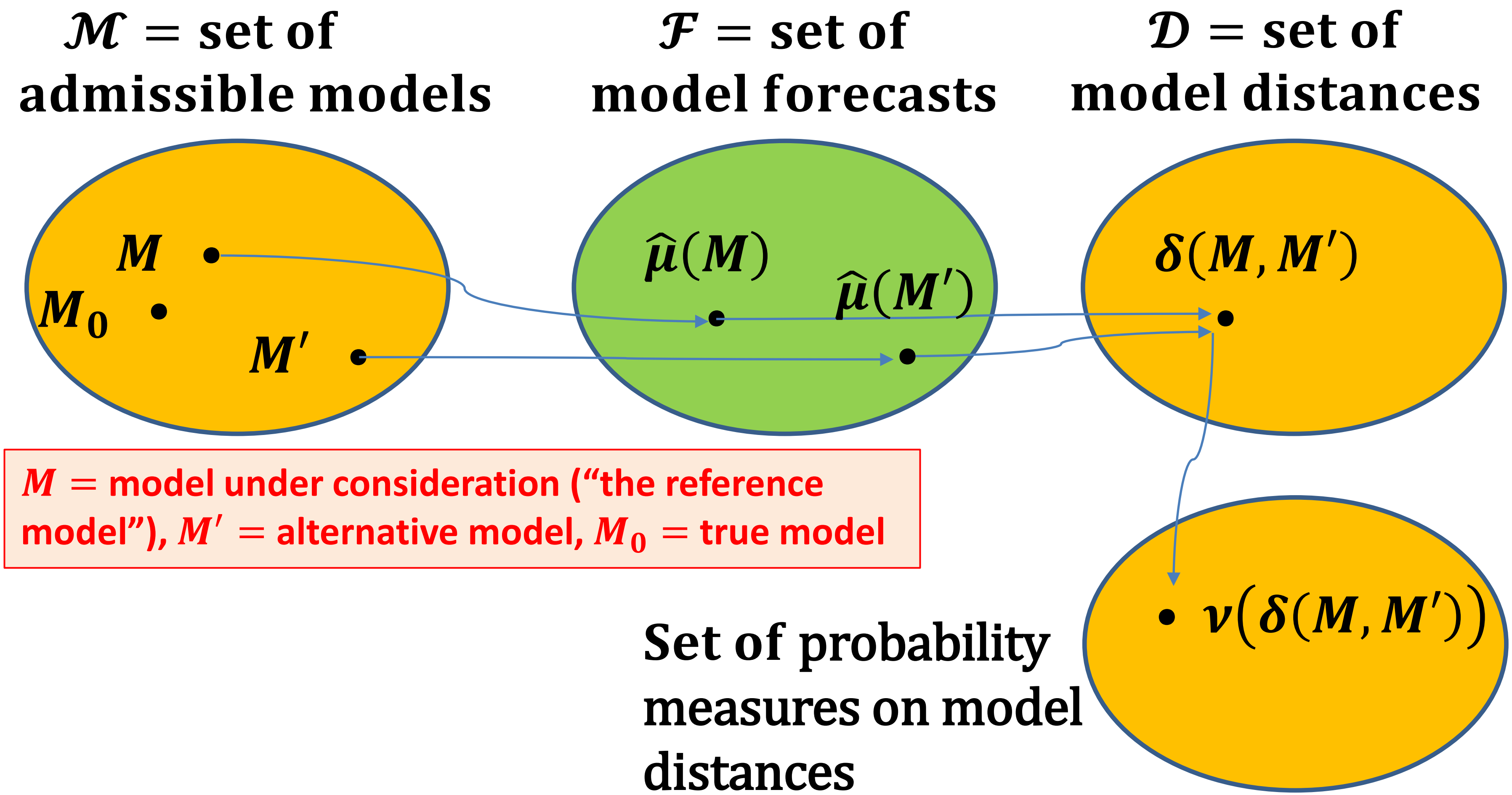
Estimation of internal model error: ingredients (1)



M = model under consideration ("the reference model"), M' = alternative model, M_0 = true model

Estimation of internal model error: ingredients (2)

- Required ingredients (Schneider and Schweizer, 2015)
  - Set of admissible models
  - Set of model forecasts
  - Set of model distances
  - Probability measure on model distances

Estimation of internal model error: ingredients (3)

- Required ingredients
  - **Set of admissible models**
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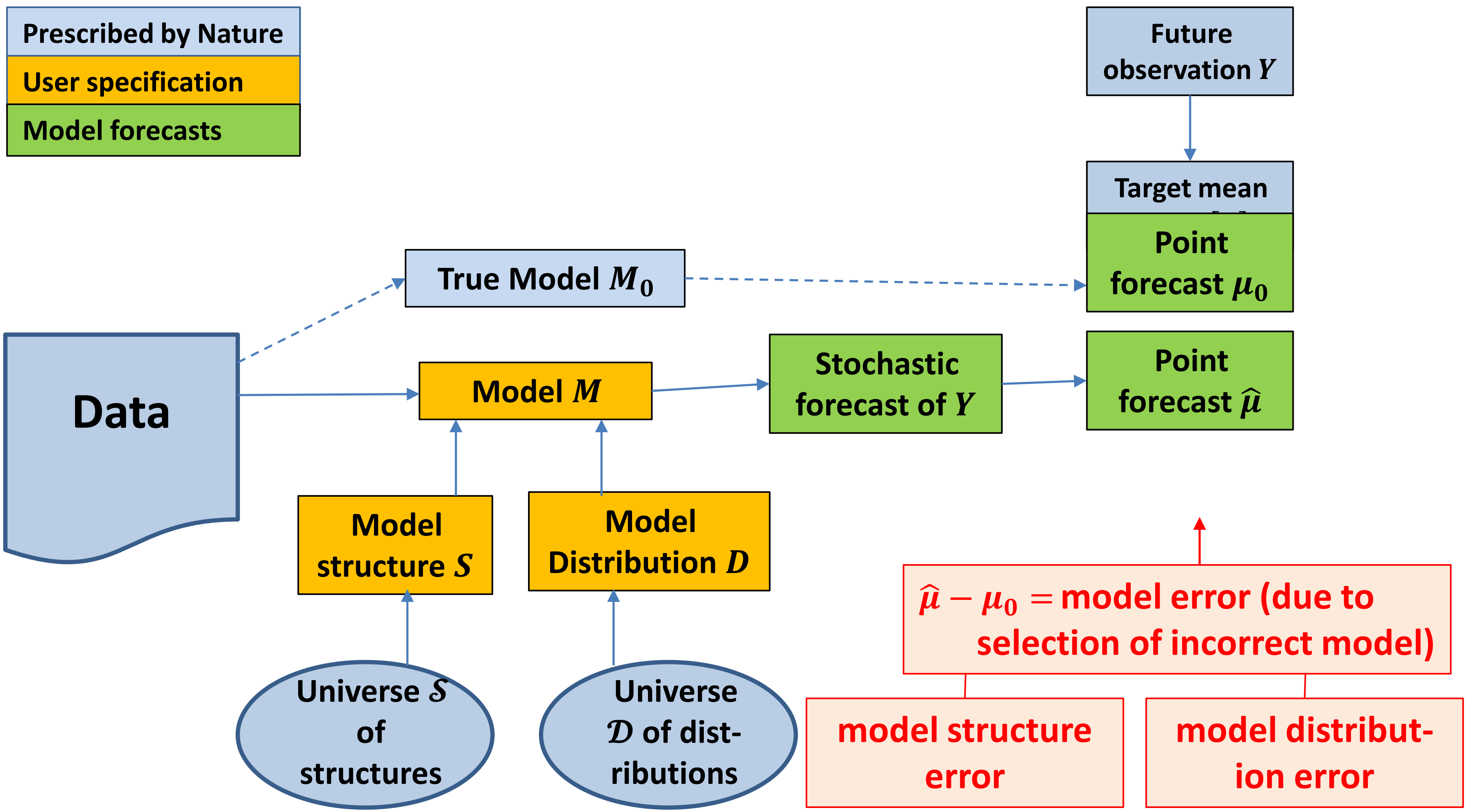
Model distribution error



