General Insurance Loss Reserving in a Network Model

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May 2021

Outline

1. Motivation
2. Evolution of loss reserving models
3. An overview of network theory
4. Loss Reserving as a network based model
5. Conclusion and discussion

Background

	Development year j						
Accident year i	0	1	2	3	$\dots$	$s-1$	s
0	$X_{0,0}$	$X_{0,1}$	$X_{0,2}$	$X_{0,3}$	$\dots$	$X_{0,s-1}$	$X_{0,s}$
1	$X_{1,0}$	$X_{1,1}$	$X_{1,2}$	$X_{1,3}$	$\dots$	$X_{1,s-1}$	$(X_{1,s})$
2	$X_{2,0}$	$X_{1,2}$	$X_{2,2}$	$X_{2,3}$	$\dots$	$(X_{2,s-1})$	$(X_{2,s})$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$s-1$	$X_{s-1,0}$	$X_{s-1,1}$	$(X_{s-1,2})$	$(X_{s-1,3})$	$\dots$	$(X_{s-1,s-1})$	$(X_{s-1,s})$
s	$X_{s,0}$	$(X_{s,1})$	$(X_{s,2})$	$(X_{s,3})$	$\dots$	$(X_{s,s-1})$	$(X_{s,s})$

Table 1: Run-off triangles of known incremental claims amounts $X_{i,j}$ with $(X_{i,j})$ to be estimated.

Objective: Find the overall estimated reserves for each individual class of business, these are called the **Incurred But Not Reported (IBNR)** reserves.

Current Methods

- A large taxonomy of loss reserving models exist. There are many ways loss reserving models vary these include:
  - Macro or Micro
  - Deterministic or Stochastic
  - Parametric or Non-Parametric
  - Machine Learning based or Algorithm based

Known limitations

Ratios are common to all the above based methods and therefore share common limitations:

- Limited capture of expert judgement
- Limited capture of underwriting cycle effects
- Limited capture of reserving cycle effects

Our aim: Introduce network modelling to capture the above dependencies and more.

What are networks?

Networks are **non-Euclidean objects**[1] that demonstrate interactions or dependence between any number of entities (nodes)[2].

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix} \Longrightarrow$$

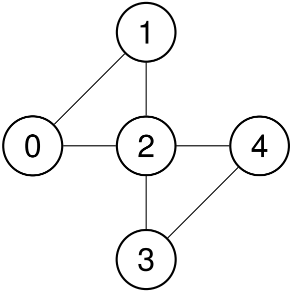

Figure 1: Adjacency Matrix and Resulting Graph

[1] Catherine Matias and Stéphane Robin. "Modeling heterogeneity in random graphs through latent space models: a selective review". In: *ESAIM: Proceedings*. Vol. 47. 2014, pp. 55–74.

[2] Eric D. Kolaczyk. *Statistical Analysis of Network Data: Methods and Models*. 1st. Springer Publishing Company, Incorporated, 2009. ISBN: 038788145X.

Random Graph Models

Structure of a simple random graph model:

Let G be a simple graph and f a known function. The adjacency matrix A of G has the known property:

$$A_{ij}|\xi \sim \mathit{Bernoulli}\,(f(\xi_i, \xi_j)) \quad (1)$$

where ξ_i, ξ_j are i.i.d uniform random variables.

Note: These are completely different to Machine Learning techniques.

Why? We view the world differently and thus use graphs and other network based structures for different purposes.

Stochastic Block Model (SBM) I

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Stochastic Block Model (SBM) is a model for generating random graphs.

Key features of the SBM:

- Clear group structure present between nodes
- Can be used as an approximation for any edge-independent graph[3,4]
- The group structure are latent (hidden)[5,6] i.i.d random variables with a given distribution function F on $\mathbb{R}^d$.

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[3] Patrick J Wolfe and Sofia C Olhede. “Nonparametric graphon estimation”. In: *arXiv preprint arXiv:1309.5936* (2013).

[4] Brian Karrer and Mark EJ Newman. “Stochastic blockmodels and community structure in networks”. In: *Physical review E* 83.1 (2011), p. 016107.

[5] P. W. Holland, K. B. Laskey, and S. Leinhardt. “Stochastic blockmodels: First steps”. In: *Social networks* 5.2 (1983), pp. 109–137.

[6] Ove Frank and Frank Harary. “Cluster inference by using transitivity indices in empirical graphs”. In: *Journal of the American Statistical Association* 77.380 (1982), pp. 835–840.

Dependencies in a run-off triangle

Several sources for dependence[7] in triangle 1:

1. Between development years
2. Between accident years
3. Between classes of business
4. Between calendar years
5. Also: external market and macro-economic effects.