Model distribution error: exploratory framework (1)

- Tweedie family: sub-family of Exponential Dispersion Family with $\boldsymbol{Var[Y] = \phi\{E[Y]\}^p}$
- Special case $p = 1$ (over-dispersed Poisson): $\boldsymbol{Var[Y] = \phi\{E[Y]\}}$
- Generalization (Constant Mean-Variance Ratio K): Tweedie with $\boldsymbol{Var[Y] = \left(\frac{1}{K\{E[Y]\}^{p-1}}\right)\{E[Y]\}^p}$
  - Resembles chain ladder
  - Consistent with some data

Dispersion parameter depends on mean

Still Tweedie with index p

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Model distribution error: exploratory framework (2)

Model distribution error ingredient	Assumption
Set of admissible models	CMVR $Tweedie_p, p > 1$, cell means unspecified but constant over all distributions
Set of model distances	Unspecified
Probability measure on model distances	Unspecified

Variation over p allows for a range of distribution tail lengths

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Model distribution error: theoretical results

- From Taylor (2020)
- For CMVR Tweedie, process error (measured by **coefficient of variation of prediction ("CoVP")**) is independent of p
- Parameter error is **NOT** independent
  - But is **asymptotically independent** as CoV of observations increases

Model distribution error: empirical results (1)

- Example triangle: workers compensation insurer 620 from Meyers and Shi (2011)

Value of p	Bootstrap CoVP					
	All replications			Top and bottom 1% censored		
	Parameter error	Process error	Total	Parameter error	Process error	Total
	%	%	%	%	%	%
1	7.5	3.4	8.3	5.6	3.2	6.4
1.35	7.5	3.4	8.3	5.5	3.2	6.3
1.45	7.8	3.7	8.6	5.6	3.4	6.5
1.55	12.2	4.9	13.1	5.6	3.3	6.5
1.65	8.2	5.1	9.7	5.6	3.2	6.5
1.75	8.2	4.1	9.2	5.7	3.2	6.5
1.85	7.5	4.9	8.9	5.5	3.1	6.3
1.95	9.1	4.5	10.1	5.7	3.2	6.5
2.05	9.8	3.0	10.2	5.8	3.1	6.6
2.15	7.4	4.5	8.6	5.8	2.5	6.4
2.25	9.9	3.3	10.4	5.9	2.3	6.3
2.35	7.8	7.4	10.7	6.2	1.7	6.4

Model distribution error: empirical results (2)

Value of p	Bootstrap percentile				
	50%	75%	90%	95%	99%
	$000	$000	$000	$000	$000
1	71880	75204	78439	80685	90479
1.35	71749	74937	78232	80610	89015
1.45	71812	75161	78519	81030	90562
1.55	71726	75090	78438	80999	90745
1.65	71735	75011	78405	80899	90884
1.75	71765	75012	78527	81033	90591
1.85	71774	75048	78234	80626	87394
1.95	71715	75009	78249	80812	90953
2.05	71619	74912	78388	80992	92555
2.15	71735	74981	78334	80696	90033
2.25	71600	74853	78221	80459	88593
2.35	71649	74893	78259	80730	89635

Model distribution error: tentative conclusions

- Variance of prediction error, for fixed model structure, little affected by choice of distribution of observations
  - Partly supported by theoretical results
  - Also supported by empirical results from a **single** data set
- Single empirical example indicates that distribution of prediction error, for fixed model structure, also little affected by choice of distribution of observations
- More work needed

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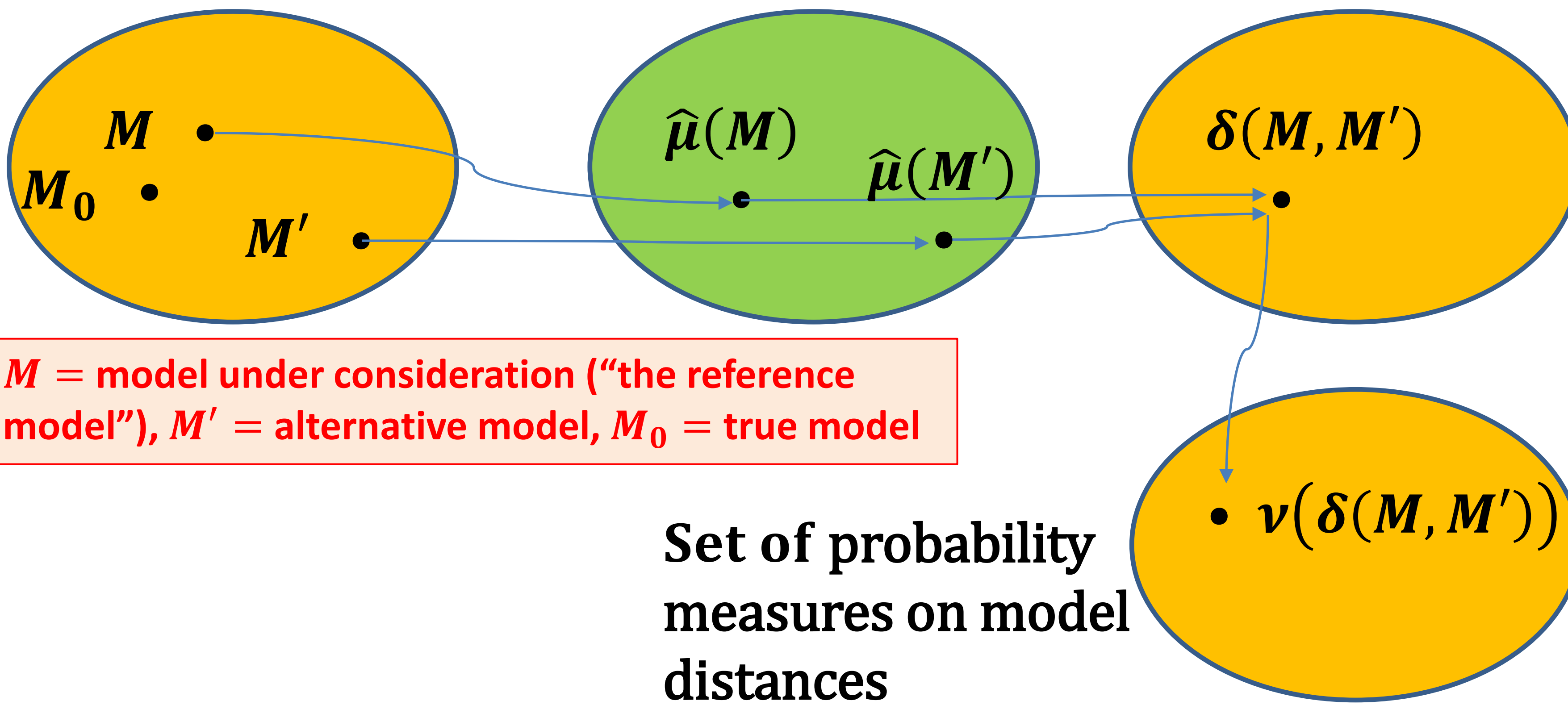
Estimation of internal model error: ingredients

$\mathcal{M}$ = set of admissible models

$\mathcal{F}$ = set of model forecasts

$\mathcal{D}$ = set of model distances



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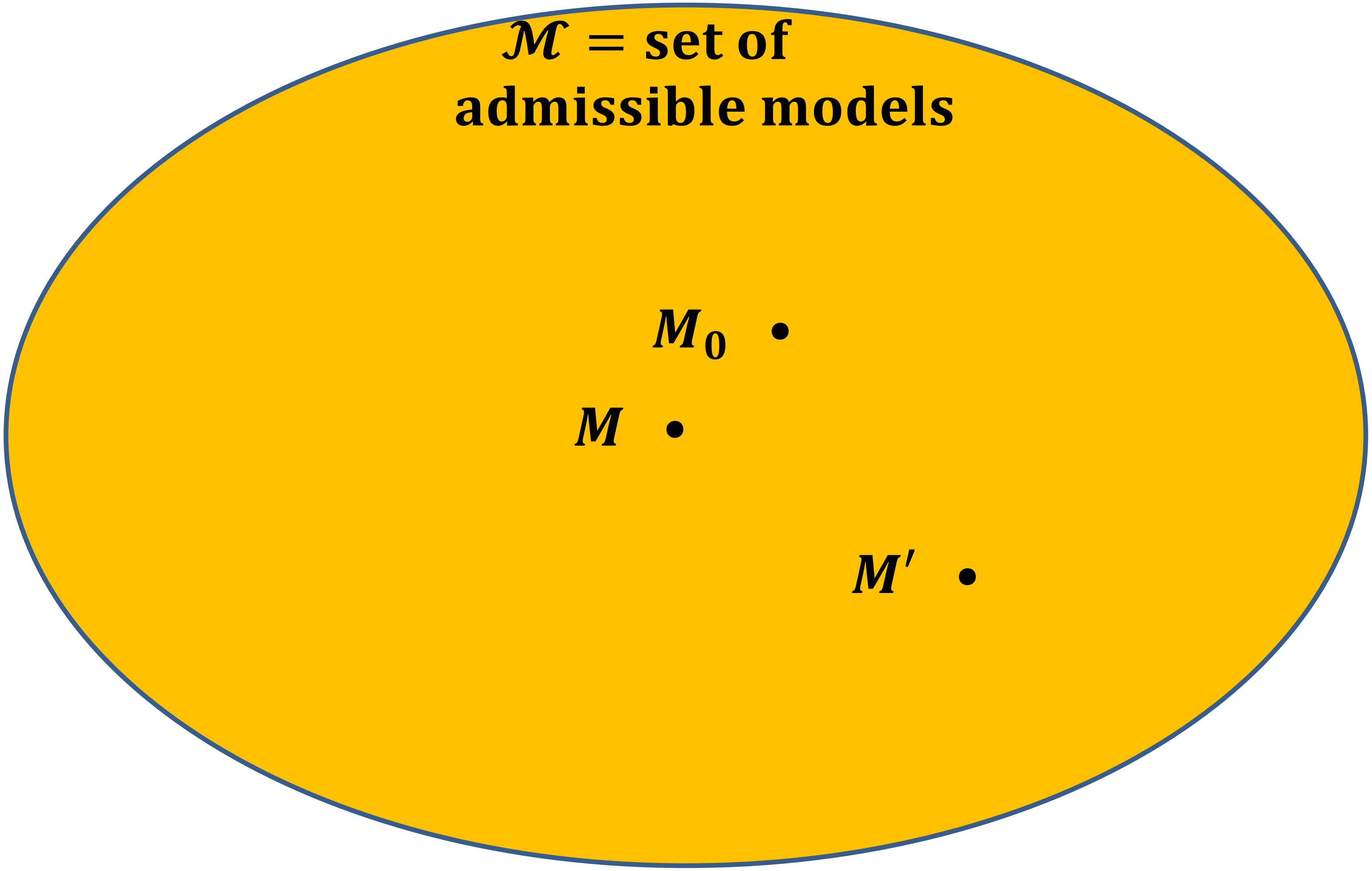
Model structure error: set-up



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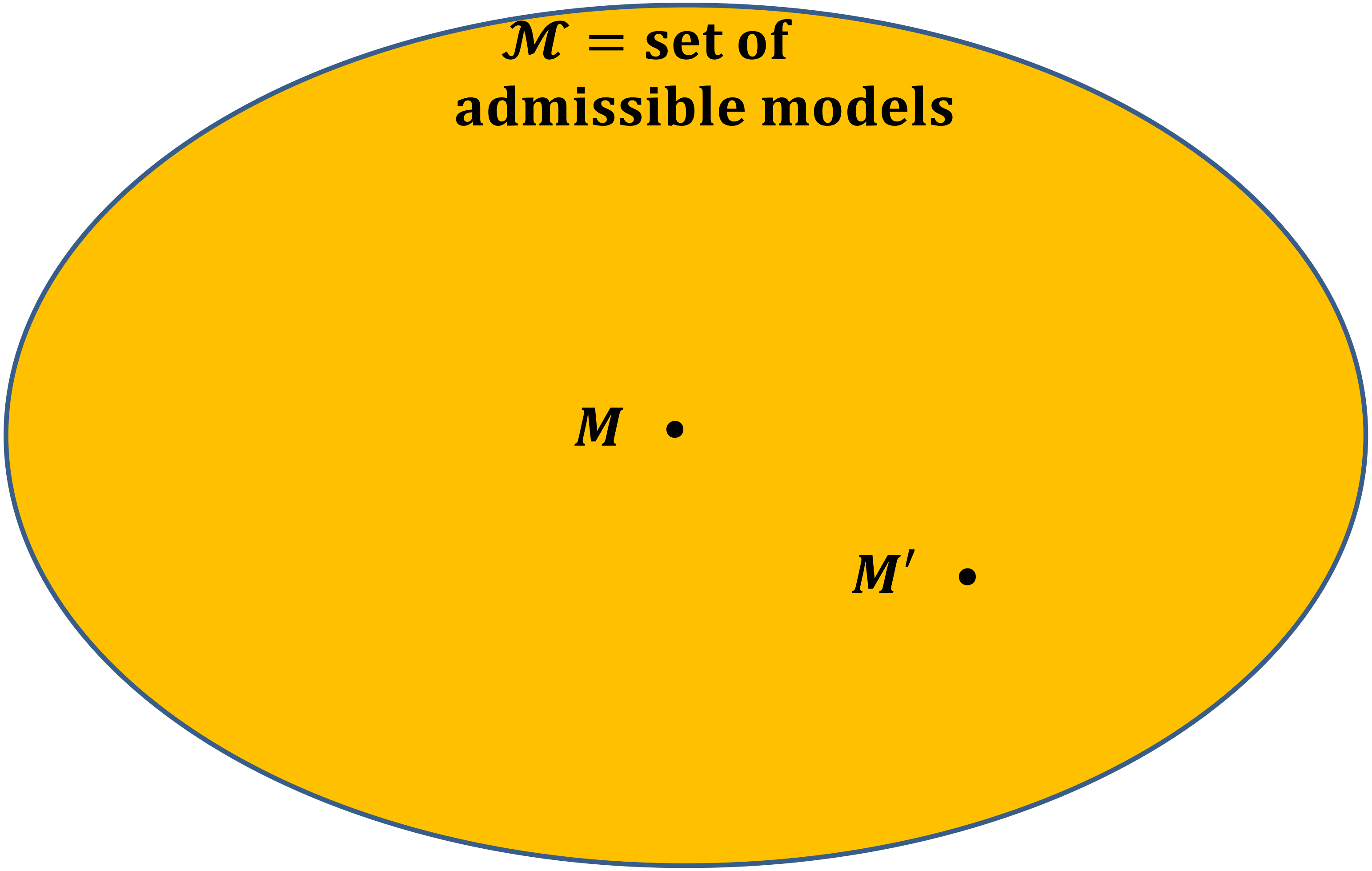