[7] Greg Taylor. "Loss Reserving Models: Granular and Machine Learning Forms". In: *Risks* 7.3 (2019). ISSN: 2227-9091. DOI: 10.3390/risks7030082. URL: https://www.mdpi.com/2227-9091/7/3/82.

SBM in loss ratio prediction I

The key part of the network-reserving model is creating a framework to capture information in the data:

$$\mathbf{A}_C = \begin{array}{c} 1988 \\ 1989 \\ 1990 \\ 1991 \\ 1992 \end{array} \overset{\begin{array}{ccccc} 1988 & 1989 & 1990 & 1991 & 1992 \end{array}}{\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}}$$

Figure 2: A_C, adjacency matrices representing dependencies between calendar years

$$\mathbf{A}_D = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

Figure 3: A_D represents dependencies between development periods

To model all the effects graphically and do predictive analysis we take the Kronecker product matrix:

$$\mathbf{A} = \mathbf{A}_C \otimes \mathbf{A}_D$$

Figure 4: **A** as an adjacency matrix

SBM in loss ratio prediction III

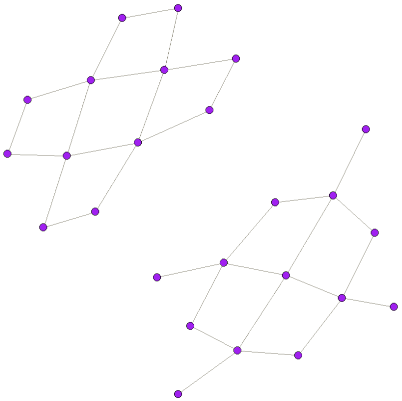

Figure 5: **A** as a graph

SBM Identification

Question: How do we model patterns in **A**?
Answer: Using the SBM or other community detection techniques.

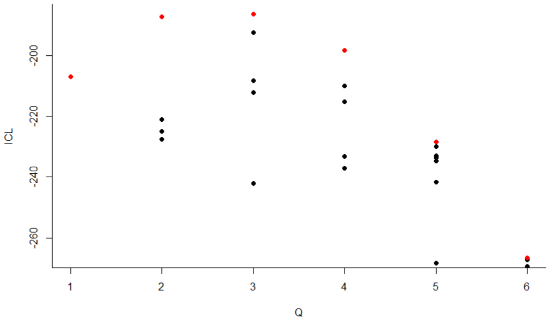

Figure 6: Optimal number of blocks detection algorithm

Results I

Predicting the next Kronecker product graph by sampling through the SBM:

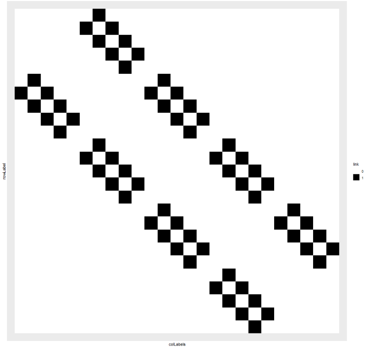

Figure 7: Data based reserve dependencies

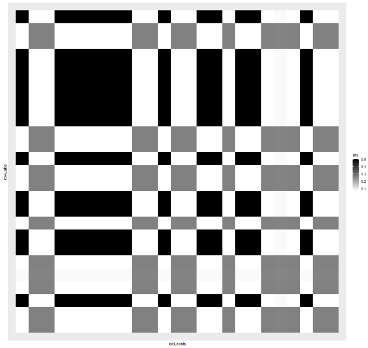

Figure 8: Predicted reserve dependencies

Results II

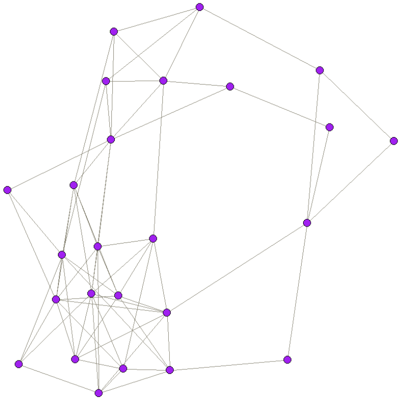

Figure 9: $\hat{\mathbf{A}}$ as a graph showing future predictions for all effects

Conclusion

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- We have discussed the reasons why a network based model may be a preferred approach to model dependencies within a run-off triangle framework and is therefore applicable to claims reserving.
- We have presented a solution, to create an adjacency matrix from a run-off triangle.
- We have demonstrated that network based models are:
  - Easy to interpret
  - Allow for clear overlays of expert judgement at several levels of the reserving process.

Thank you for listening to this presentation and I am happy to take questions.

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