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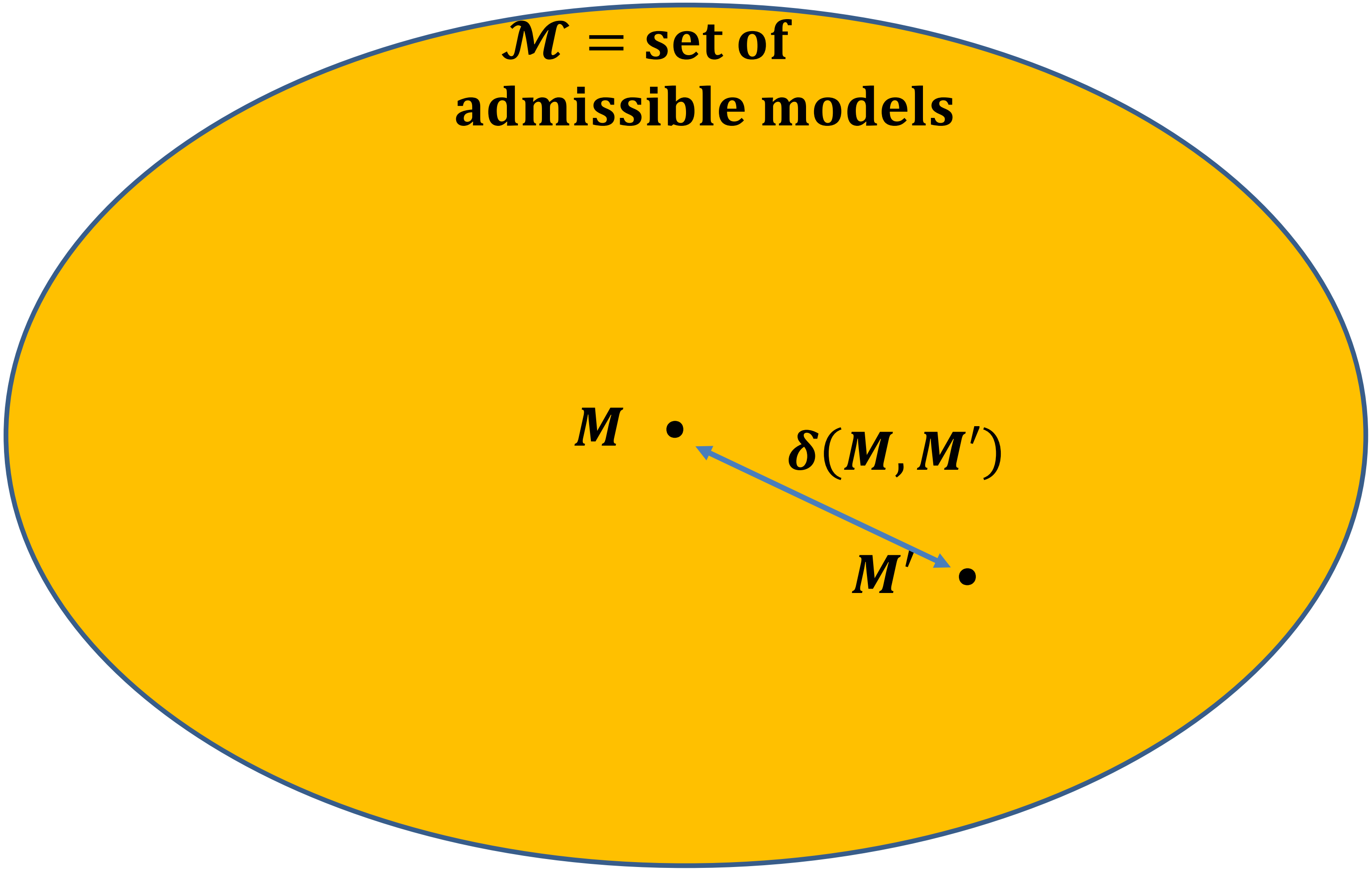

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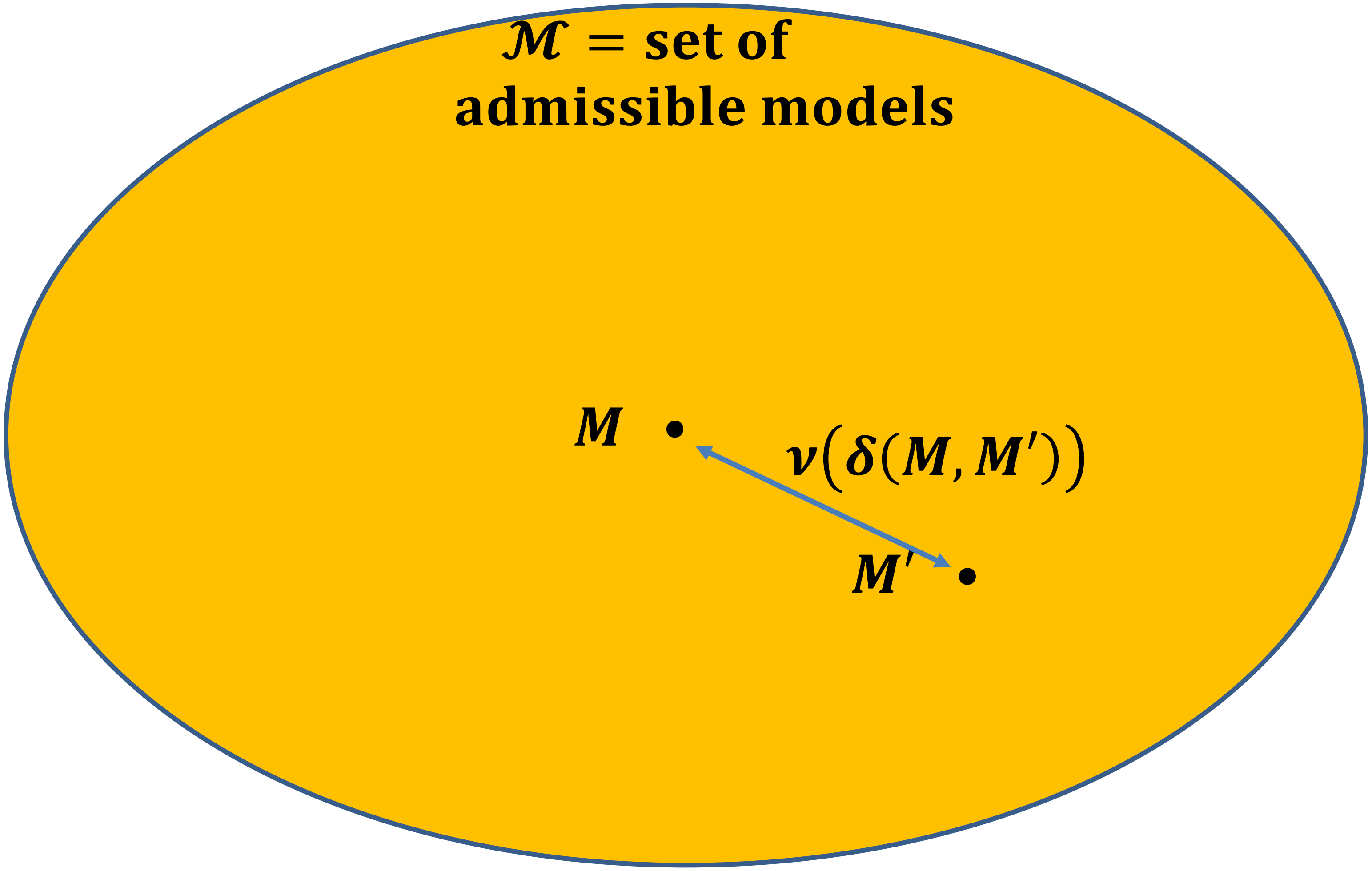
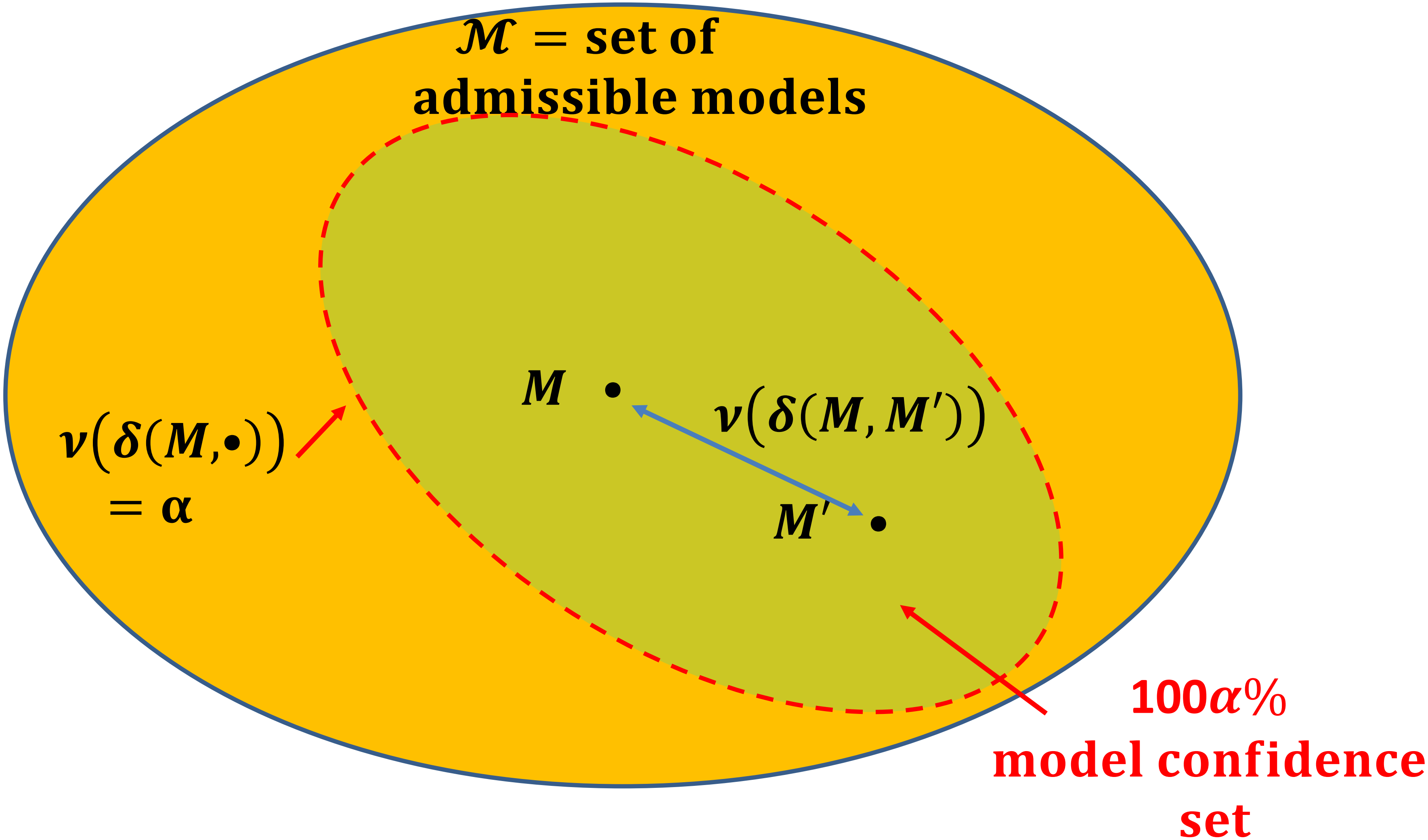


Model confidence set (Hansen et al (2011))



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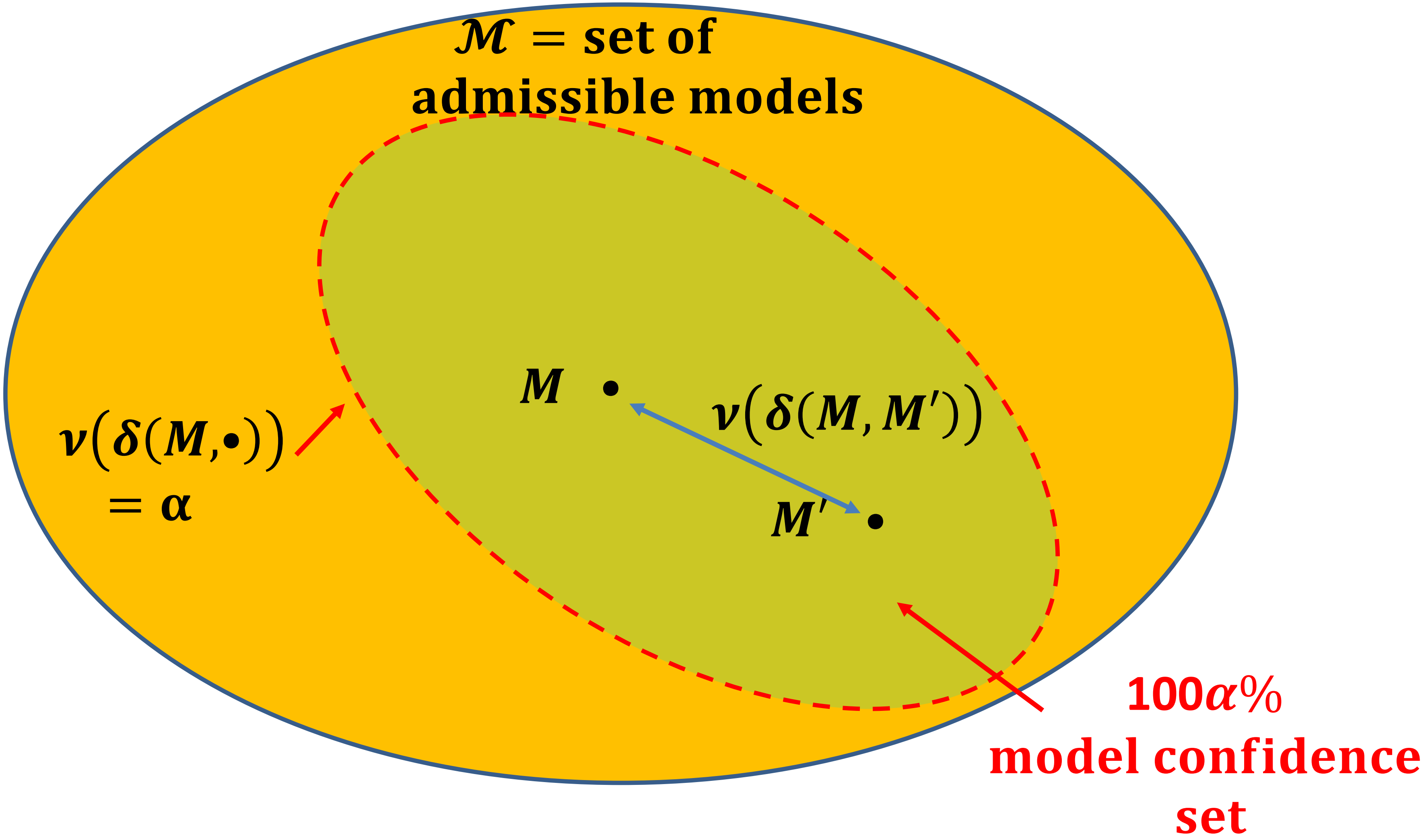
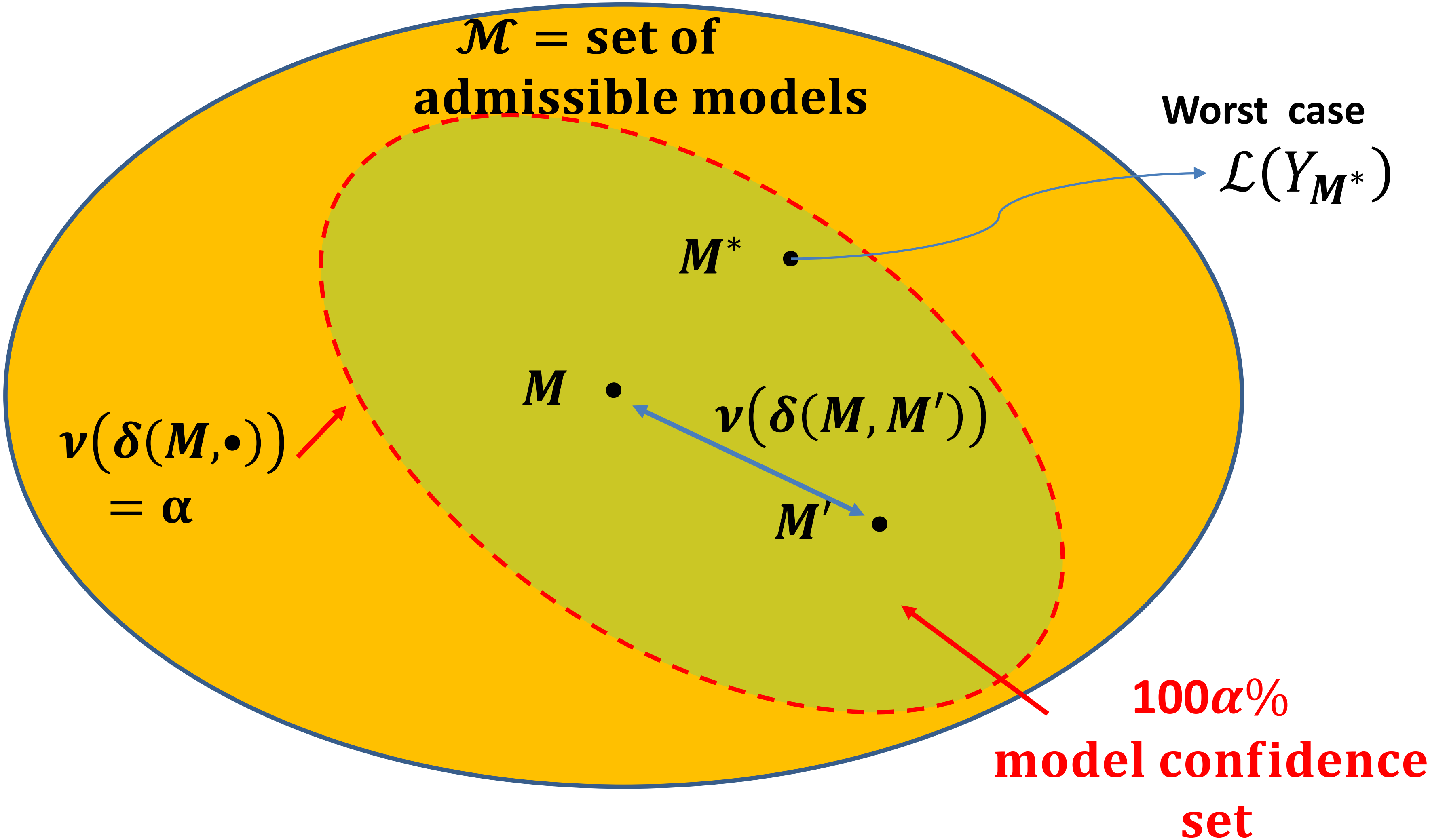


Worst-case scenario

- Now suppose that a loss function $\mathcal{L}$ can be associated with each stochastic forecast $Y_{M'}$ from model M'
  - e.g. $Y_{M'} \mapsto \mathcal{L}(Y_{M'})$ = **forecast loss reserve**
- Call the confidence set $\mathcal{C}(M, \alpha)$
- One might select the **maximum loss reserve (worst case)** Y_{M^*} within confidence set $\mathcal{C}(M, \alpha)$ (e.g. Bignozzi and Tsanakas, 2016), i.e.

$$M^* = \underset{M' \in \mathcal{C}(M,\alpha)}{\arg\max}\, \mathcal{L}(Y_{M'})$$

Model confidence set (Hansen et al (2011))



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Bayesian model averaging: ingredients

- Set of admissible models
- Set of model distances
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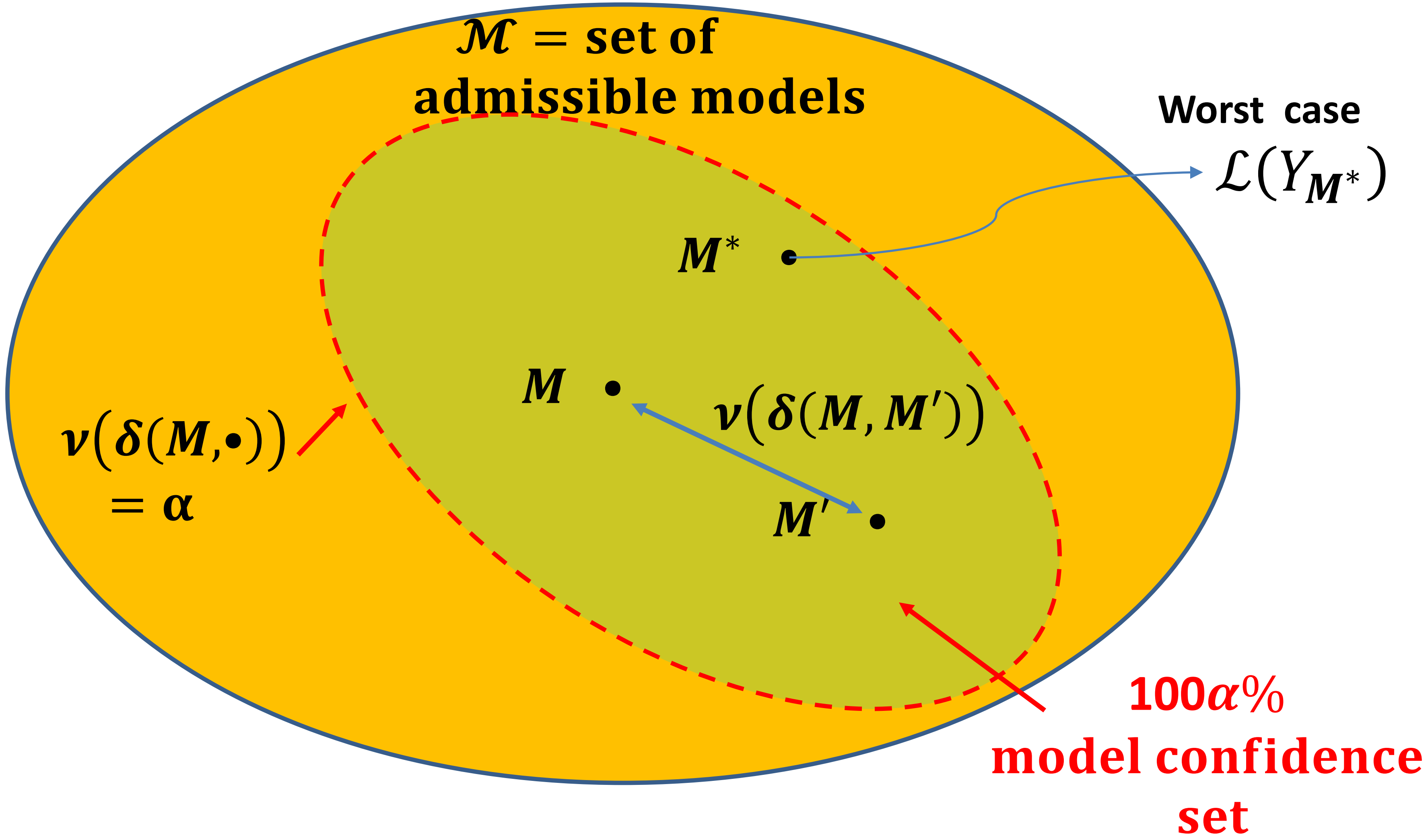
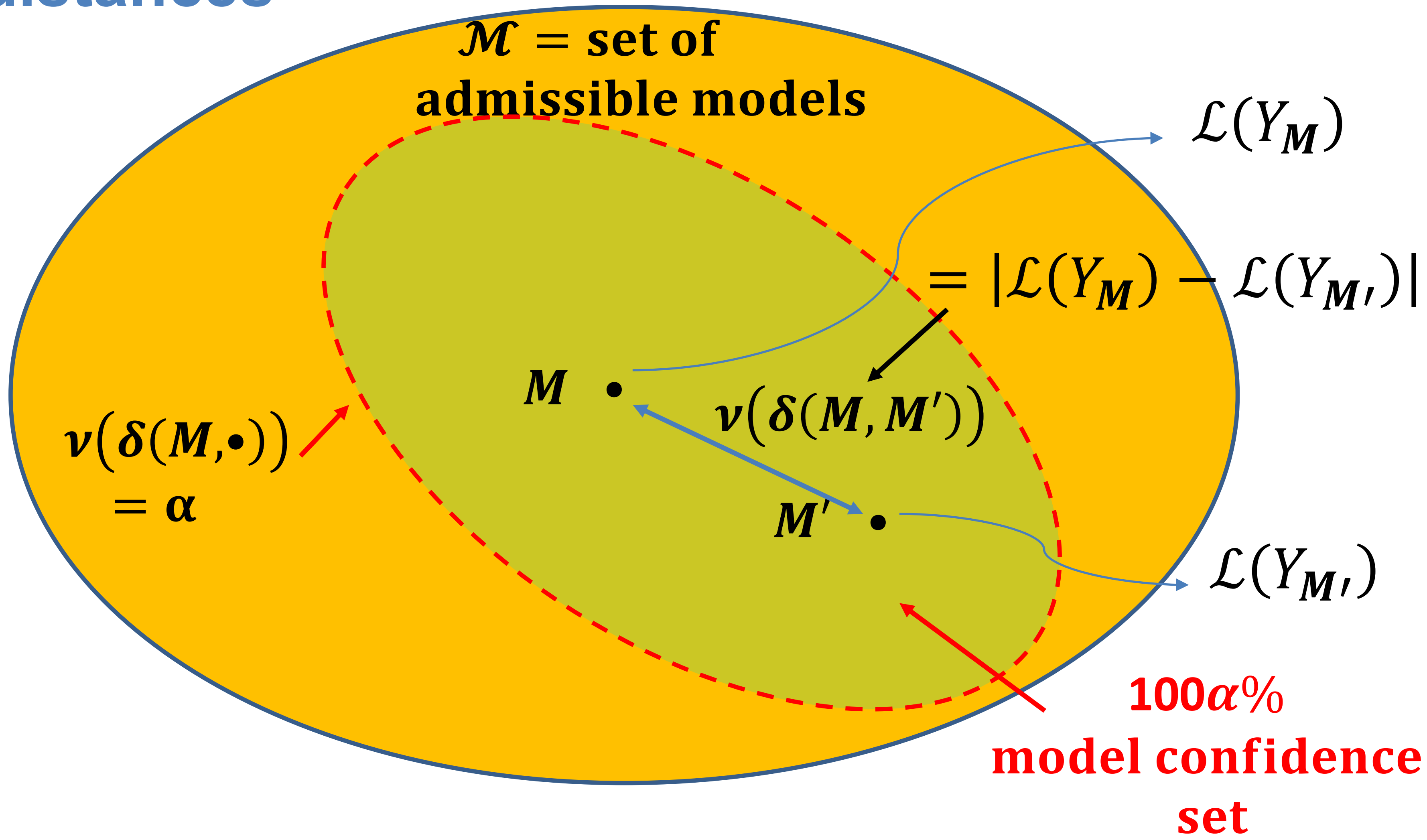
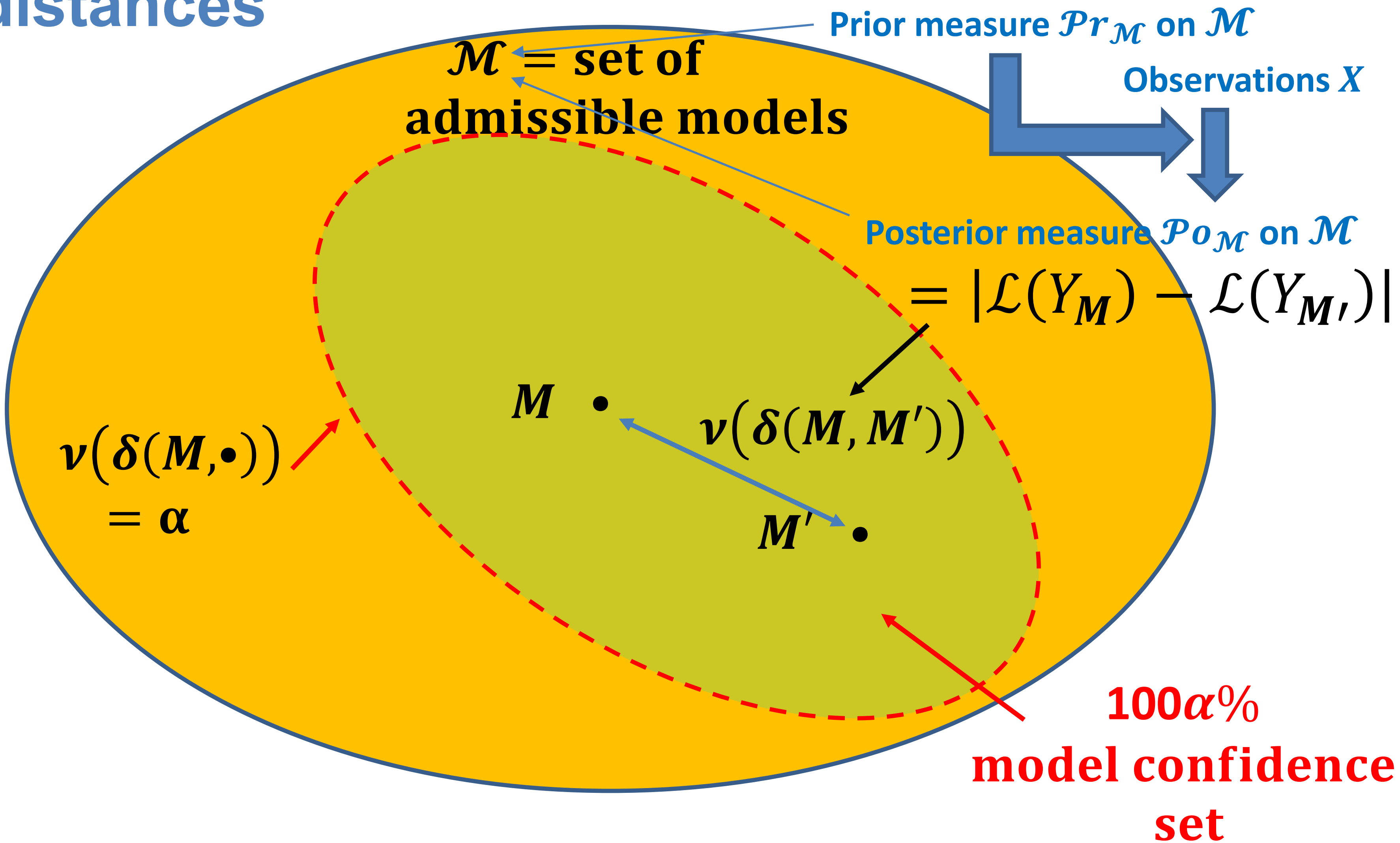
Model confidence set with loss function



Bayesian model confidence set: model distances

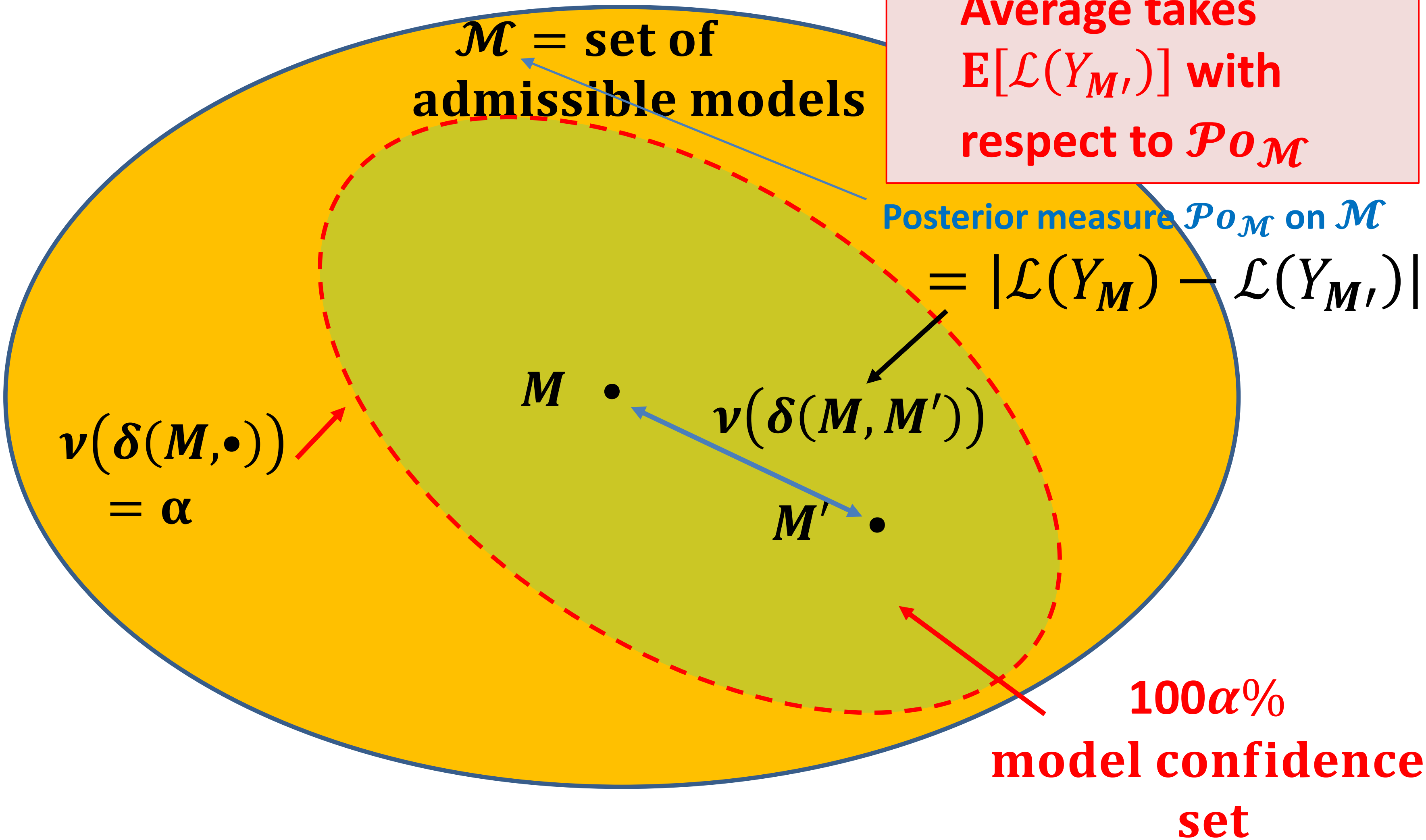


Bayesian model confidence set: measure on distances



Bayesian model averaging

Bayesian Model Average takes $\mathbf{E}[\mathcal{L}(Y_{M'})]$ with respect to $\mathcal{P}o_{\mathcal{M}}$



Conclusions

- Model error is a highly relevant component of loss reserve forecast error
- Model error itself consists of several components
  - Internal model error
  - External model error
  - Each case decomposing into:
    - Structural error
    - Distributional error
    - Some evidence that distributional error is the minor component
  - We reviewed the available methodology for quantifying structural error
  - Next step to implement that methodology

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Thank you

Questions?